

Biofilm mechanics and patterns

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BIRS Workshop on Coupled Mathematical Models for Physical and Biological Systems and Their Applications





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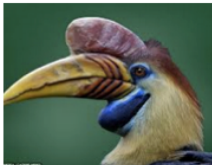
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European Union funding
for Research & Innovation



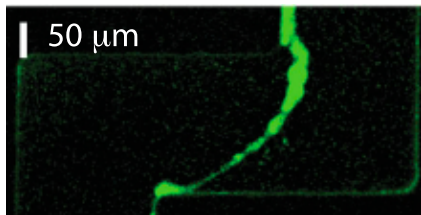


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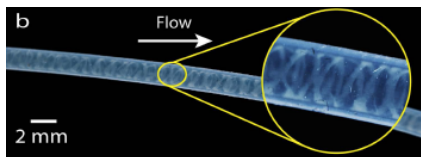


Biofilm shapes

Biofilms in flows

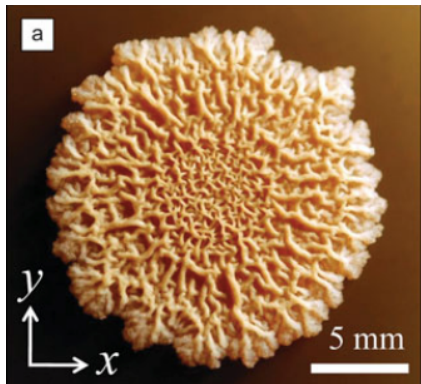


Rusconi, Lecuyer, Guglielmini, Stone, J. R. Soc. Interface 2010



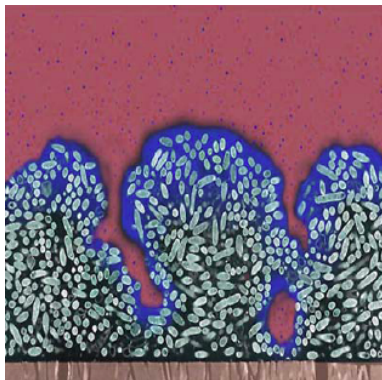
Espeso, Carpio, Martínez-García, de Lorenzo, Sci Rep 2016

Biofilms on agar/air interfaces



Wilking, Angelini, Seminara, Brenner, Weitz, MRS Bulletin 2011

What is a biofilm?



Flemming, Wingender, Nat Rev Microb 2010

Cells stick to each other and to a surface, embedded within a self produced matrix of extracellular polymeric substance (EPS).

Biofilms in flows: filamentary structures

streamer

**Nucleation: Secondary
corner vortices**

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graph TD; A[Nucleation: Secondary corner vortices] --> B[Elongation: Flow streamlines]; B --> C[Final shape: Equilibrium of elastic rod in a corner flow];
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**Elongation: Flow
streamlines**

**Final shape: Equilibrium of
elastic rod in a corner flow**

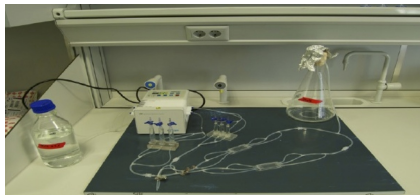
Drescher, Shen, Bassler, Stone, Proc Natl Acad Sci 2013

Laminar flow, $Re \ll 1$.

Time: seconds. Space: microns.

Bacteria: *Pseudomonas Aeruginosa*.

Biofilms in tubular circuits



Espeso, Carpio, Martínez-García, de Lorenzo, Sci Rep 2016

Laminar flow $Re \sim 1$.

Time: hours. Space: cm.

Bacteria: *Pseudomonas Putida*.

Nucleation: Vortices at junctions/stenosis



Elongation: Flow streamlines



Final shape: Helical equilibrium of an elastic rod

Helical biofilms

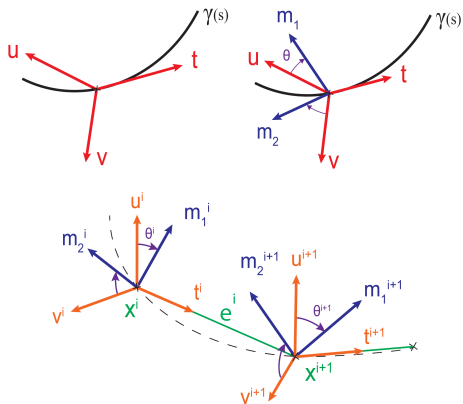


helix



Espeso, Carpio, Martínez-García, de Lorenzo, Sci Rep 2016, Chouaieb, Goriely, Maddocks, PNAS 2006

Discrete elastic rods



Filaments are represented by:

- a nodes x in a curve Γ (the centerline),
- a reference system $\{\mathbf{t}, \mathbf{m}_1, \mathbf{m}_2\}$ (the material frame).

The material frame at each node is constructed rotating the untwisted Bishop frame $\{\mathbf{t}, \mathbf{u}, \mathbf{v}\}$ an angle θ .

Discrete elastic rods

Energy:

$$E = \sum_{i=1}^n \beta \frac{(\theta^i - \theta^{i-1})^2}{\bar{\ell}^i} + \sum_{i=1}^n \frac{\alpha}{2\bar{\ell}^i} \sum_{j=i-1}^i \|\mathbf{w}_i^j - \bar{\mathbf{w}}_i^j\|^2,$$

α , β bending and torsion moduli, $\mathbf{w}(\theta, \mathbf{x})$ material curvatures, $\bar{\ell}$, $\bar{\mathbf{w}}$ reference lengths and curvatures.

Equation for the angles:

$$\frac{\partial E}{\partial \theta^i} = 0,$$

Equation for the positions:

$$\mathbf{M} \frac{d^2 \mathbf{x}}{dt^2} = - \frac{dE}{d\mathbf{x}} + \mathbf{f}$$

\mathbf{f} asymptotic value of fluid force on the filament

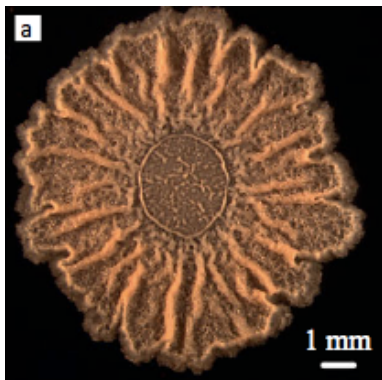
Bergou et al, ACM Trans Graphics 2008, Cox JFM 1970, Goldenthal et al, ACM Trans Graphics 2007, Espeso et al, Sci Rep 2016

Biofilms in corner flows

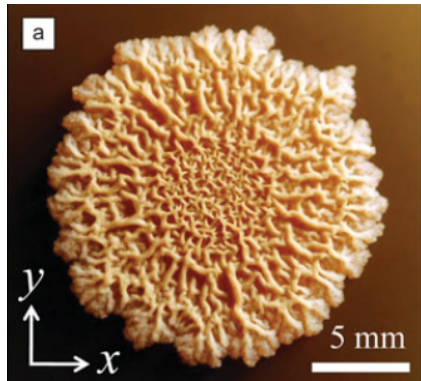
bending

growing

Biofilms on agar: wrinkled shapes



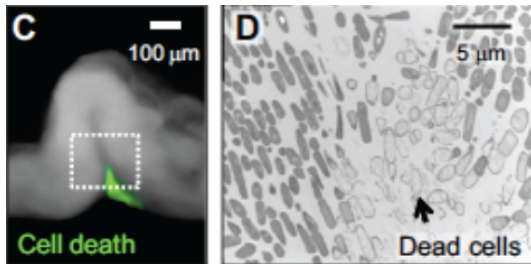
Chai et al, MRS Bulletin 2011



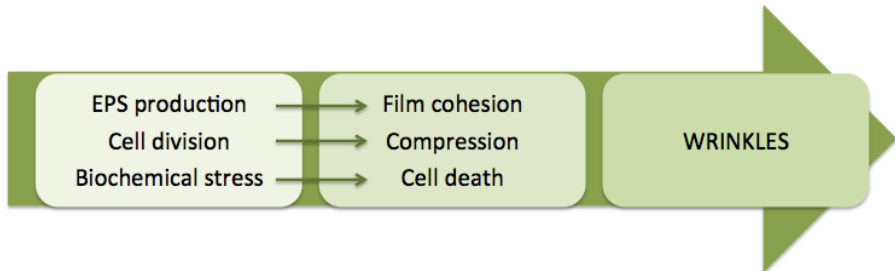
Wilking et al, MRS Bulletin 2011

Bacterial genus: *Bacillus Subtilis*

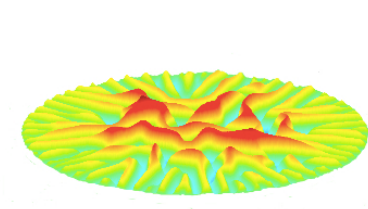
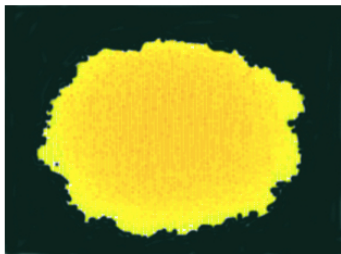
Cell death and wrinkle onset



Asally et al, Proc Natl Acad Sci 2013



Elastic film coupled to a viscoelastic substratum



$$\frac{\partial \xi}{\partial t} = \frac{1 - 2\nu_v}{2(1 - \nu_v)} \frac{h_v}{\eta_v} \left[D(-\Delta^2 \xi + \Delta C_M) + h \frac{\partial}{\partial x_\beta} \left(\sigma_{\alpha, \beta}(\mathbf{u}) \frac{\partial \xi}{\partial x_\alpha} \right) \right] - \frac{\mu_v}{\eta_v} \xi = 0,$$

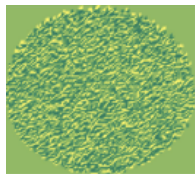
$$\frac{\partial \mathbf{u}}{\partial t} = \frac{h_v h}{\eta_v} \nabla \cdot \sigma(\mathbf{u}) - \frac{\mu_v}{\eta_v} \mathbf{u} = 0,$$

$$D = \frac{Eh^3}{12(1-\nu^2)}, \quad \varepsilon_{\alpha, \beta} = \frac{1}{2} \left(\frac{\partial u_\alpha}{\partial x_\beta} + \frac{\partial u_\beta}{\partial x_\alpha} + \frac{\partial \xi}{\partial x_\alpha} \frac{\partial \xi}{\partial x_\beta} \right) + \boxed{\varepsilon_{\alpha, \beta}^0}$$

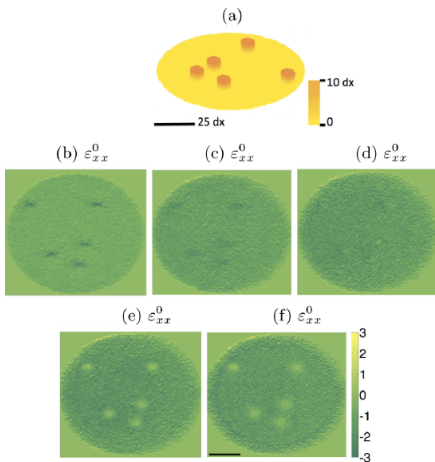
$$\sigma_{xx} = \frac{E}{1 - \nu^2} (\varepsilon_{xx} + \nu \varepsilon_{yy}), \quad \sigma_{yy} = \frac{E}{1 - \nu^2} (\varepsilon_{yy} + \nu \varepsilon_{xx}), \quad \sigma_{xy} = \frac{E}{1 + \nu} \varepsilon_{xy}$$

Residual stresses and averaged elastic constants

Residual stresses
from a growth tensor tracking
cell division and
death

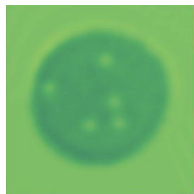


Single trial



Ensemble average

Smoothed using
image processing
techniques

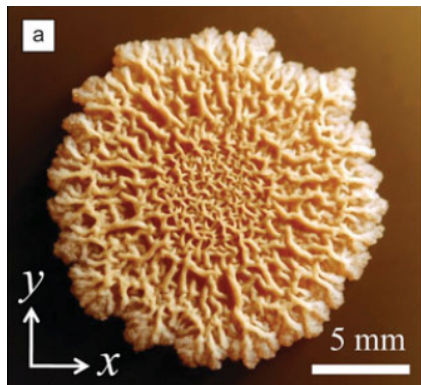


Filtered average

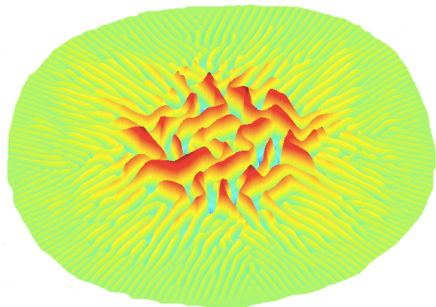
Wrinkle branching

wrinkle

Successive branching



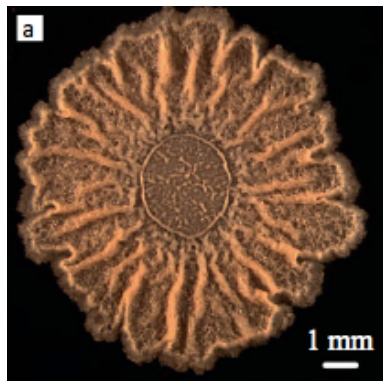
Wilking, Angelini, Seminara, Brenner, Weitz, MRS Bulletin 2011



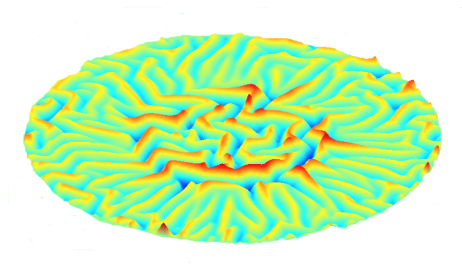
Espeso, Carpio, Einarsson, Phys Rev E 2015

Residual stresses: compression front expanding outwards fast enough.

Wrinkled coronas



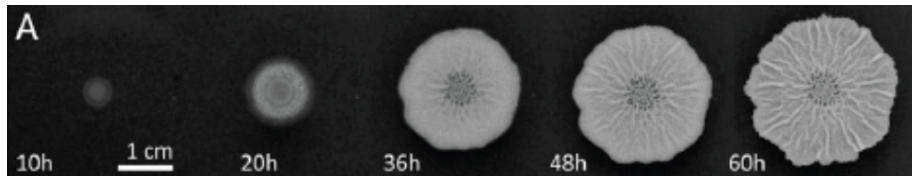
Chai, Vlamakis, Kolter, MRS Bulletin 2011



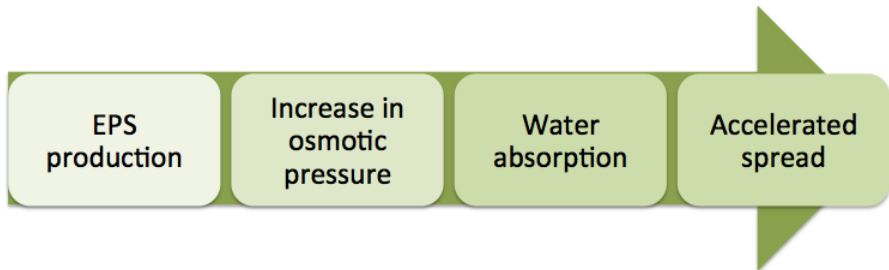
Espeso, Carpio, Einarsson, Phys Rev E 2015

'Corona instability': Swollen outer corona with diminished Young modulus.

Water absorption



Seminara et al, Proc Natl Acad Sci 2012



Conclusions and future work

