Token Sliding on chordal graphs

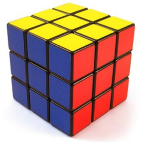
Nicolas Bousquet
joint work with Marthe Bonamy (LaBRI, Bordeaux, France)

Banff - Reconfiguration Workshop

Reconfiguration of Independent Sets

Introduced by Hearn and Demaine in 2005 in a general study of one-player games :

> *A one-player game is a puzzle : one player makes a series of moves, trying to accomplish some goal.*



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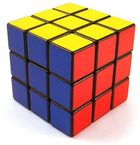

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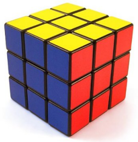

Question :
Giving my current position, can I reach my target position ?

- Equivalence with reconfiguration of satisfiability constraints.
- Generalize the Warehouseman's problem (motion of robots).
- Introduced for colorings, satisfiability problems, dominating sets, cliques, list colorings, bases of matroids...

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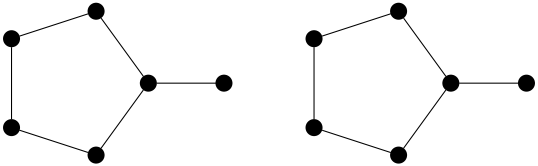

Definition (TS-sequence)

A TS-sequence $I_1, \ldots, I_\ell$ of independent sets is a sequence such that there exist $v \in I_{j+1}$ and $u \in I_j$ such that $I_{j+1} = I_j \cup \{v\} \setminus \{u\}$ and uv is an edge.

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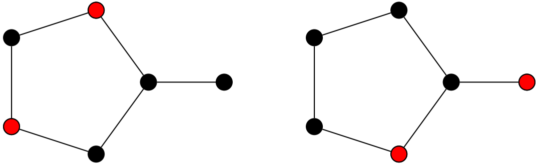

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Equivalent formulation :

We slide tokens along the edges in such a way the set remains an independent set at any step.

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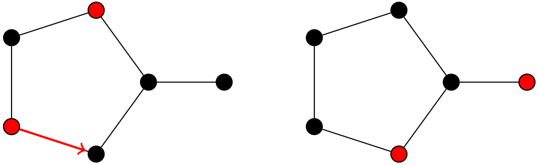

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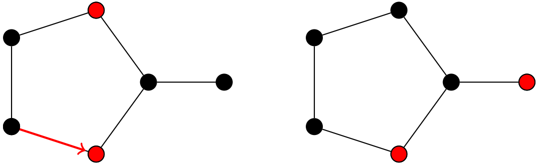

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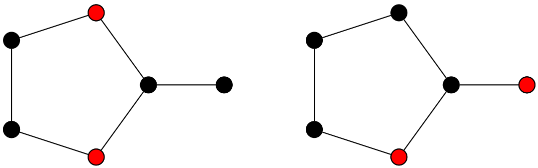

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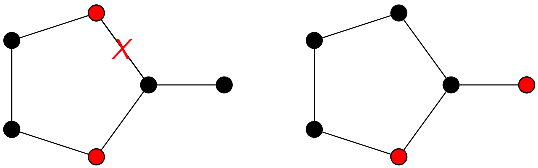

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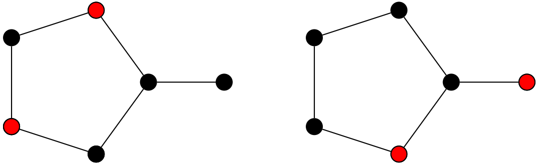

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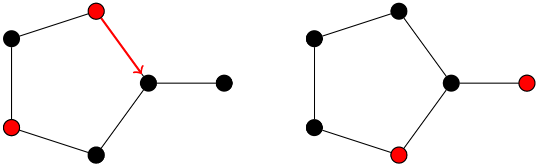

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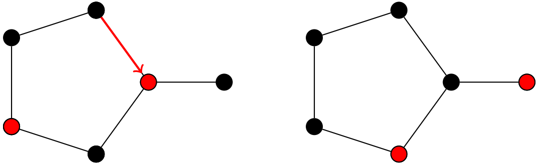

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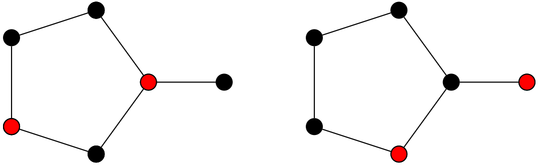

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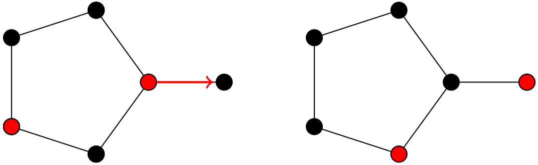

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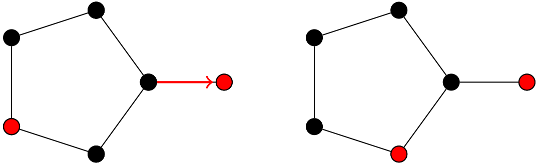

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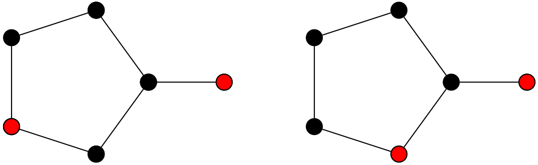

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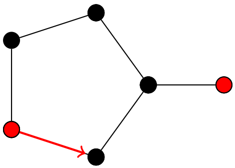

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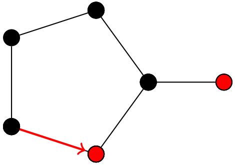
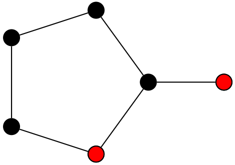

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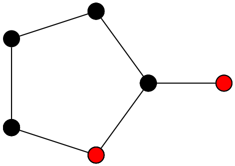

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Main questions

- **Reachability problem.** Given two configurations, is it possible to transform one into the other ?
- **Connectivity problem.** Given any pair of configurations, is it possible to transform one into the other ?
- **Minimization.** Given two configurations, what is the length of a shortest sequence ?

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- **Minimization.** Given two configurations, what is the length of a shortest sequence ?
- **Algorithmics.** Can we efficiently solve these questions ? (In polynomial time, FPT-time...).

Formal definition of the problems

TS-Reachability

Input : A graph G, $k \in \mathbb{N}$, two independent sets I, J of size k.
Output : YES iff there exists a TS-sequence from I to J.

TS-Connectivity

Input : A graph G, an integer k.
Output : YES iff it is possible to transform any independent set of size k into any other via a TS-sequence.

Theorem (Hearn, Demaine '05)

TS-Reachability is PSPACE-complete on planar graphs.

Polynomial time algorithms for :

- Demaine et al. Trees.
- Kamiński, Medvedev, Milanič. Cographs.
- Bonsma, Kamiński, Wrochna. Claw-free graphs.
- Fox-Epstein et al. Bipartite permutation graphs.

Our results

Question (Demaine et al.)

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- YES on interval graphs.
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- Maybe No on split graphs.
  Deciding TS-Connectivity is co-NP hard and co-W[2]-hard.
  (split graph $= V = V_1 \cup V_2$ where V_1 induces a clique and V_2 a stable set)

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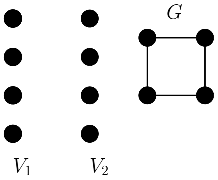
Remark :

With a similar construction $\Rightarrow$ TS-connectivity is co-NP hard and co-W[2]-hard on bipartite graphs.

Hardness result on split graphs

Let G be a graph. Create a graph H :

- Create two copies V_1, V_2 of $V(G)$.



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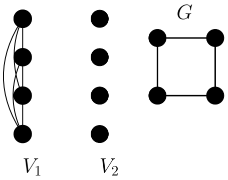

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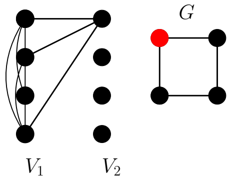

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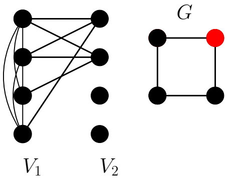

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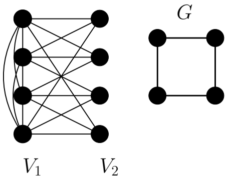

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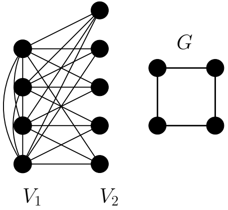

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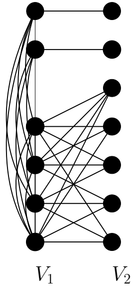

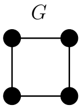

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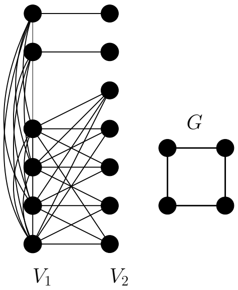

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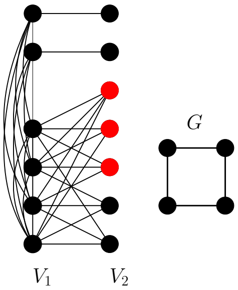

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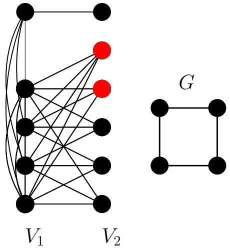

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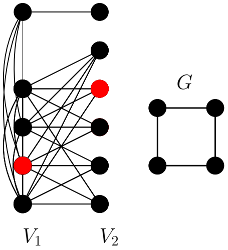

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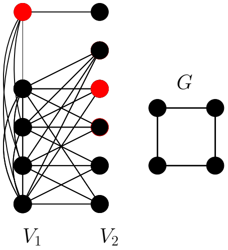

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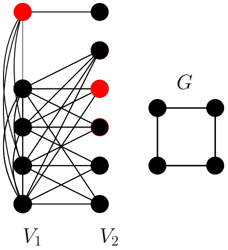

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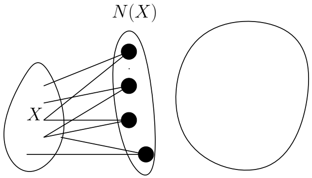
$\Leftarrow$ Move one by one vertices to the top. **Not Always Possible !**

Fix the problem



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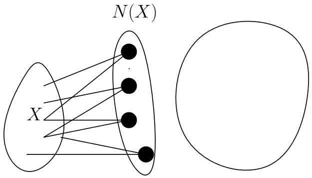

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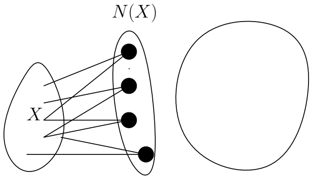

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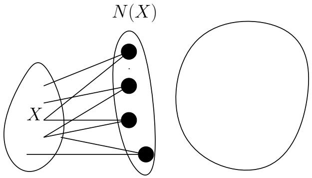

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Lemma

We can transform any independent set of H' of size $k+1$ into any other iff there is no dominating set of size k in G'.

Conclusion

k-Dominating Set is NP-hard and W[2]-hard.

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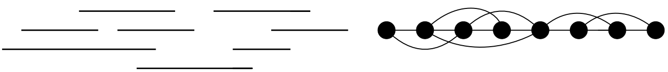
k-TS-Connectivity is co-NP-hard and co-W[2]-hard.

Interval graphs



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A geometric representation can be obtained in polynomial time.

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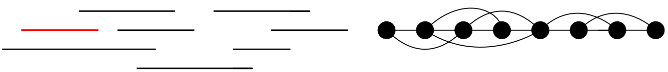

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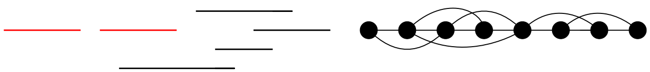

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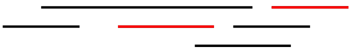
Informal goal

Decide if an Independent Set of size k can be transformed into the LIS.

First try : Naive Method



Lemma

I can be transformed into the LIS iff

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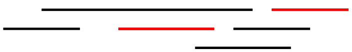

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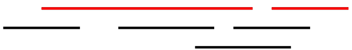

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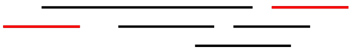

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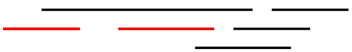

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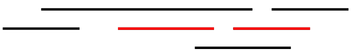

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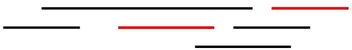

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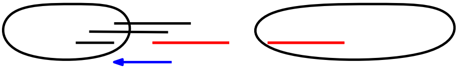

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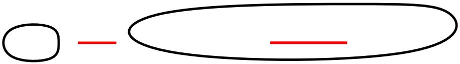

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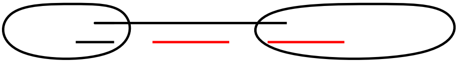

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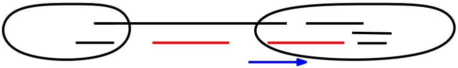

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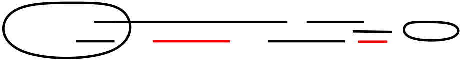

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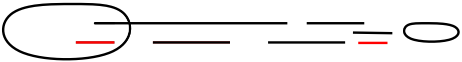

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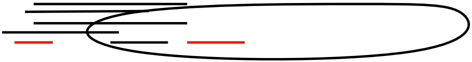

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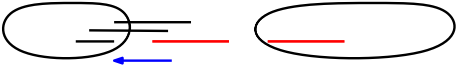
Algorithm for TS-Reachability



We repeat the following procedure on each independent set I and J :

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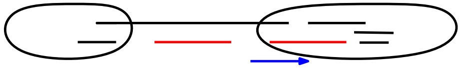

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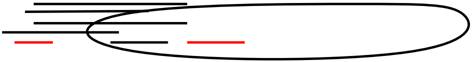

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- If no vertices of the independent sets have moved, compare the first vertices of I' and J' :
  - If they are different : answer NO.
  - If they are the same : delete their first vertices and their neighborhoods and repeat.

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Questions

- Given two independent sets, does there exist a polynomial P such that a minimum transformation between I and J, if it exists, has length at most $P(n)$?
- If yes, is the sequence of this algorithm polynomial ?

Dynamic programming

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- without vertices strictly before u,
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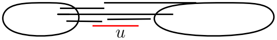

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Complexity : $\mathcal{O}(n \cdot m)$.

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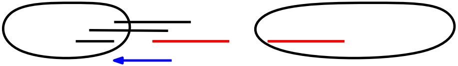

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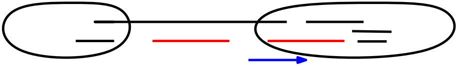

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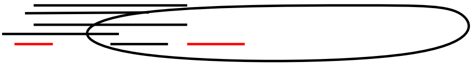

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Computation ?

Using a slightly more complicated dynamic programming algorithm.

Conclusion and open problems

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Thanks for your attention !