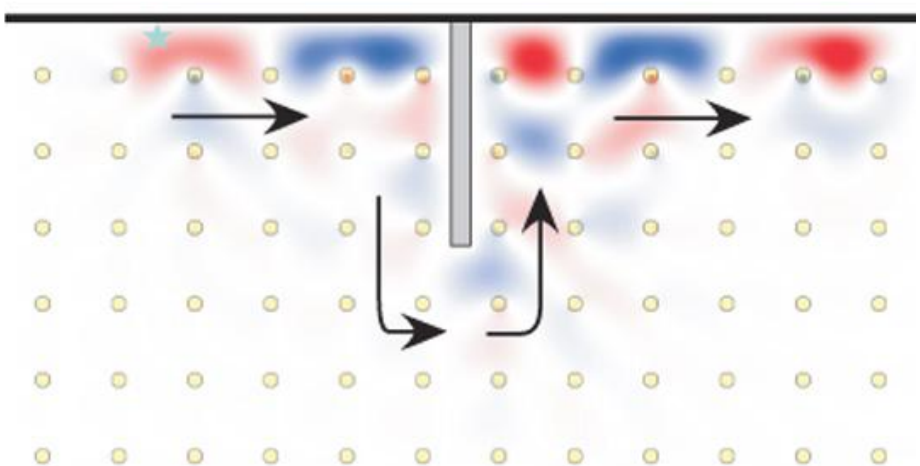
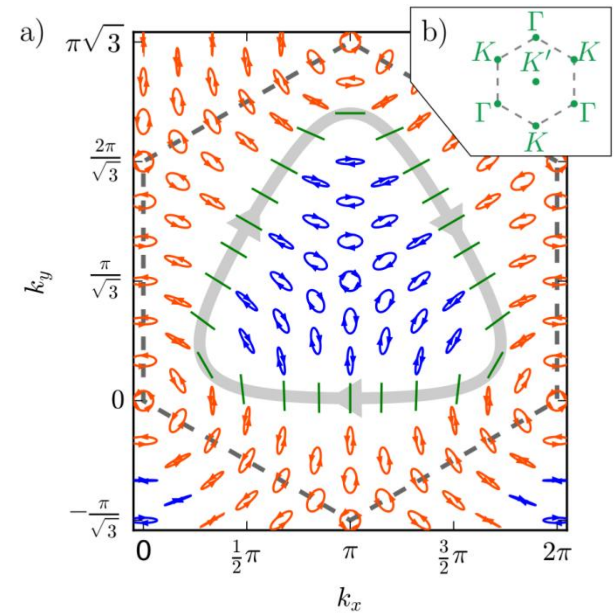
# Topological phases and topological photonics

Daniel Leykam



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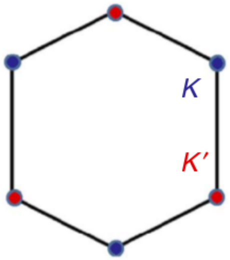
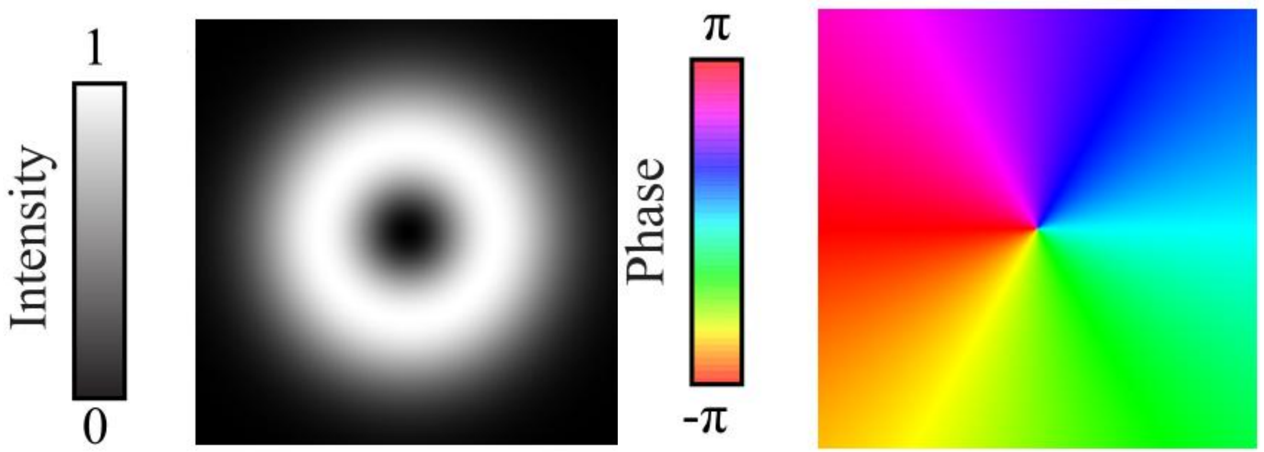
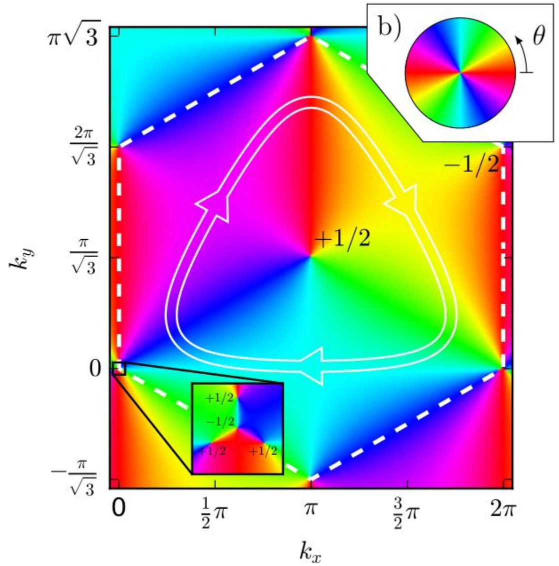
# Motivation

- **Structured light**: optical fields shaped in *real* space
- **Topological phases**: Bloch wave eigenmodes shaped in *momentum* space
- Common language: topological charges & invariants
- What can each tell us about the other?



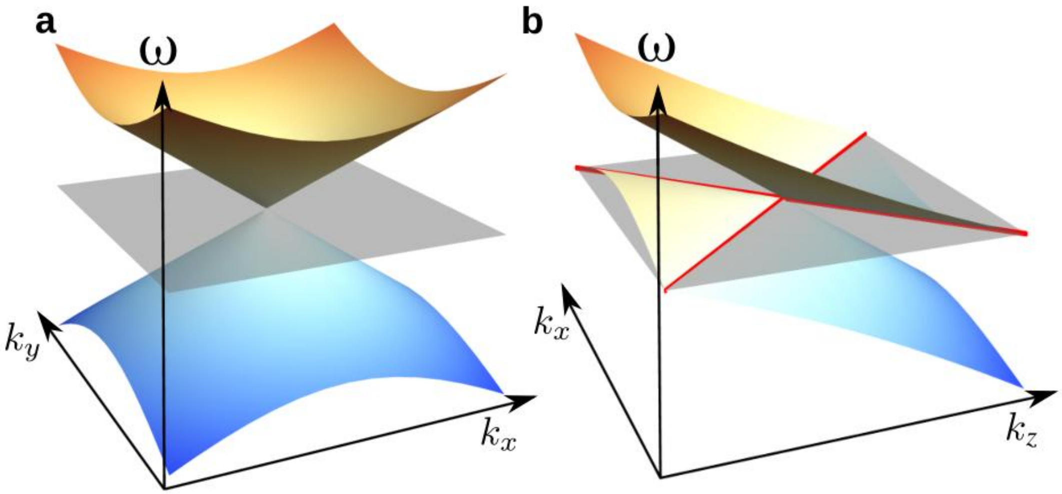




T. Foesel, V. Peano, F. Marquardt, arXiv:1703.08191

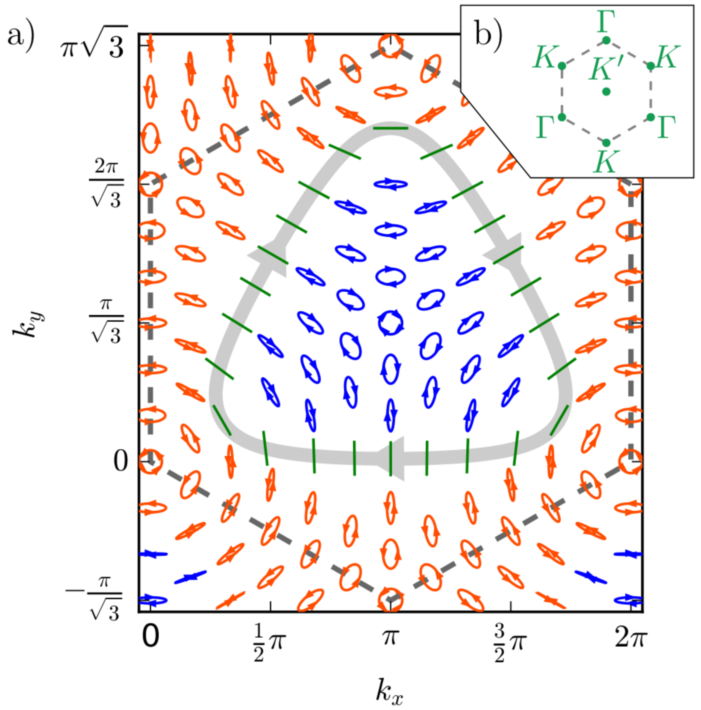
# Outline

1. Bloch band vortices: gapless topological phases

2. Pseudospins and structured light




Noh et al, Nature Physics 4072 (2017)



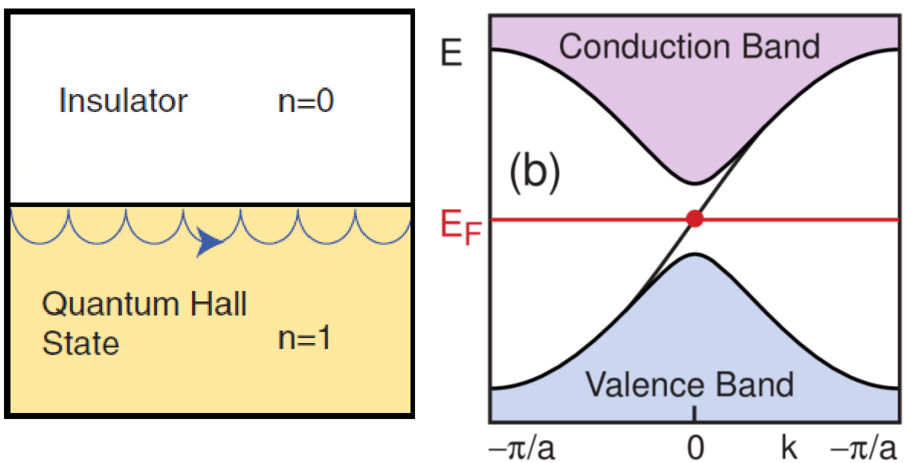

T. Foesel, V. Peano, F. Marquardt, arXiv:1703.08191

# 1. Gapless topological phases

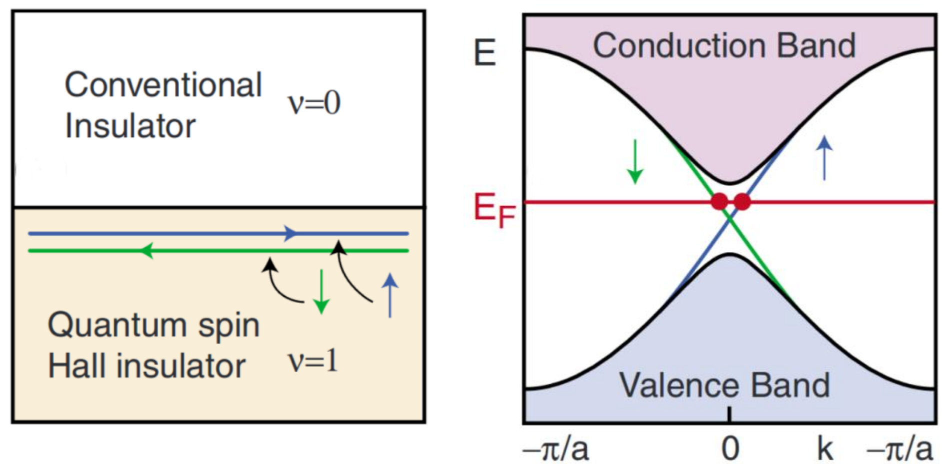
# Topological insulators

- Characterized by topological invariants: Z or $Z_2$
- Bulk-edge correspondence: edge modes whenever bulk invariants change
- Topological protection: invariants cannot change if band gap remains open
- Defined for non-interacting systems

Z: integer charge




$Z_2$: parity (even/odd)




Hasan & Kane, Rev. Mod. Phys. 82, 3045 (2010)

# Fermionic vs bosonic topological insulators

- Fermions: Spin 1/2, $T^2 = -1$, $Z_2$ topological insulators (spin-momentum locking)
- Bosons: Spin 1, $T^2 = +1$, no $Z_2$ topological *insulating* phases!

| | | TRS | PHS | SLS | $d=1$ | $d=2$ | $d=3$ |
|---|---|---|---|---|---|---|---|
| Standard | A (unitary) | 0 | 0 | 0 | - | $\mathbb{Z}$ | - |
| (Wigner-Dyson) | AI (orthogonal) | +1 | 0 | 0 | - | - | - |
| | AII (symplectic) | −1 | 0 | 0 | - | $\mathbb{Z}_2$ | $\mathbb{Z}_2$ |
| Chiral | AIII (chiral unitary) | 0 | 0 | 1 | $\mathbb{Z}$ | - | $\mathbb{Z}$ |
| (sublattice) | BDI (chiral orthogonal) | +1 | +1 | 1 | $\mathbb{Z}$ | - | - |
| | CII (chiral symplectic) | −1 | −1 | 1 | $\mathbb{Z}$ | - | $\mathbb{Z}_2$ |
| BdG | D | 0 | +1 | 0 | $\mathbb{Z}_2$ | $\mathbb{Z}$ | - |
| | C | 0 | −1 | 0 | - | $\mathbb{Z}$ | - |
| | DIII | −1 | +1 | 1 | $\mathbb{Z}_2$ | $\mathbb{Z}_2$ | $\mathbb{Z}$ |
| | CI | +1 | −1 | 1 | - | - | $\mathbb{Z}$ |

Kitaev, arXiv:0901.2686; Hasan & Kane, Rev. Mod. Phys. 82, 3045 (2010)

# Photons are massless

- Low energy electrons: vacuum is topologically trivial insulator, with $m_e >> E$
- Photons: vacuum is gapless; not an “insulator” for light
- Topological invariants are ill-defined or marginal
- How is the bulk-edge correspondence modified for photons?
- Similarly, for relativistic electrons?

Low energy electron

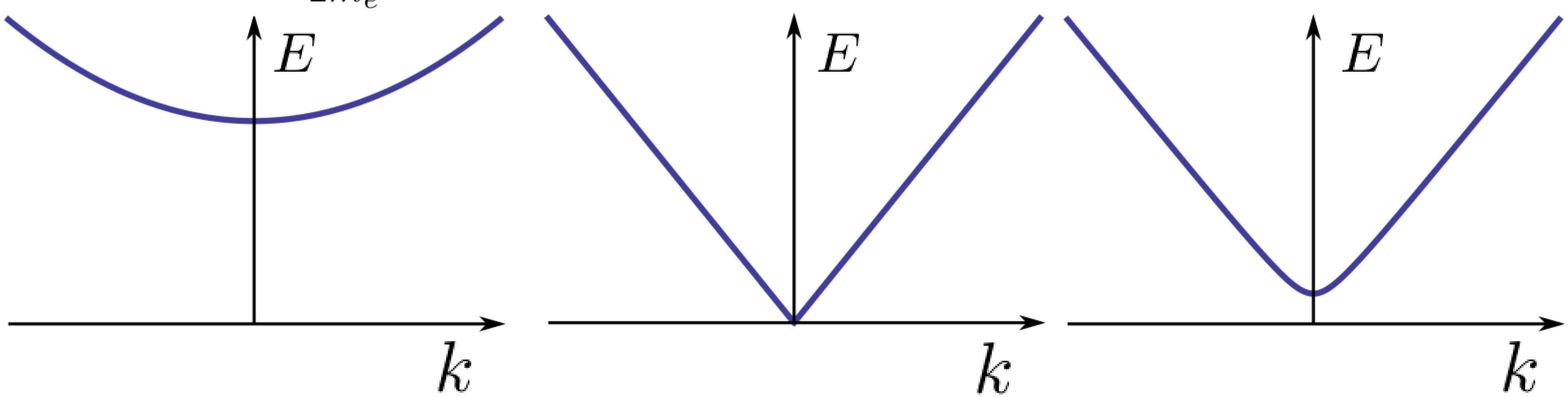
$$E = m_e + \frac{\hbar^2 k^2}{2m_e}$$

Photon

$$E = \hbar k$$

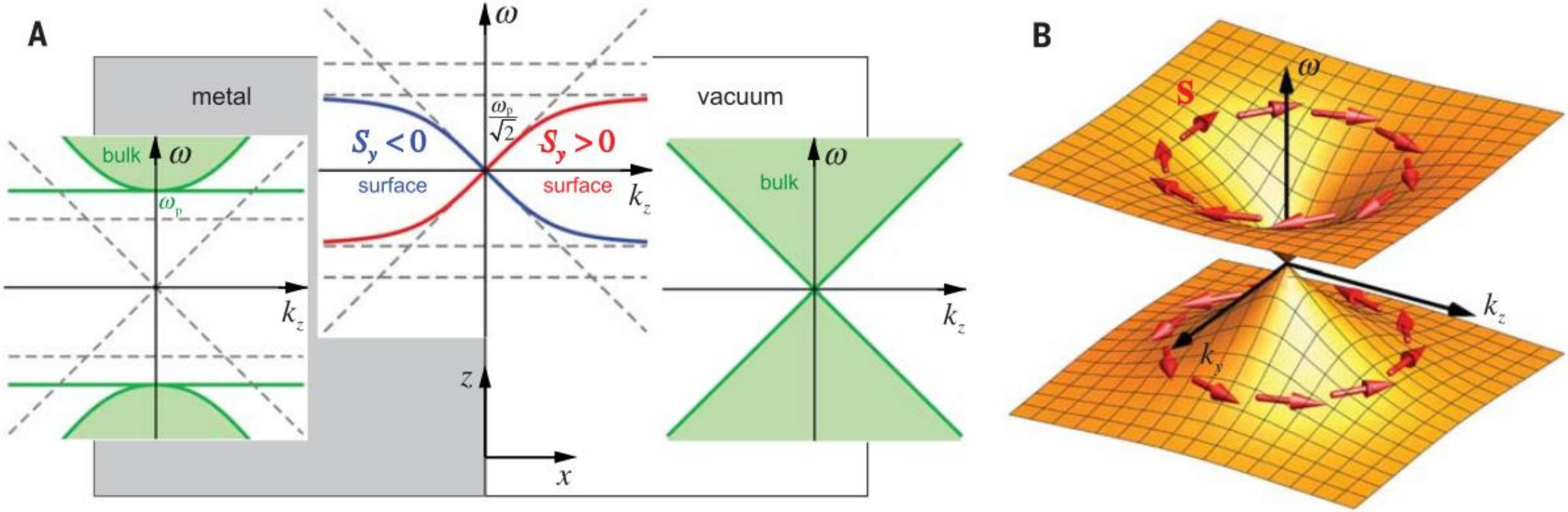
Relativistic electron

$$E = \sqrt{m_e^2 + \hbar^2 k^2}$$



# Quantum spin Hall effect of light

- Free space light: degeneracy in 3D, but integer spin
- T-symmetric & trivial $Z_2$ topological invariant
- Spin-momentum locking of evanescent waves
- E.g. free space (gapless) – metal (gapped) interfaces
- Gapless systems can host *new classes* of topological edge states

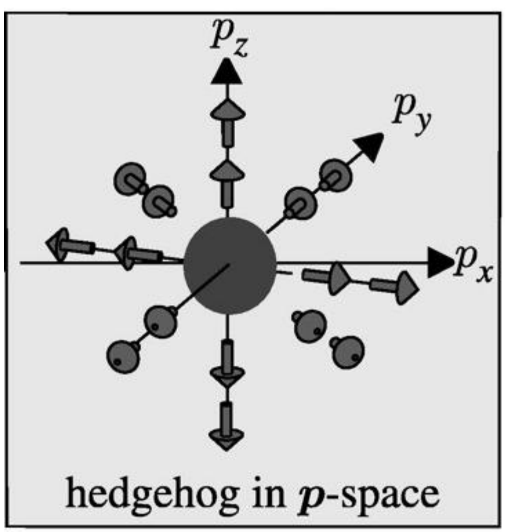



K. Y. Bliokh, D. Smirnova, F. Nori, Science 348, 1448 (2015)

# Degeneracies as topological defects

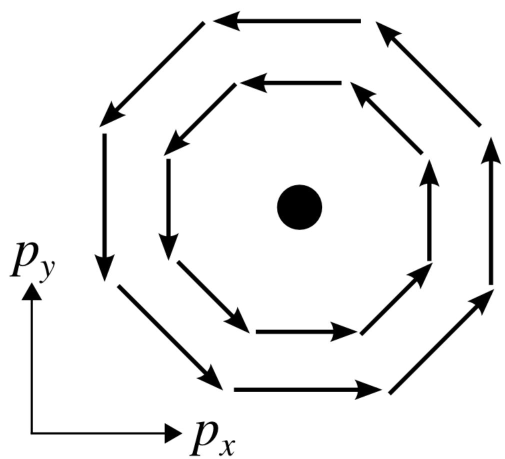
- Simplest example: two band Hermitian systems
- Two level Bloch Hamiltonian: $\hat{H}(\boldsymbol{p}) = \boldsymbol{d}(\boldsymbol{p}) \cdot \hat{\boldsymbol{\sigma}} = \begin{pmatrix} d_z & d_x - i d_y \\ d_x + i d_y & -d_z \end{pmatrix}$
- Gap closing points (degeneracies) have co-dimension 3: $d_x(\boldsymbol{p}) = d_y(\boldsymbol{p}) = d_z(\boldsymbol{p}) = 0$
- 3D momentum space: **Weyl points** (hedgehogs)
- 2D + symmetry $d_z = 0$ : **Dirac points** (vortices)
- 3D + symmetry $d_z = 0$ : **Nodal lines** (vortex lines)

Weyl point



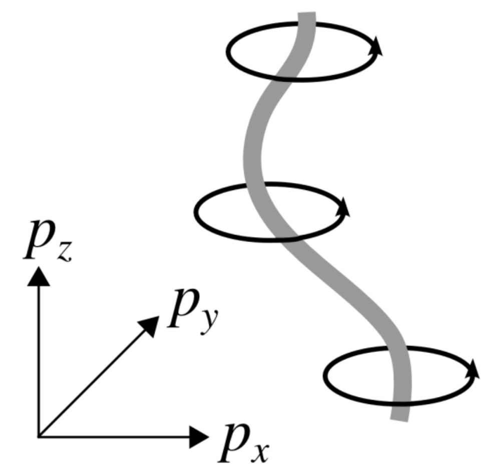

Volovik, Proc. R. Soc. A (2008)

Dirac point




vortex in *p*-space

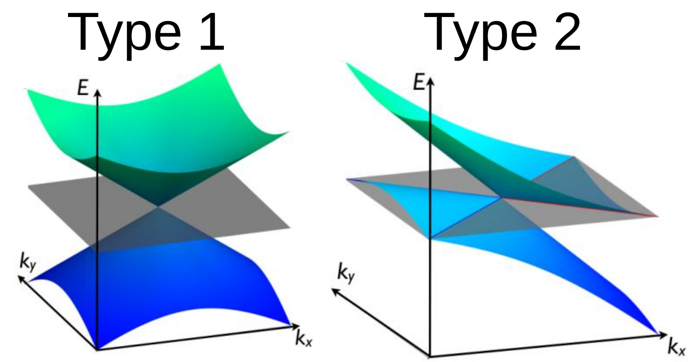
Nodal line



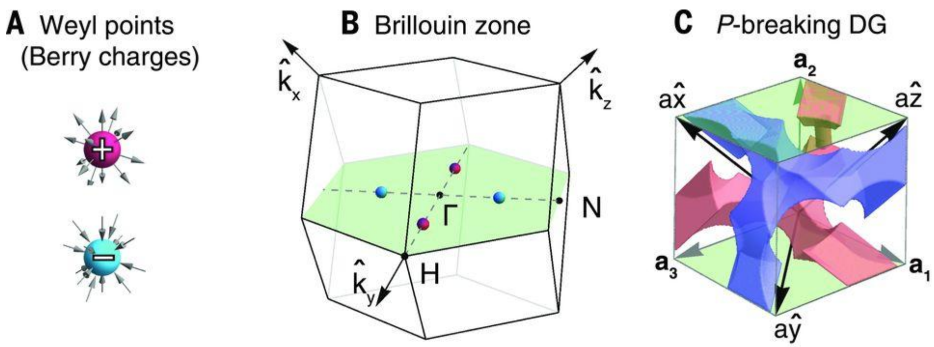

vortex tube in *p*-space

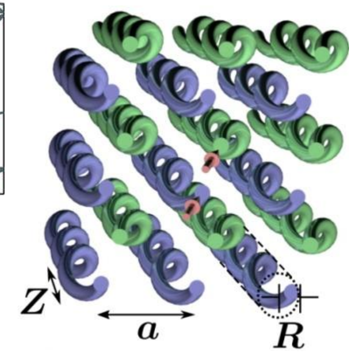
# Photonic Weyl point degeneracies

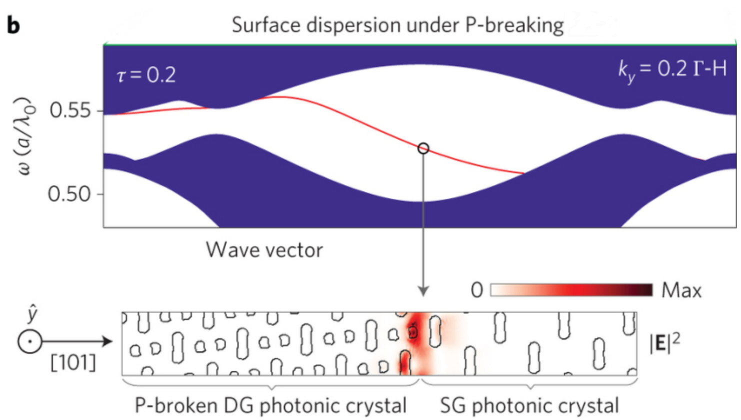
- •Fermi arcs of edge modes link pairs of Weyl points
- •Gyroid photonic crystal: isotropic type 1 Weyl points
- •Helical photonic lattice: tilted type 2 Weyl points

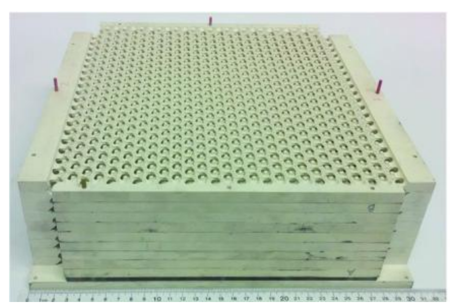


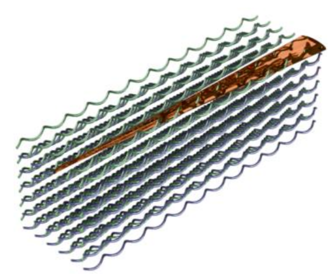

Xu et al., Phys. Rev. Lett. (2015)
Soluyanov et al., Nature (2015)

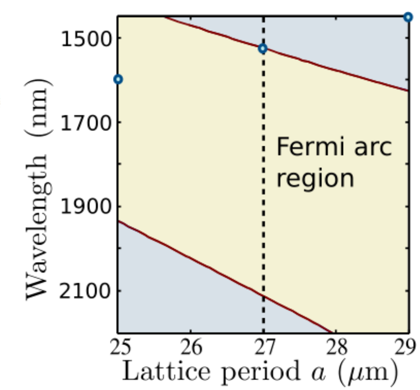




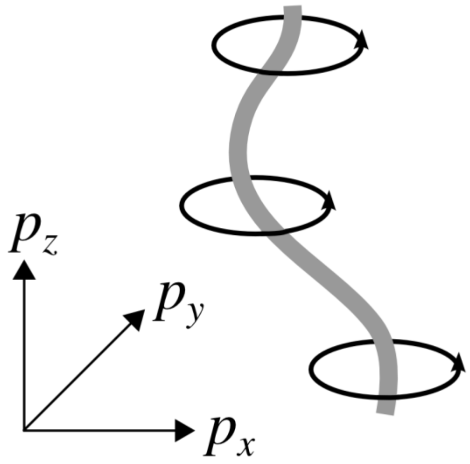



Lu et al., Nature Photon. (2013); Lu et al., Science (2015)

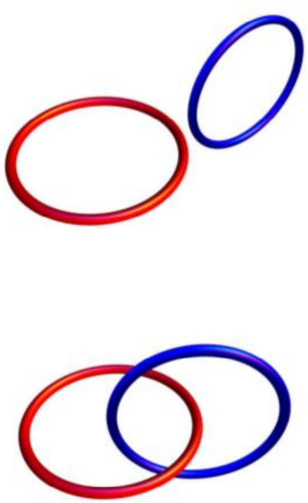
Leykam et al, Phys. Rev. Lett. (2016)
Noh et al, Nature Physics (2017)

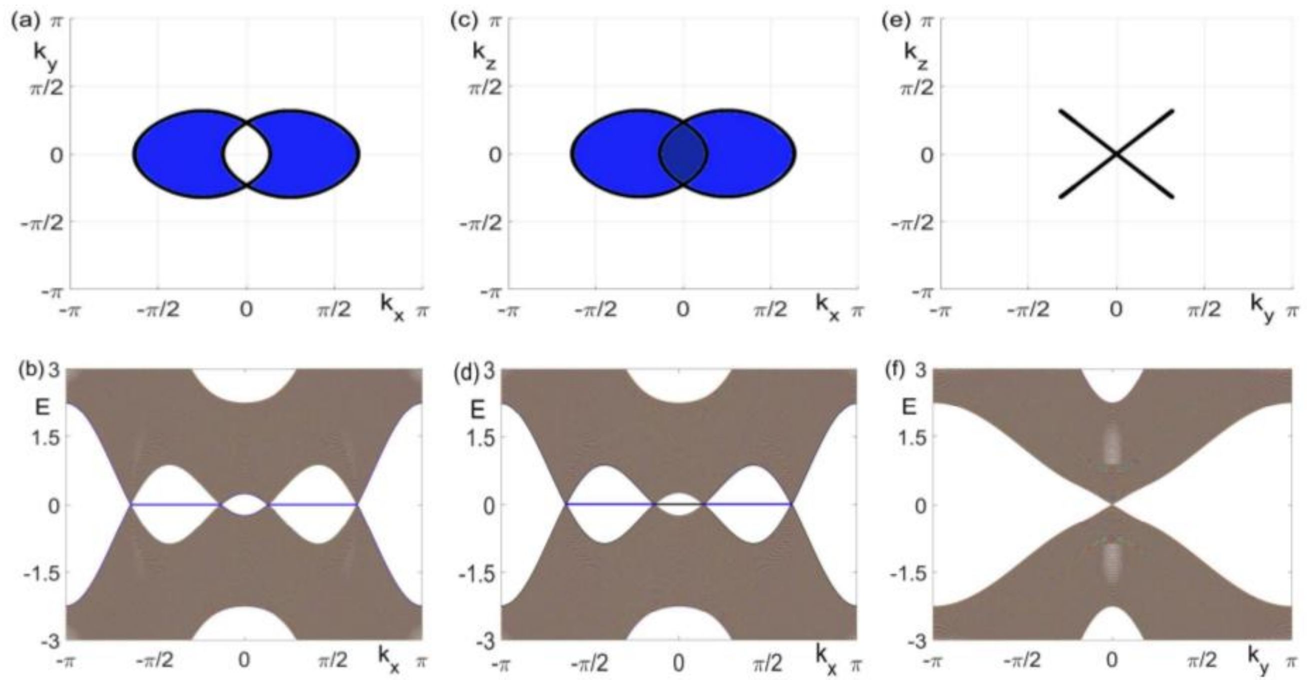
# Hopf link degeneracies

Line node




- Line nodes can form linked rings with toroidal Berry phase
- Analogous to isolated optical vortex knots & links
- Zero energy edge modes & shifted Landau levels
- Optical lattice realizations challenging
- Alternative: drive a system with structured light?

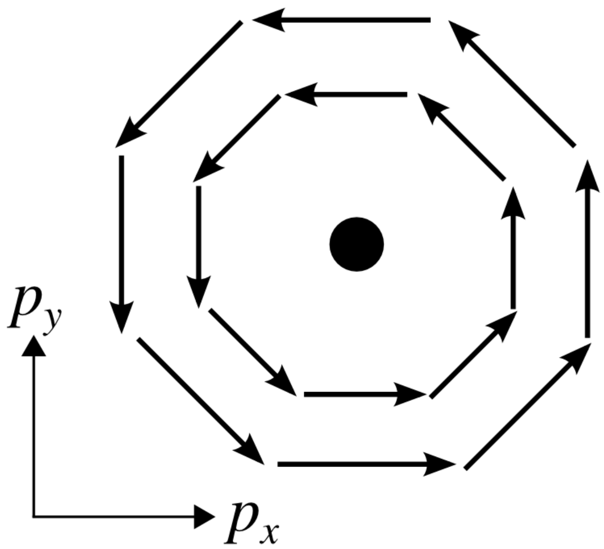


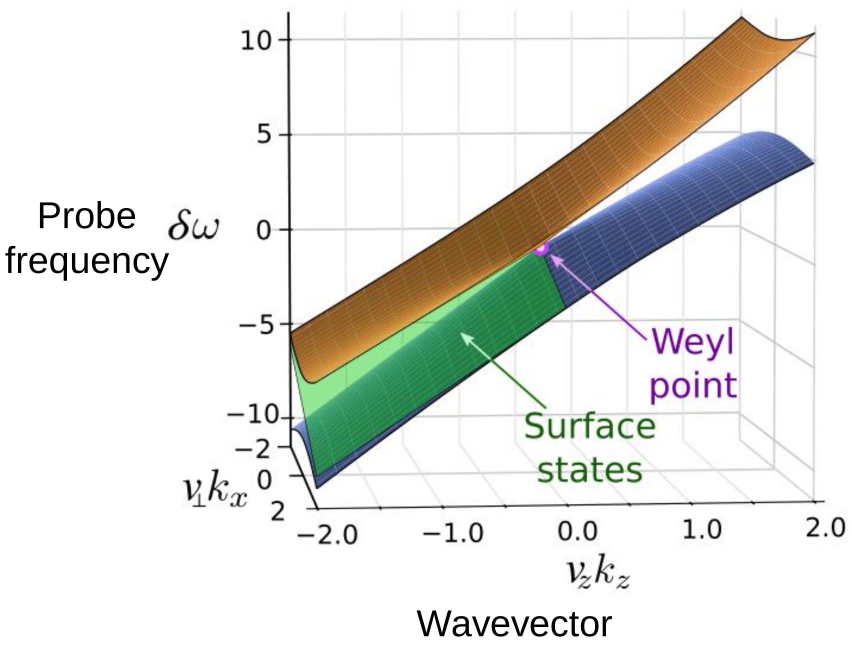



“Topological Hopf-link semimetal,” arXiv:1703.10886
“Nodal-link semimetals,” arXiv:1704.00655
“Weyl-link semimetals,” arXiv:1704.01948

# Summary (part 1)

- Gapless systems also host topological phases
- E.g. massless photons, Weyl point photonic crystals
- Point degeneracies analogous to 2D vortices & 3D topological defects
- Line nodes can be linked or knotted



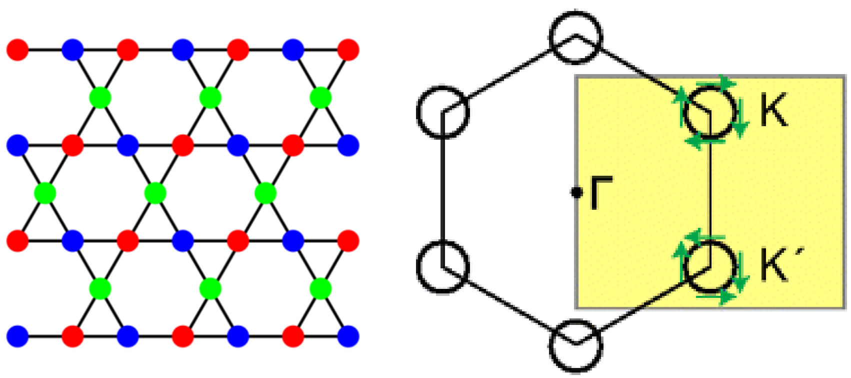



# 2. Pseudospins and structured light

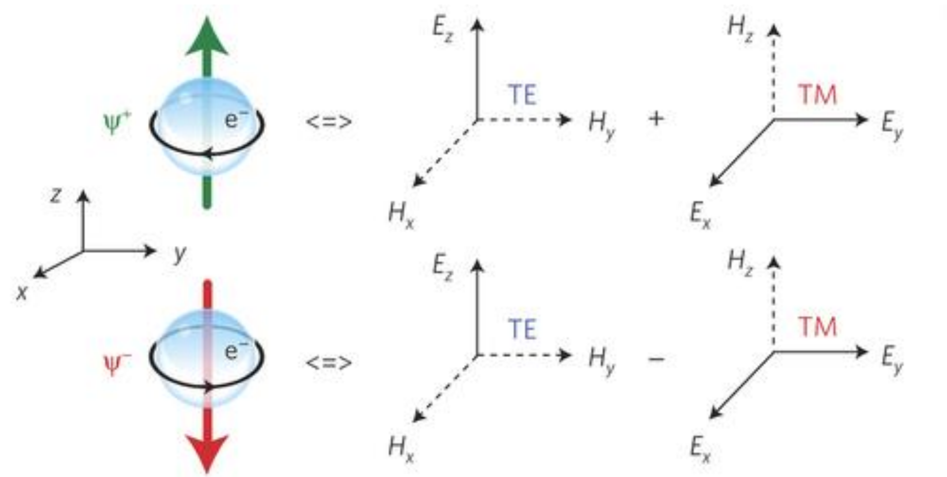
# Pseudospins

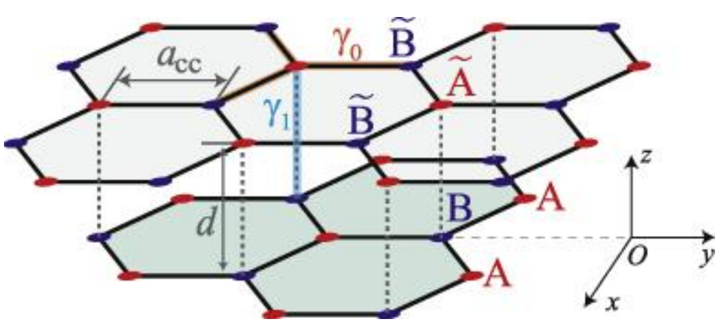
$$\hat{H}(\boldsymbol{p}) = \boldsymbol{d}(\boldsymbol{p}) \cdot \hat{\boldsymbol{\sigma}} \equiv \mathbf{B}_{\text{eff}} \cdot \hat{\boldsymbol{\sigma}}$$

- Any internal / microscopic states of system with spin-like behaviour
- Eg. sublattices, orbitals, layers, valleys, polarisations, helicities
- Analogue of “real” spin, carries angular momentum? Mecklenburg & Regan, PRL **106**, 116803 (2011)
- Symmetry-protected pseudospin-momentum locked edge modes

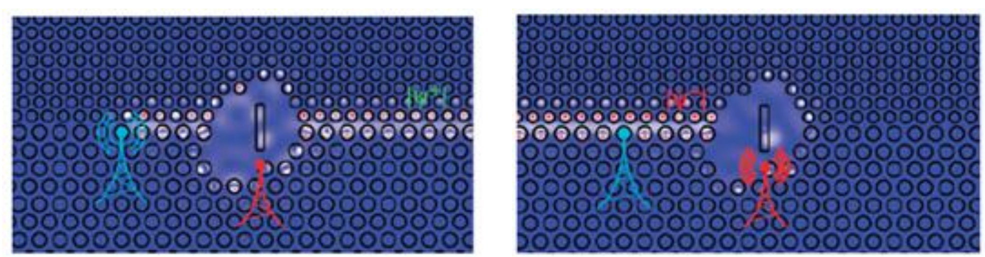



Park et al, PNAS 108, 18622 (2011)




Y.-H. Hyun et al , J. Phys.: Cond. Mat. **24**, 045501 (2012)



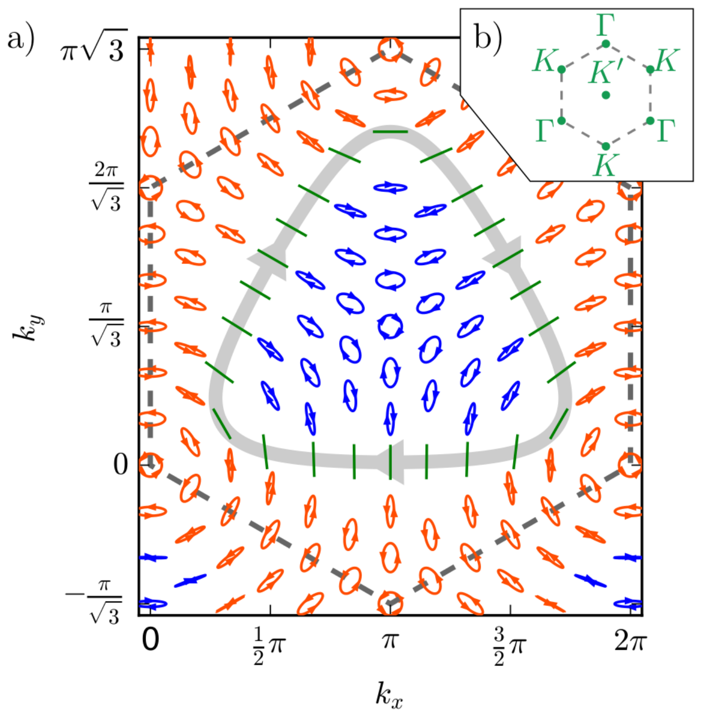
Khanikaev et al, Nature Mater. 12, 233 (2013)

# Polarization textures of Bloch functions

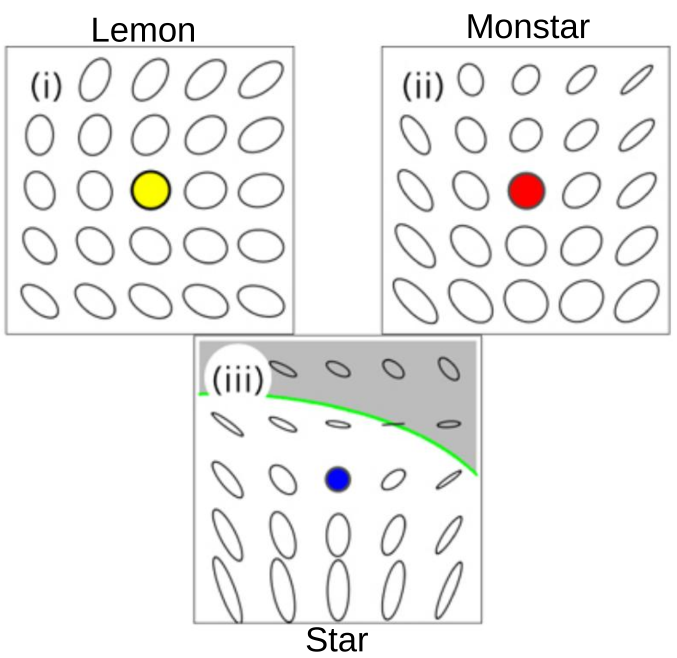
- Can characterize pseudospin of eigenmodes with Stokes parameters
- Positions of C points, L lines is basis-dependent
- Chern number: basis-independent winding number of L lines or sum of C points
- Significance of different C point morphologies?

$$\vec{\psi}^{(n)}(\vec{k}) = \sqrt{S_0}\mathrm{e}^{\mathrm{i}\varphi} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \cos\chi \\ \mathrm{i}\sin\chi \end{pmatrix}$$

$$S_0 = |\psi_1|^2 + |\psi_2|^2$$
$$S_1 = |\psi_1|^2 - |\psi_2|^2 = S_0\cos(2\chi)\cos(2\theta)$$
$$S_2 = 2\mathrm{Re}(\psi_1^*\psi_2) = S_0\cos(2\chi)\sin(2\theta)$$
$$S_3 = 2\mathrm{Im}(\psi_1^*\psi_2) = S_0\sin(2\chi)$$



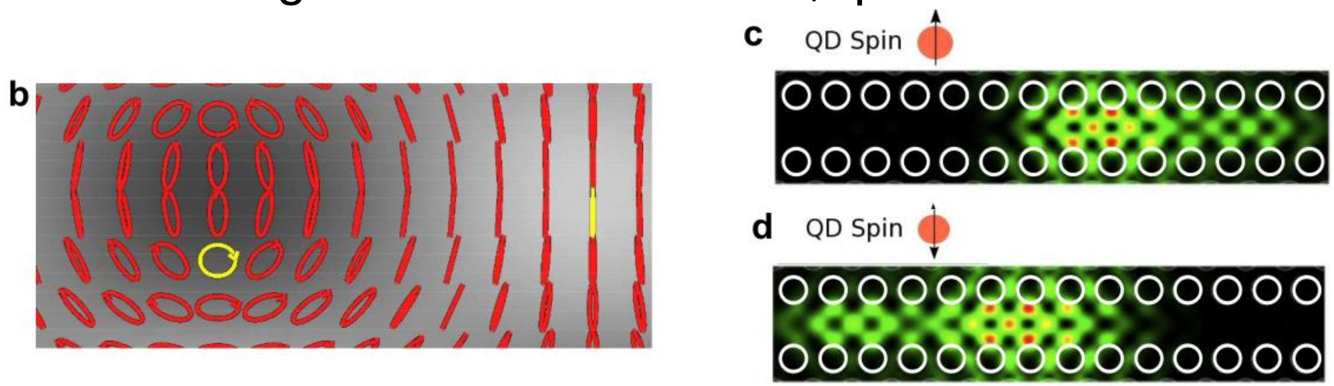

T. Foesel, V. Peano, F. Marquardt, arXiv:1703.08191



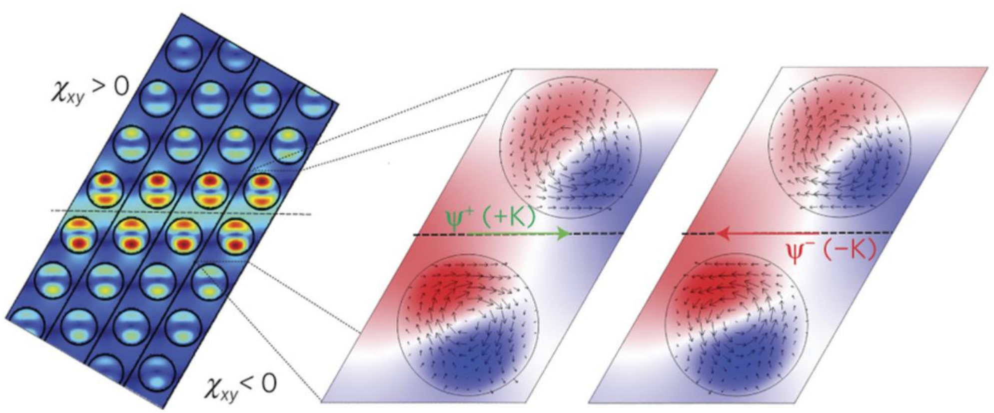

Dennis, O'Holleran, & Padgett, Prog. Opt. (2009)

# Not all pseudospins are equal

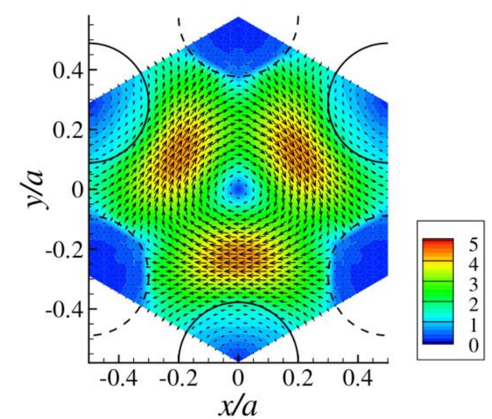
- Linear topological phases & edge modes “blind” to form of pseudospin
- Optical forces, angular momentum sensitive to microscopic details!
- Strong spatial variations on scale of unit cell ~ λ, sensitive to disorder?
- Important for local light-matter interactions, quantum effects




Young et al, Phys. Rev. Lett. 115, 153901 (2015)




Khanikaev et al, Nature Mater. 12, 233 (2013)



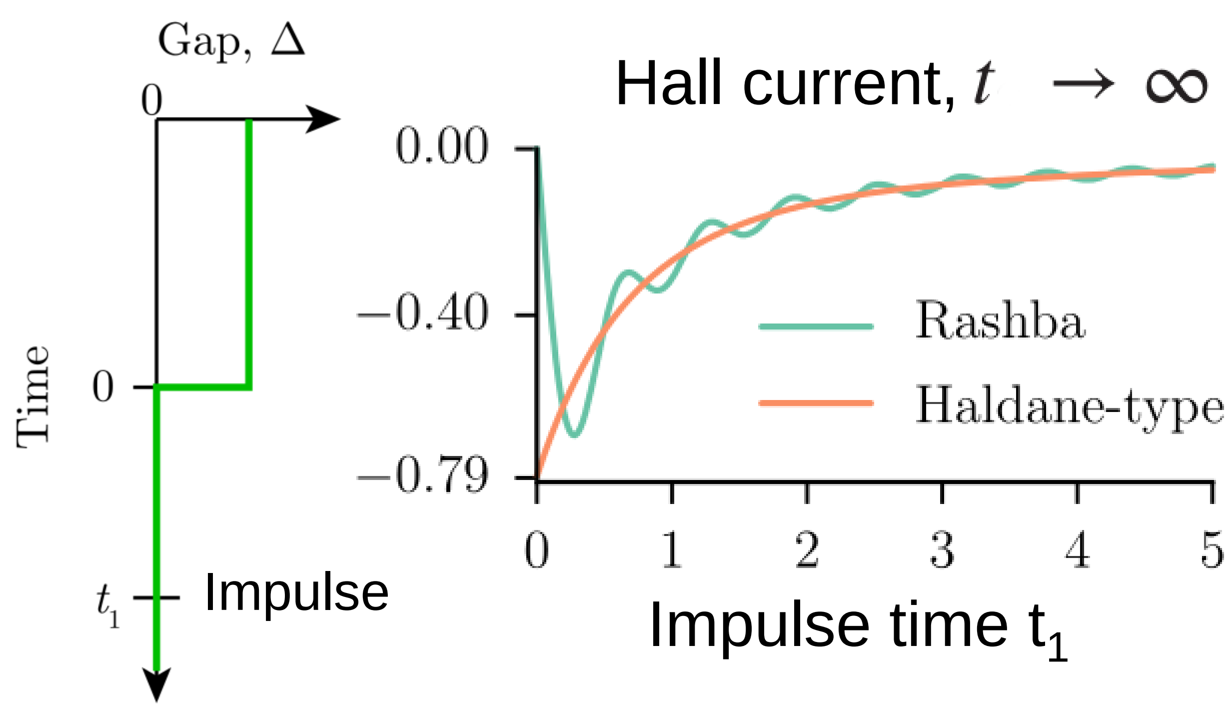

Onoda & Ochiai, Phys. Rev. Lett. 103, 033903 (2009)

# Quenching topological systems

- Rapidly change a control parameter
- Can switch between trivial, nontrivial, gapless phases
- Does wavefunction retain memory of original topological phase?
- Eg. Chern insulator: rapidly switch off magnetic field
- Memory of initial topological state: nonzero Hall current

$$h(\mathbf{p}, \Delta) = \mathbf{d}[\mathbf{p}, \Delta(t)] \cdot \boldsymbol{\sigma}$$

$$\mathbf{d}[\mathbf{p}, \Delta(t)] = (p_x, p_y, \Delta\Theta(-t))$$

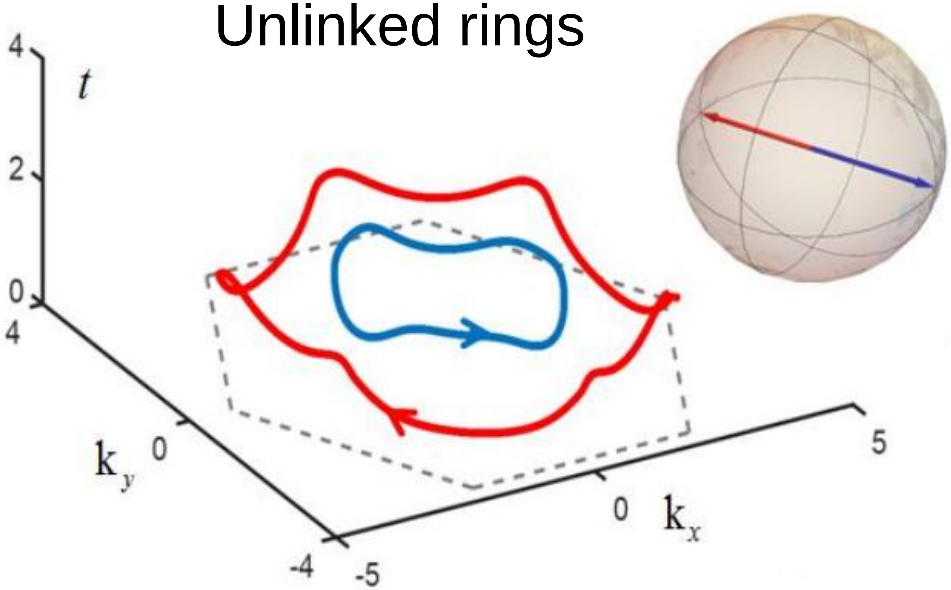



Wilson, Song, & Refael, "Remnant geometric Hall response in a quantum quench," Phys. Rev. Lett. (2016)

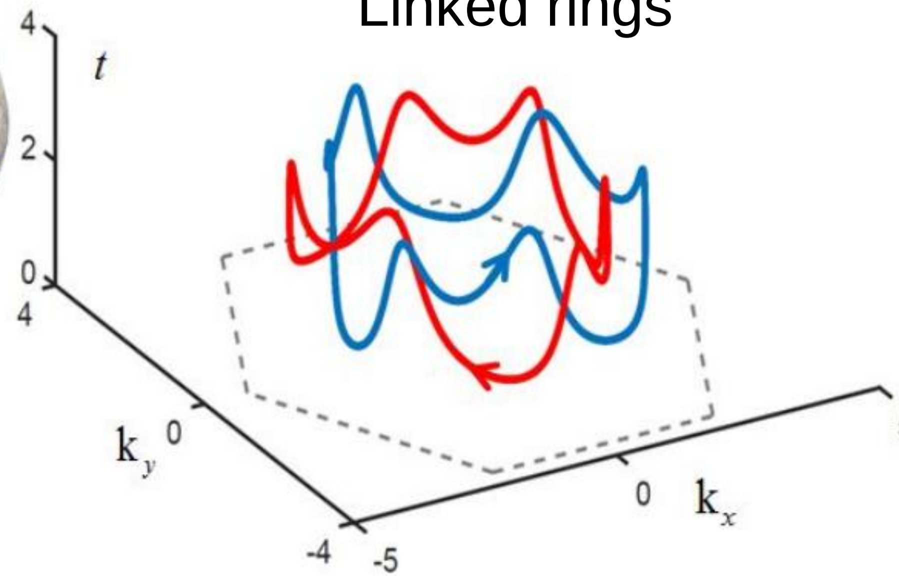
# Linking numbers of pseudospins

- Pseudospin textures sensitive to quenching between phases
- After quench, pseudospins evolve as $n(k_x,k_y, t)$
- Lines of fixed pseudospin form closed curves
- Linking number of any two curves sensitive to Chern number
- E.g. linking of left-handed and right-handed C lines $(k_x,k_y,t)$

Trivial quench
Unlinked rings




Nontrivial quench
Linked rings

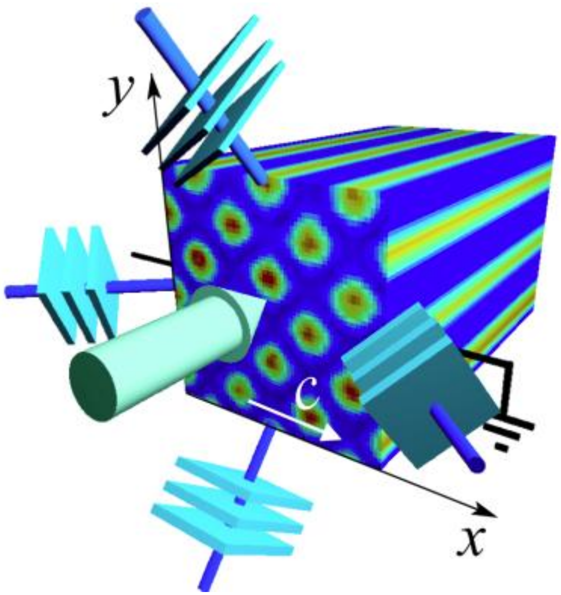



Wang et al., Phys. Rev. Lett. in press, arXiv:1611.03304 (2016); J. Yu, arXiv:1611.08917 (2016)

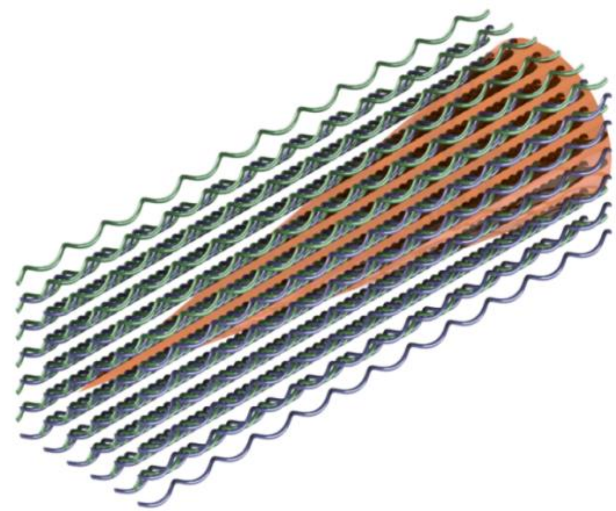
# “Quenching” a photonic lattice

- Injecting light into a photonic lattice naturally acts as a quench!
- z<0: free space propagation Δn =0, z>0 lattice potential Δn
- Non-equilibrium propagation dynamics & precession of pseudospin

$$i\partial_z \psi = -\frac{1}{2k_0}\nabla^2\psi - \frac{k_0 \Delta n(x,y,z)}{n_0}\psi,$$

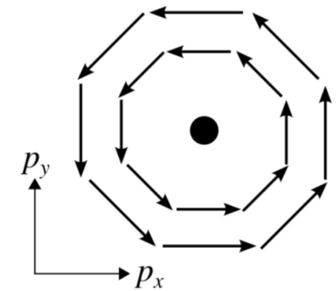



Trompeter et al., Phys. Rev. Lett. (2006)



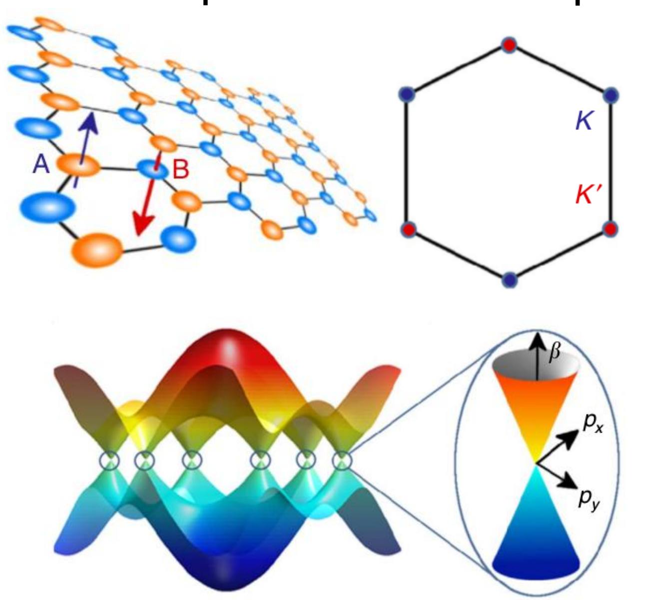
Noh et al, Nature Physics (2017)

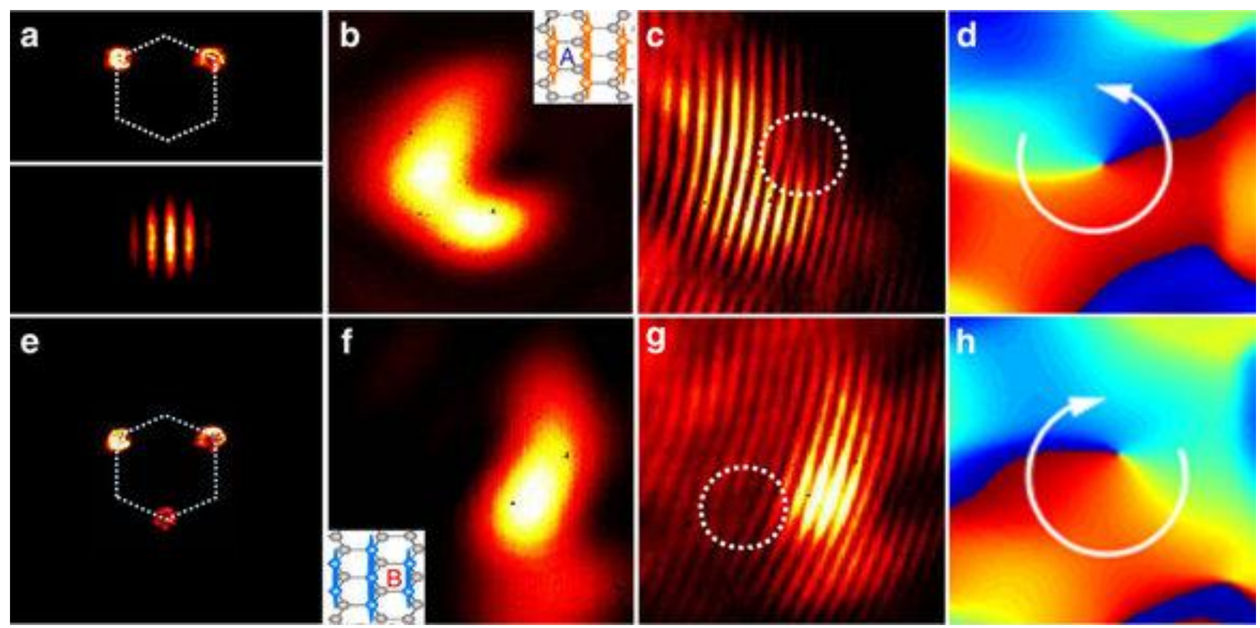
# Optical vortices from Dirac points




- Honeycomb lattice: vortex generation from Dirac point chirality
- Momentum space vortex of eigenmodes generates real space vortex
- Lieb lattice: double charge Dirac point => charge 2 vortex generation
- Gapped photonic topological insulators: linked vortex rings observable?
- Challenge: measuring 3D vortex lines

Real space Fourier space

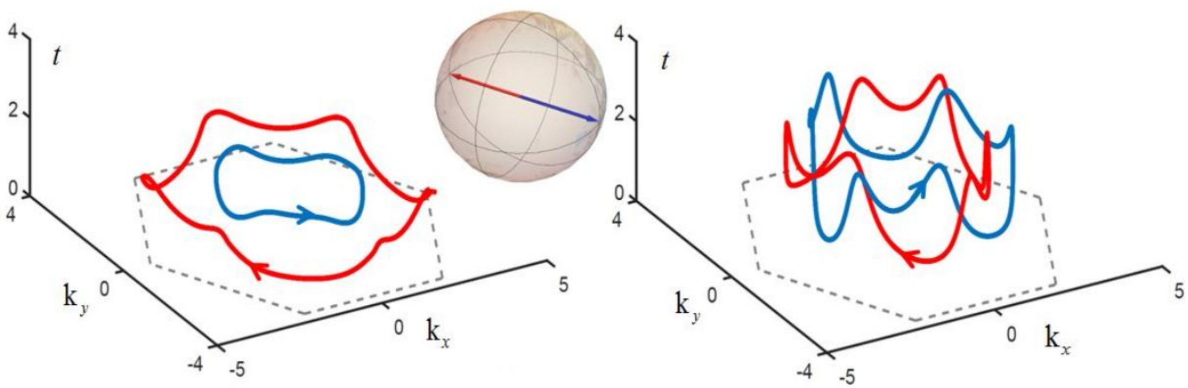


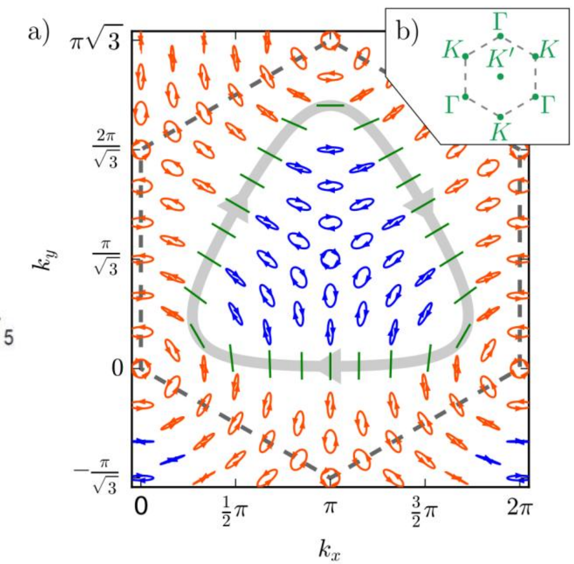

Song et al, Nature Comms. 6, 6272 (2015); Diebel et al, Phys. Rev. Lett. 116, 183902 (2016)

# Summary (part 2)

- Interesting analogies between pseudospins and spins
- Chern number as a basis-independent sum over C points or L lines
- Topological phase vs microscopic currents & momentum
- Quenches between topological phases generate linked pseudospin vortices
- Linking difficult to observe in condensed matter, easier in photonics?




Wang et al., Phys. Rev. Lett. in press, arXiv:1611.03304 (2016)
J. Yu, arXiv:1611.08917 (2016)




T. Foesel, V. Peano, F. Marquardt, arXiv:1703.08191