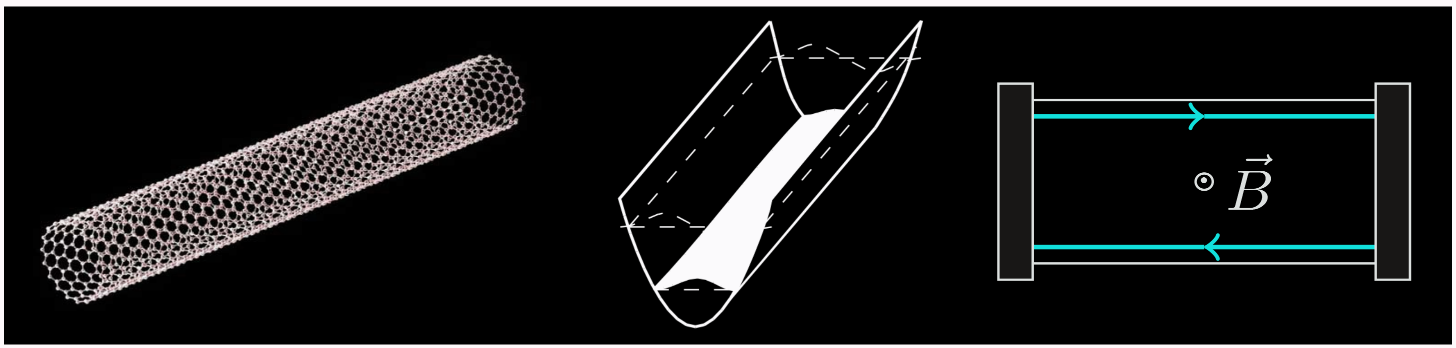
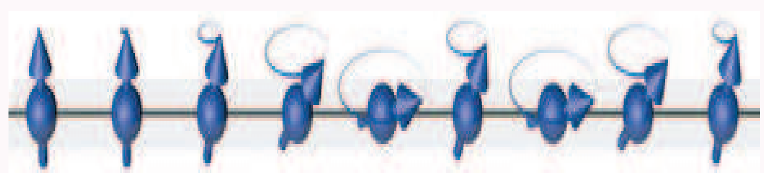
Heat waves in 1+1-dimensional Conformal Field Theory

Krzysztof Gawędzki, Banff, August 2018

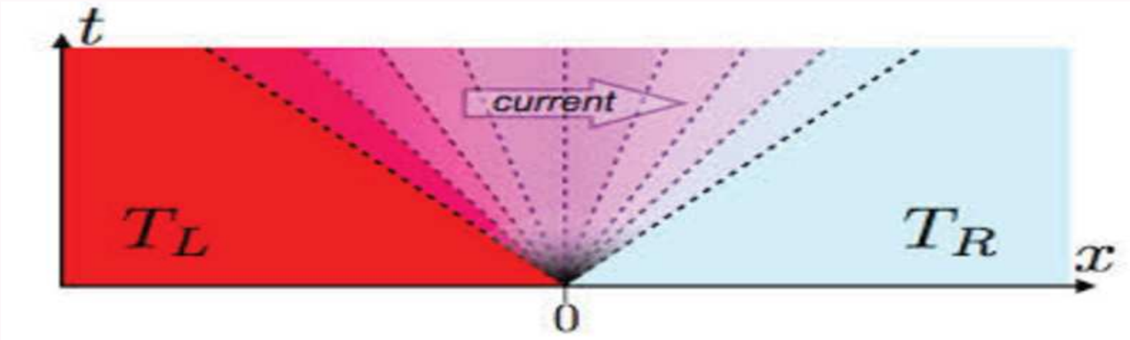
- Many one-dimensional quantum systems have massless low-energy excitations described by **Conformal Field Theory**

  Examples: carbon nanotubes, electrons or cold atoms trapped in $1d$ potential wells, quantum Hall edge currents, XXZ spin chains





- **1+1**-D **CFT** describes the low temperature equilibrium physics of such systems but also some of nonequilibrium situations as
  - the "partitioning protocol" after two halves of a system prepared in different equilibrium states are joined together (reviewed by **Bernard-Doyon** in J. Stat. Mech. (2016), 064005, see also **Hollands-Longo**, CMP 357 (2018))



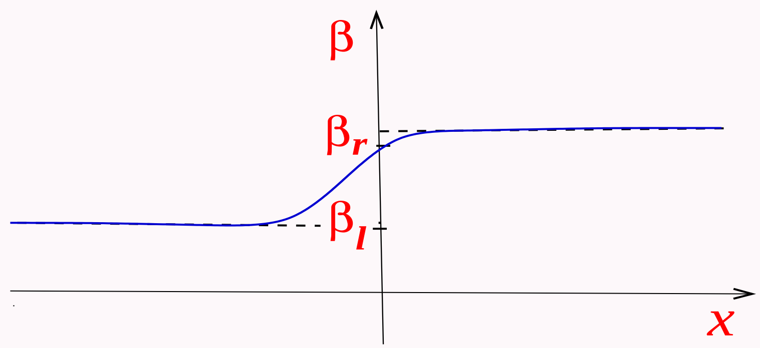
- The purpose of this talk is to show how **CFT** describes the dynamics of states with a preimposed smooth temperature profile
- Based on joint work with **E. Langmann** and **P. Moosavi**, J. Stat. Phys. 172 (2018), 353-378, and on my ongoing research

- Inspired by the paper by **Lebowitz-Langmann-Mastropietro-Moosavi**, Phys. Rev. B 95 (2017)

LLMM studied in the **Luttinger** model of interacting $1d$ electrons the time evolution of the nonequilibrium state

$$\omega^{\rm neq}(A) = \frac{\mathrm{Tr}\big(\mathrm{e}^{-G}A\big)}{\mathrm{Tr}\big(\mathrm{e}^{-G}\big)} \qquad \text{for} \qquad G = \int \beta(x)\,\mathcal{E}(0,x)\,dx$$

where $\mathcal{E}(t,x)$ is the energy density and $\beta(x)$ is a smooth inverse-temperature profile with the values β_ℓ and (β_r) far on the left (right)



- By resuming the perturbation series in powers of $(\beta_r - \beta_\ell)$, **LLMM** showed that for the model with local interactions (which is a **CFT**)

$$\omega^{\mathrm{neq}}(\mathcal{E}(t,x)) = \frac{1}{2}\big(F(x-vt) + F(x+vt)\big)$$
$$\omega^{\mathrm{neq}}(\mathcal{J}(t,x)) = \frac{v}{2}\big(F(x-vt) - F(x+vt)\big)$$

where $\mathcal{J}(t,x)$ is the heat current, v is the effective **Fermi** velocity, and

$$F(x) = \frac{\pi}{6v\beta(x)^2} - \frac{v}{12\pi}\mathcal{S}_\beta(x)$$

for

$$\mathcal{S}_\beta(x) = -\frac{\beta''(x)}{\beta(x)} + \frac{1}{2}\Big(\frac{\beta'(x)}{\beta(x)}\Big)^2$$

- They noticed that $\mathcal{S}_\beta(x)$ is the **Schwarzian** derivative

$$\{f(x),x\} = \frac{f'''(x)}{f'(x)} - \frac{3}{2}\left(\frac{f''(x)}{f'(x)}\right)^2$$

of the map

$$x \mapsto \int_0^x \frac{dx'}{\beta(x')}$$

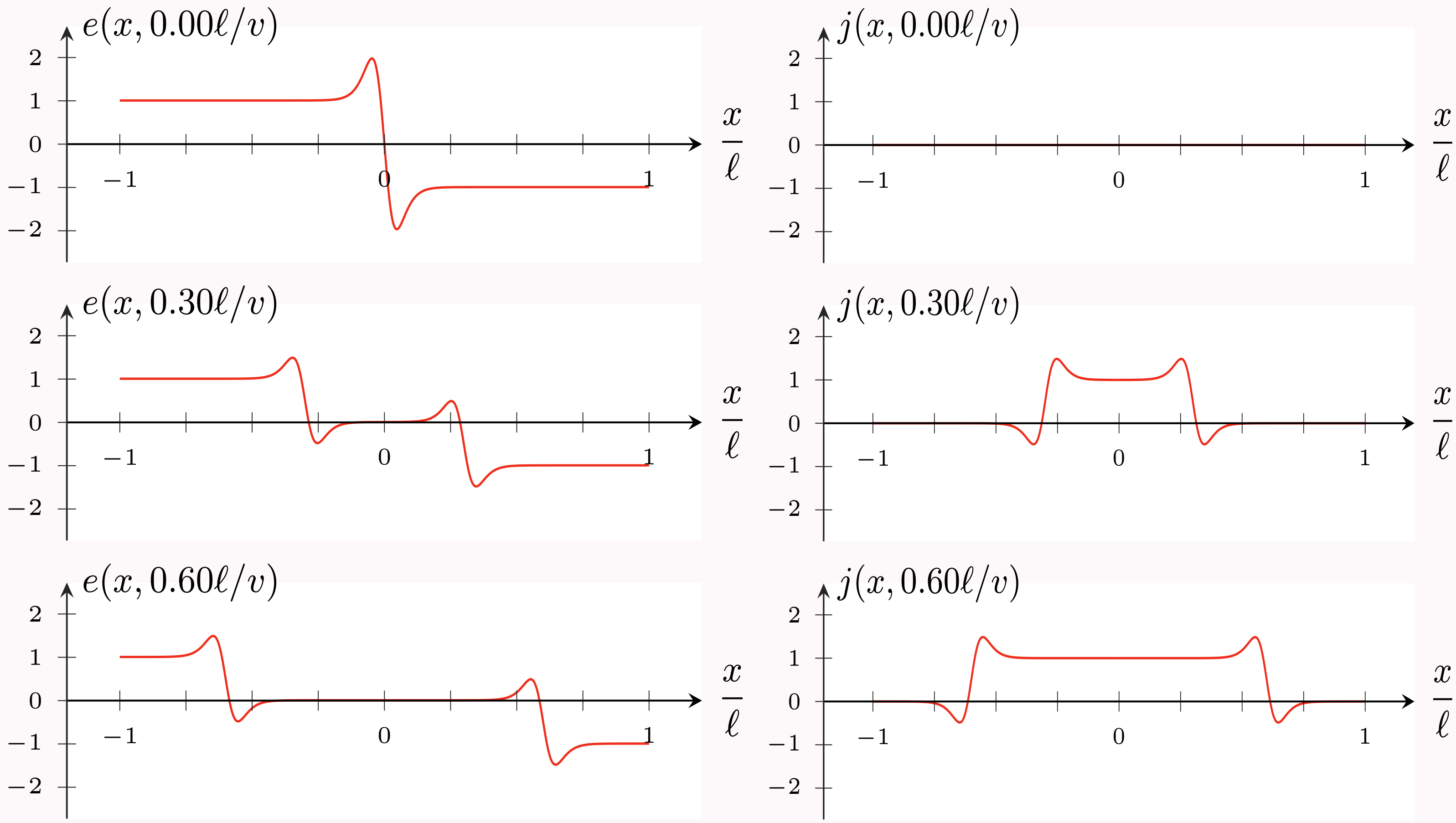
and $\{f(x),x\}$ appears in the **CFT** formula for the transformation of the energy-momentum tensor suggesting a **CFT** origin of their result

- The formulae of **LLMM** imply that

$$\omega^{\rm neq}(\mathcal{E}(t,y)) \underset{t\to\infty}{\longrightarrow} \frac{\pi}{12v}\left(\beta_\ell^{-2} + \beta_r^{-2}\right) \equiv \mathcal{E}_0$$

$$\omega^{\rm neq}(\mathcal{J}(t,y)) \underset{t\to\infty}{\longrightarrow} \frac{\pi}{12}\left(\beta_\ell^{-2} - \beta_r^{-2}\right) \equiv \mathcal{J}_0 \neq 0$$

but also shows a nontrivial evolution of the nonequilibrium expectations of $\mathcal{E}(t,x)$ and $\mathcal{J}(t,x)$ with traveling heat waves



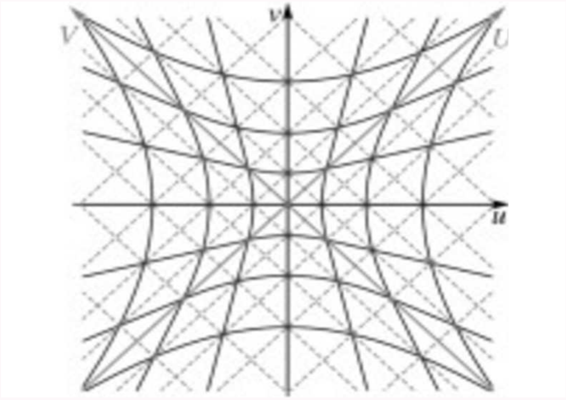
Evolution of the mean energy density minus $\mathcal{E}_0$ (left) and of the mean heat current (right)

- **General theory**
  - Set $x^{\pm} \equiv x \pm vt$. The conformal transformations in $1+1$-D spacetime are:

$$(x^-, x^+) \mapsto (f_+(x^-), f_-(x^+))$$

since

$$v^2 dt^2 - dx^2 = dx^- dx^+ \mapsto d\,f_+(x^-) df_-(x^+) = f'_+(x^-) f'_-(x^+) dx^- dx^+$$



- In a **CFT** the infinitesimal action of conformal symmetries in the **Hilbert** space $\mathbb{H}$ of states is generated by the components $T_{--}(x^-)$ and $T_{++}(x^+)$ of the energy-momentum tensor s.t.

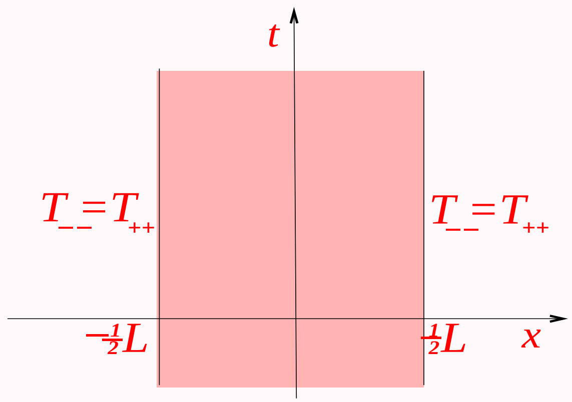
$$[T_{\substack{--\\++}}(x), T_{\substack{--\\++}}(x')] = \mp 2\mathrm{i}\,\delta'(x-x')T_{\substack{--\\++}}(x') \pm \mathrm{i}\,\delta(x-x')T'_{\substack{--\\++}}(x') \pm \frac{c\,\mathrm{i}}{24\pi}\,\delta'''(x-x')$$

  where c is the **central charge** of the theory

- The energy density and heat current in a **CFT** are

$$\mathcal{E}(t,x) = v\big(T_{--}(x^-) + T_{++}(x^+)\big)$$
$$\mathcal{J}(t,x) = v^2\big(T_{--}(x^-) - T_{++}(x^+)\big)$$

- It is convenient to work in a finite box $[-\frac{1}{2}L, \frac{1}{2}L]$ with the boundary conditions that guarantee that $T_{--}(x^-) = T_{++}(x^+)$ for $x = \pm\frac{1}{2}L$



- There is then only one independent component of the energy-moment. tensor $T_{--}(x^-) = T_{--}(x^- + 2L)$ with $T_{++}(x^+) = T_{--}(x^+ \pm L)$

$$T_{--}(x) = \frac{\pi}{2L^2}\sum_{n=\infty}^{\infty} \mathrm{e}^{\frac{\pi \mathrm{i}}{L}(x+\frac{1}{2}L)}\left(L_n - \frac{c}{24}\right) \equiv T(x)$$

where L_n satisfy the **Virasoro** algebra:

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}(n^3 - n)\delta_{m+n,0}$$

- T generates a unitary projective representation $f \mapsto U_f$ of $Diff_+ S^1$ for $f(x+2L) = f(x) + 2L$ with $f'(x) > 0$ such that

$$U_f \, T(x) \, U_f^{-1} \; = \; f'(x)^2 \, T(f(x)) \; - \; \frac{c}{24\pi} \{f(x), x\}$$

- If f_s is the flow of a vector field $-\zeta(x)\partial_x$ with $\zeta(x+2L) = \zeta(x)$, i.e.

$$\partial_s f_s(x) = -\zeta(f_s(x)) \, , \qquad f_0(x) = x$$

then

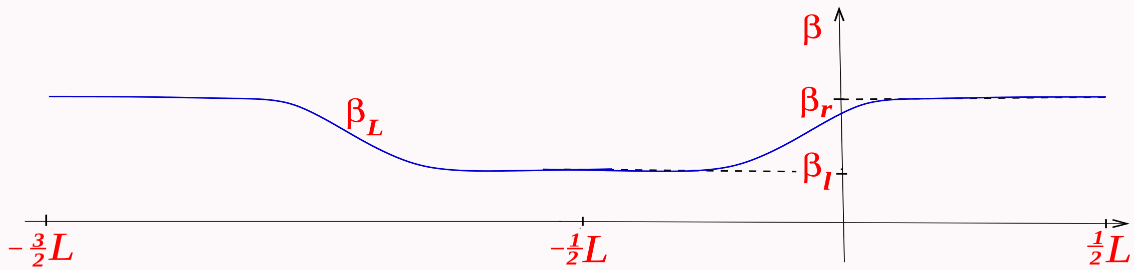
$$U_{f_s} = \exp\left[\mathrm{i} s \int_{-L}^{L} \zeta(x) \, T(x) \, dx \right]$$

E.g. for translations $f_s(x) = x - s$

$$U_{f_s} \; = \; \mathrm{e}^{\frac{\pi \mathrm{i}}{L} s (L_0 - \frac{c}{24})}$$

- For L big enough let $\beta_L(x) = \beta_L(x+2L)$ be defined by

$$\beta_L(x) \;=\; \begin{cases} \beta(x) & \text{for} \quad x \in [-\tfrac{1}{2}L, \tfrac{1}{2}L] \\ \beta(-x-L) & \text{for} \quad x \in [-\tfrac{3}{2}L, -\tfrac{1}{2}L] \end{cases}$$



- Consider for $G_L = \int_{-\frac{1}{2}L}^{\frac{1}{2}L} \beta(x)\, \mathcal{E}(0,x)\, dx \;=\; v \int_{-\frac{3}{2}L}^{\frac{1}{2}L} \beta_L(x)\, T(x)\, dx$

  the finite-box nonequilibrium state

$$\omega_L^{\text{neq}}(A) \;=\; \frac{\text{Tr}\big(\text{e}^{-G_L} A\big)}{\text{Tr}\big(\text{e}^{-G_L}\big)}$$

- Let $f = f_L \in Diff_+ S^1$ be such that $f'_L(x) = \frac{\beta_{0,L}}{\beta_L(x)}$ with $\beta_{0,L}$ fixed by the requirement that $f_L(x+2L) = x+2L$. Then

$$\boxed{U_{f_L}\, G_L\, U_{f_L}^{-1}} = v \int_{-\frac{3}{2}L}^{\frac{1}{2}L} \beta_L(x)\, U_{f_L} T(x)\, U_{f_L}^{-1}\, dx$$

$$= v \int_{-\frac{3}{2}L}^{\frac{1}{2}L} \beta_L(x)\, f'_L(x)^2\, T(f_L(x))\, dx - \frac{cv}{24\pi} \int_{-\frac{3}{2}L}^{\frac{1}{2}L} \beta_L(x)\, \{f_L(x), x\}\, dx$$

$$\overset{y=f_L(x)}{=} v\,\beta_{0,L} \int_{-\frac{3}{2}L}^{\frac{1}{2}L} T(y)\, dy - \underbrace{\frac{cv}{24\pi} \int_{-\frac{3}{2}L}^{\frac{1}{2}L} \beta_L(x) \{f_L(x), x\}\, dx}_{\mathbf{c-number}}$$

$$= \boxed{\beta_{0,L} H_L + \text{const.}}$$

$\Rightarrow$ **the conjugation by** U_{f_L} **flattens the temperature profile !!!**

- This allows to compare the non-equilibrium and equilibrium finite-volume states:

$$\boxed{\omega_L^{\rm neq}(A) = \omega^{\rm eq}_{\beta_{0,L},L}\big(U_{f_L} A\, U_{f_L}^{-1}\big)}$$

- That relation may be applied to $A=\prod_i T_{--}(x_i^-)\prod_j T_{++}(x_j^+)$ for which one has the identity

$$U_{f_L}\, T_{\substack{--\\++}}(x^\mp)\, U_{f_L}^{-1} = \big(\tfrac{\beta_{0,L}}{\beta_L(x^\mp)}\big)^2 T_{\substack{--\\++}}\big(f_L(x^\mp)\big) - \tfrac{c}{24\pi}\{f_L(x^\mp),x^\mp\}$$

- The thermodynamic limit $L\to\infty$ is easily controlled using standard **CFT** techniques leading to the infinite-volume relations

$$\begin{aligned}
&\omega^{\rm neq}\Big(\prod_i T_{--}(x_i)\prod_j T_{++}(x_j)\Big)\\
&= \omega^{\rm eq}_{\beta_0}\bigg(\prod_i\Big(\tfrac{\beta_0^2}{\beta(x_i)^2}T_{--}(f_\beta(x_i)) - \tfrac{c}{24\pi}\{f_\beta(x_i),x_i\}\Big)\\
&\qquad\times\prod_j\Big(\tfrac{\beta_0^2}{\beta(x_j)^2}T_{++}(f_\beta(x_i)) - \tfrac{c}{24\pi}\{f_\beta(x_j),x_j\}\Big)\bigg)
\end{aligned}$$

where $f_\beta(x)=\int_0^x\frac{\beta_0}{\beta(x')}\,dx'$ with arbitrary β_0

- For 1-point functions of $T_{\substack{--\\++}}(x^{\mp})$ the above relations together with the infinite-volume **CFT** identity $\omega^{\rm eq}_{\beta_0}\big(T_{\substack{--\\++}}(x^{\mp})\big)=\frac{\pi c}{12(v\beta_0)^2}$ give

$$\boxed{\omega^{\rm neq}\Big(T_{\substack{--\\++}}(x^{\mp})\Big)\;=\;\frac{\pi\,c}{12(v\beta(x_i)^2}\;-\;\frac{c}{24\pi}\{f_\beta(x^{\mp}),x^{\mp}\}}$$

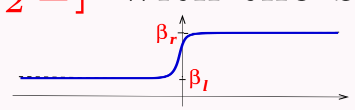
extending the result of **LLMM** about the nonequilibrium expectations of the energy density and the heat current to any unitary **CFT**

- The 1-point expressions are the simplest example of the general relations that hold for the nonequilibrium expectations in any **CFT** model

- The expectations with insertions of primary fields may be treated similarly leading to analogous infinite-volume identities

- **Full counting statistics for the heat transfer**

  - For the profile states, one may obtain an exact expression for the full counting statistics (**FCS**) of the heat transfers across the kink in a $\beta(x)$-profile

  - Consider a **CFT** on $[-\frac{1}{2}L, \frac{1}{2}L]$ with the boundary conditions as before. If the kink in $\beta(x)$ 

 is narrow then

$$G_L = \int_{-\frac{1}{2}L}^{\frac{1}{2}L} \beta(x)\, \mathcal{E}(0,x)\, dx \;=\; \beta_\ell E_\ell + \beta_r E_r$$

where E_ℓ and E_r are the energies to the left and to the right of the kink, respectively

  - One gets access to the **FCS** of the heat transfers by performing two measurement of G_L in the nonequilibrium state ω_L^{neq} separated by time t

- By spectral decomposition

$$G_L = \sum_i g_i P_i\,, \qquad G_L(t) \equiv \mathrm{e}^{\mathrm{i}tH_L} G_L \mathrm{e}^{-\mathrm{i}tH_L} = \sum_i g_i P_i(t)$$

If the $1^{\rm st}$ measurement gives the value g_i and the $2^{\rm nd}$ one the value g_j then the transfer of the energy across the kink in time t is

$$\Delta E = E_r(t) - E_r(0) = -(E_\ell(t) - E_\ell(0)) = \frac{g_j - g_i}{\Delta\beta}$$

where $\Delta\beta = \beta_r - \beta_\ell$

- By the **QM** rules the probability of getting the results (g_i, g_j) is

$$p_{ij} = \omega_L^{\rm neq}\Big(P_i P_j(t)\Big)$$

giving for the **PDF** of the energy transfers

$$p_{t,L}(\Delta E) = \sum_{ij} \delta\Big(\Delta E - \tfrac{g_j - g_i}{\Delta\beta}\Big)\, \omega^{\rm neq}\Big(P_i P_j(t)\Big)$$

- The characteristic function of the probability distribution of ΔE is

$$F_{t,L}(\lambda) \equiv \int \mathrm{e}^{\mathrm{i}\lambda\Delta E}\, p_{t,L}(\Delta E)$$
$$= \sum_{i,j} \mathrm{e}^{\frac{\mathrm{i}\lambda}{\Delta\beta}(g_j - g_i)}\, \omega^{\mathrm{neq}}\Big(P_i P_j(t)\Big) = \omega_L^{\mathrm{neq}}\Big(\mathrm{e}^{-\frac{\mathrm{i}\lambda}{\Delta\beta}G_L}\, \mathrm{e}^{\frac{\mathrm{i}\lambda}{\Delta\beta}G_L(t)}\Big)$$
$$= \omega^{\mathrm{eq}}_{\beta_{0,L},L}\Big(U_{f_L}\mathrm{e}^{-\frac{\mathrm{i}\lambda}{\Delta\beta}G_L}\, \mathrm{e}^{\frac{\mathrm{i}\lambda}{\Delta\beta}G_L(t)} U_{f_L}^{-1}\Big)$$

using our relation between the nonequilibrium and equilibrium states

$$U_{f_L} G_L\, U_{f_L}^{-1} = \beta_{0,L} H_L - \underbrace{\frac{cv}{24\pi}\int_{-\frac{3}{2}L}^{\frac{1}{2}L} \beta_L(x)\{f_L(x), x\}\, dx}_{\mathbf{c-number}}$$

$$U_{f_L} G_L(t)\, U_{f_L}^{-1} = \int_{-\frac{3}{2}L}^{\frac{1}{2}L} \zeta_{t,L}(y)\, T(y)\, dy - \underbrace{\frac{cv}{24\pi}\int_{-\frac{3}{2}L}^{\frac{1}{2}L} \beta_L(x^+)\{f_L(x), x\}\, dx}_{\mathbf{c-number}}$$

where $\zeta_{t,L}(y) = v\beta_{0,L}\frac{\beta_L(f_L^{-1}(y)+vt)}{\beta_L(f_L^{-1}(y))}$. Since $H_L = \frac{\pi}{L}(L_0 - \frac{c}{24})$,

$$F_{L,t}(\lambda) = \omega_L^{\mathrm{neq}}\Big(\mathrm{e}^{-\frac{\mathrm{i}\lambda}{\Delta\beta}G_L}\,\mathrm{e}^{\frac{\mathrm{i}\lambda}{\Delta\beta}G_L(t)}\Big)$$

$$= \boxed{\frac{\mathrm{Tr}\Big(\mathrm{e}^{2\pi\mathrm{i}\tau_s(L_0-\frac{c}{24})}\,U_{f_s}\Big)}{\mathrm{Tr}\Big(\mathrm{e}^{2\pi\mathrm{i}\tau_0(L_0-\frac{c}{24})}\Big)}}\ \mathrm{e}^{\mathrm{i}s\,\frac{cv}{24\pi}\int_{-\frac{3}{2}L}^{\frac{1}{2}L}\big(\beta_L(x)-\beta_L(x^+)\big)\{f_L(x),x\}\,dx}$$

for $s = \frac{\lambda}{\Delta\beta}$, $\tau_s = \frac{(\mathrm{i}-s)\beta_{0,L}}{2L}$, and $f_s \in Diff_+S^1$ solving the flow equation equation $\partial_s f_s(y) = -\zeta_{t,L}(f_s(y))$, $f_0(y) = y$

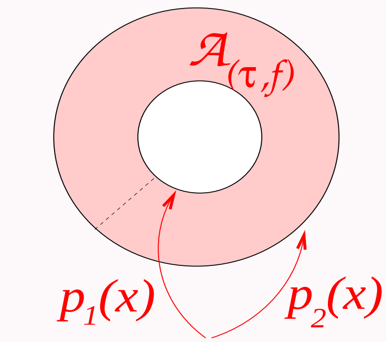
- One usually views the denominator $\mathrm{Tr}\Big(\mathrm{e}^{2\pi\mathrm{i}\tau(L_0-\frac{c}{24})}\Big)$ as the character of the **Virasoro** algebra representation in the space of states of **CFT**
- Similarly, the numerator $\mathrm{Tr}\Big(\mathrm{e}^{2\pi\mathrm{i}\tau(L_0-\frac{c}{24})}\,U_f\Big)$ may be viewed as the character of the corresponding representation of $Diff_+(S^1)$

- **Characters of** $Diff_{+}(S^{1})$

  - The characters of $Diff_{+}(S^{1})$ may be reduced to those of the respective **Virasoro** representation (this did not seem to exist in the literature)

  - According to **G. Segal**, the operator $e^{2\pi i\tau(L_0 - \frac{c}{24})}U_f$ is proportional to the chiral **Euclidian CFT** amplitude of the complex annulus

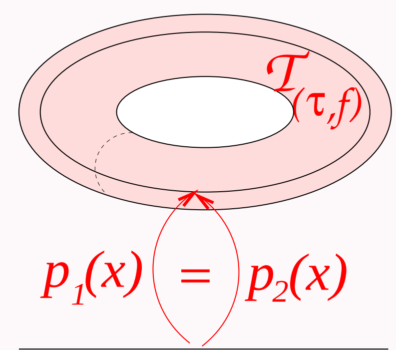
$$\mathcal{A}_{\tau,f} = \{\, z \,\big|\, |e^{2\pi i\tau}| \le |z| \le 1 \,\}$$



with the boundary components parameterized by

$$p_1(x) = e^{2\pi i\tau} e^{-\frac{\pi i}{L} f(x)}, \qquad p_2(x) = e^{-\frac{\pi i}{L} x}$$

  - Characters are **class functions** invariant under the adjoint action. What it means here is that (up to a scalar factor) $\mathrm{Tr}\left(e^{2\pi i\tau(L_0 - \frac{c}{24})}U_f\right)$ depends only on the torus $\mathcal{T}_{\beta,f}$ obtained from $\mathcal{A}_{\tau,f}$ by gluing its parameterized boundaries



- Indeed, $\mathrm{Tr}\left(\mathrm{e}^{2\pi\mathrm{i}\tau(L_0-\frac{c}{24})}\,U_f\right)$ is proportional to the **CFT** amplitude of the torus $\mathcal{T}_{\beta,f}$ with its natural complex structure

- The complex torus $\mathcal{T}_{\beta,f}$ is isomorphic to $\mathcal{T}_{\widehat{\tau},f_0}$ for $f_0(x)\equiv x$ and some $\widehat{\tau}$ in the upper half plane. This implies the relation

$$\mathrm{Tr}\left(\mathrm{e}^{2\pi\mathrm{i}\tau(L_0-\frac{c}{24})}\,U_f\right) = C_{\tau,f}\ \mathrm{Tr}\Big(\mathrm{e}^{2\pi\mathrm{i}\widehat{\tau}(L_0-\frac{c}{24})}\Big)$$

  where on the right-hand-side is the trace of the **CFT** amplitude of the annulus $\mathcal{A}_{\widehat{\tau},f_0}$ and $C_{\tau,f}$ is a complex number due to the projective character of the chiral **CFT** amplitudes

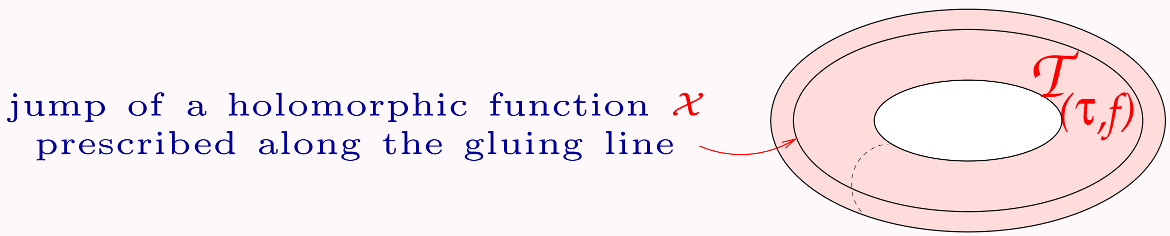
- The constant $C_{\tau,f}$ may be expressed in terms of determinants of **Fredholm** operators on $L^2([-\frac{3}{2}L,\frac{1}{2}L])\equiv\mathcal{H}$ that appear in the context of a **Riemann-Hilbert**-type problem on the torus $\mathcal{T}_{\tau,f}$

- $\widehat{\tau}$ may be obtained by solving a related **Fredholm** equation

• The Riemann-Hilbert problem on $\mathcal{T}_{\tau,f}$

- Given a function $X \in \mathcal{H}$ one searches for a holomorphic function $\mathcal{X}$ on $\mathcal{A}_{\tau,f}$ such that

$$X = X_1 - X_2 \qquad \text{for} \qquad X_i = \mathcal{X} \circ p_i$$



- Let $P_>$ and $P_<$ be the orthogonal projectors in $\mathcal{H}$ on the subspaces spanned by functions $\mathrm{e}^{-\frac{\pi \mathrm{i}}{L} nx}$ with $n > 0$ and $n < 0$, respectively

- Let $Q_{\tau,f} : \mathcal{H}_0 \longrightarrow \mathcal{H}_0$, for $\mathcal{H}_0 \subset \mathcal{H}$ composed of functions with vanishing integral, be the operator

$$(P_> + P_<)(X_1 - X_2) \xrightarrow{Q_{\tau,f}} P_> X_1 - P_< X_2$$

- $Q_{\tau,f}$ is a **traceclass**. Explicitly

$$Q_{\tau,f} = (K_{11}+K_{12}-K_{21})(I-K_{11}-K_{12}-K_{21})^{-1}(P_{<}-K_{12})-K_{12}$$

where $K_{ij} : \mathcal{H} \longrightarrow \mathcal{H}$ have smooth kernels

$$(K_{11}X)(x) = \frac{1}{2\pi \mathrm{i}} \int_{-\frac{3}{2}L}^{\frac{1}{2}L} X(y) \Big(\frac{dp_1(y)}{p_1(y)-p_1(x)} - \frac{dp_2(y)}{p_2(y)-p_2(x)} \Big)$$

$$(K_{12}X)(x) = -\frac{1}{2\pi \mathrm{i}} \int_{-\frac{3}{2}L}^{\frac{1}{2}L} X(y) \, \frac{dp_2(y)}{p_2(y)-p_1(x)}$$

$$(K_{21}X)(x) = \frac{1}{2\pi \mathrm{i}} \int_{-\frac{3}{2}L}^{\frac{1}{2}L} X(y) \, \frac{dp_1(y)}{p_1(y)-p_2(x)}$$

and as such are traceclass

- The theory of determinant bundles of **Quillen** and **Segal** implies that

$$\boxed{\mathrm{Tr}\Big(\mathrm{e}^{2\pi\mathrm{i}\tau L_0}\,U_f\Big) \;=\; \underbrace{\Big(\frac{\det(I-Q_{\tau,f})}{\det(I-Q_{\widehat{\tau},f_0})}\Big)^{\frac{c}{2}}\,\big\langle 0\big|U_f\big|0\big\rangle}_{C_{\tau,f}}\;\mathrm{Tr}\Big(\mathrm{e}^{2\pi\mathrm{i}\widehat{\tau}L_0}\Big)}$$

  where $\big\langle 0\big|U_f\big|0\big\rangle$ is the vacuum expectation of U_f

- That reduces the characters of $Diff_+(S^1)$ to the more standard ones of the **Virasoro** algebra

- In particular, this permits to reduce to the latter the formula for the **FCS** characteristic function $F_{L,t}(\lambda)$ in which $\mathrm{Tr}\Big(\mathrm{e}^{2\pi\mathrm{i}\tau(L_0-\frac{c}{24})}\,U_{f_s}\Big)$ was the only nonexplicit entry

- **FCS for the heat transfer in the thermodynamic limit**

  - The formula for characteristic function of the **FCS** heat transfer simplifies in the limit $L \to \infty$ giving

$$F_t(\lambda) = \lim_{L\to\infty} F_{L,t}(\lambda) = \mathrm{e}^{-\frac{c}{2}\sum\limits_{+,-}\int_0^s \mathrm{Tr}\Big((I-\mathcal{Q}^{\pm}_{t,s'})^{-1}\partial_{s'}\mathcal{Q}^{\pm}_{t,s'} - \mathcal{R}^{\pm}_{t,s'}\Big)ds'} \times \mathrm{e}^{is\,\frac{cv}{24\pi}\int\Big(2\beta(x)-\beta(x^+)+\beta(x^-)\Big)\{f(x),x\}\,dx}$$

where operators $\mathcal{Q}$ in $L^2(\mathbb{R})$ are related to the integral operators $\mathcal{K}_{ij}$ obtained in the $L \to \infty$ limit from K_{ij}

$\mathcal{Q}^{\pm}_{t,s}$ correspond to $f(y) = f_{\pm s}(\pm y)$ for $\partial_s f_{\pm s}(\pm y) = \mp\zeta_{\pm t}(f_{\pm s}(\mp y))$ with $\zeta_{\pm t}(y) = v\beta_0 \frac{\beta(f_\beta^{-1}(y)\pm vt)}{\beta(f_\beta^{-1}(y))}$ (right- and left-movers contributions)

Operators $\mathcal{R}^{\pm}_{t,s}$ are obtained from

$$\mathcal{R} = P_{>}\Phi_f P_{<}\zeta\partial_x P_{>}(P_{>}\Phi_f P_{>})^{-1}$$

where $\Phi_f\varphi = \varphi\circ f$ by setting $\zeta(y) = \zeta_{\pm t}(\pm y)$ and $f(y) = f_{\pm s}(\pm y)$

- The contribution of $\mathcal{R}^{\pm}_{t,s'}$ comes from $\langle 0|U_{f_{s,L}}|0\rangle$ and may be easily obtained from the **Fredholm**-determinant expression for the latter for free massless bosons worked out in **Bruneau**-**Dereziński** (2005)

- It follows that $F_t(\lambda)$ is universal depending only on the profile $\beta(x)$ and the central charge of the **CFT**

- One should be able to extract the large deviations asymptotics of **Bernard**-**Doyon** (2012)

$$\lim_{t\to\infty}\frac{1}{t}\,\ln F_t(\lambda) \;=\; \frac{\pi c}{12}\left(\frac{1}{\beta_\ell - \mathrm{i}\lambda} - \frac{1}{\beta_\ell} + \frac{1}{\beta_r + \mathrm{i}\lambda} - \frac{1}{\beta_r}\right)$$

from our exact formula for $F_t(\lambda)$

Conclusions

- In a **CFT** conformal symmetries may be used to map inhomogeneous situations to homogeneous ones
- That allowed to express nonequilibrium expectations in states with temperature profile in terms of equilibrium ones
- The states where one imposes also the profiles of chemical potential can be treated similarly in theories with current-algebra symmetries
- The general results confirmed and extended the particular ones obtained by **LLMM** for the **Luttinger** model through perturbative calculations
- The **FCS** statistics of energy transfers in such states was expressed using characters of $Diff_+(S^1)$ and was shown to exhibit in the thermodynamic limit a universal dependence on the temperature profile