Geometrical Methods, non Self-Adjoint Spectral Problems, and Stability of Periodic Structures

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1 Overview of the Field

Eigenvalue problems occur in many contexts in applied mathematics. It is well-understood that the solution of certain linear partial differential equations in physically-relevant geometries leads to eigenvalue problems via the procedure of separation of variables. However, similar problems also arise in the nonlinear context, for example via the linearized stability analysis of traveling waves and other fully-nonlinear coherent structures, or in the study of completely integrable soliton equations by means of inverse-scattering theory based on an associated spectral problem. In many cases of practical interest, the spectral problem cannot be viewed simply as the eigenvalue problem for a self-adjoint operator but rather more complicated forms occur like non self-adjoint operators or so-called "pencils" of operators whose spectral theory must be understood. This has necessitated the development of new analytical and numerical methods for understanding such spectral problems. It is becoming clear that geometrical ideas are important in the study of these spectral problems and, moreover, that these geometrical ideas can be applied in all of these seemingly disparate contexts. These observations have led to new understanding and rigorous justification for modulation theory of traveling waves in dispersive equations, linear and nonlinear optics, pattern-formation and reaction-diffusion systems, and many other applied mathematical models.

In summary, our goal with this meeting was to bring together researchers from different subfields where geometric techniques for eigenvalue problems have proved fruitful, as well as to promote interactions and cross-fertilization.

Our workshop brought together different kinds of researchers: those working on various types of spectral problems in many different mathematical contexts; those developing techniques of applied geometry; those focused on the consequences of stability properties on particular problems in the natural sciences; and those who gravitate around all of the above. The meeting provided a unique opportunity to promote cross-fertilization between these areas.

2 Presentation Highlights

Ben Akers (Air Force Institute of Technology): *"Overturned traveling interfacial waves"*

Ben Akers talked about periodic traveling waves are computed on parameterized interfaces, which are not functions of the horizontal coordinate(s). These overturned traveling waves are computed on one and two-dimensional interfaces, on a classic interface between two fluids as well as on boundary formed by a hydroelastic ice sheet. Numerical continuation procedures are coupled with local and global bifurcation theorems. Extreme wave types and bifurcation surfaces were presented. The prospects for stability of overturned traveling waves were also discussed. Akers mentioned that there are very few works on the stability of these overturned waves (notably, that of Tiron and Choi [14]), opening up, in this fashion, a very interesting line of research.

Jaime Angulo-Pava (Universidade de São Paulo): *"Stability theory of bump standing waves for NLS equations with point interactions"*

In his talk, Jaime Angulo-Pava demonstrated the effectiveness of an extension theory for investigation of stability of standing waves for semi-linear Schrödinger equations with δ- and δ'-interaction on the line and on the star graph. One of the focus topics was the standing waves with a bump profile. Angulo mentioned the intrinsic difficulties of the problem, which pertain to the computation of the Morse index.

Andrea Barreiro (Southern Methodist University): *"A geometric method for analyzing operators with low-rank perturbations"*

In her presentation, Andrea Barreiro talked about her joint work with Tom Anastasio and Jared Bronski (UIUC). Barreiro considered the problem of finding the spectrum of an operator taking the form of a low-rank (rank one or two) non-normal perturbation of a self-adjoint operator. She used a simple idea of classical differential geometry (the envelope of a family of curves) to analyze the spectrum. When the rank of the perturbation is two, this allows us to view the system in a geometric way through a "phase plan" in the perturbation strengths. Barreiro showed how to apply this technique to two problems: a neural network model of the oculomotor integrator [1], and a nonlocal model of phase separation [13]. To sum up, Barreiro presented a novel approach to compute the spectrum of low-rank perturbations of self-adjoint operators in finite- and infinite-dimensional spaces and discussed potential applications.

Sylvie Benzoni-Gavage (University of Lyon): *"Stability of periodic waves in Hamiltonian PDEs"*

For Hamiltonian systems of PDEs the stability of periodic waves is encoded by the Hessian of an action integral, as shown in earlier work. In this presentation, Sylvie Benzoni-Gavage dealt with two asymptotic regimes, namely for waves of small amplitude, and for waves of long wavelength. In both cases stability criteria can be investigated analytically, thanks to the asymptotic expansions of the Hessian of the action and their special structure. The stability results thus obtained apply to various models of mathematical physics for nonlinear dispersive waves. In these cases, the Hamiltonian structure of the system plays a key role.

Antonio Capella-Kort (IPICyT): *"On the stability of Bloch walls in a dynamical model with eddy currents"*

One of the main features of micro-magnetic materials is the formation of magnetic domain separated by the so called magnetic walls. These walls are transition layers that move upon the application of external magnetic fields. In bulk material the main type of walls is the Bloch wall. There are two models for micro-magnetic precession dynamics with damping. One is based on the Landau-Lifshitz-Gilbert (LLG) equation, and the other is based on eddy current damping. In this talk, Antonio Capella showed some stability results for the Bloch wall under both the eddy current damping model and the LLG model. In addition, Capella presented micro-magnetic dynamics as a paradigm of a physically relevant theory in which non-local problems arise naturally from first principles, opening up a rich line of research for the stability of nonlinear waves community.

Graham Cox (Memorial University of Newfoundland): *"Constructing a generalized Maslov index for non-Hamiltonian systems"*

The Maslov index is a powerful and well known tool in the study of Hamiltonian systems, providing a generalization of Sturm-Liouville theory to systems of equations. For non-Hamiltonian systems, one no longer has the symplectic structure needed to define the Maslov index. In this talk, Graham Cox described a recent construction of a generalized Maslov index for a very broad class of differential equations. The key observation is that the manifold of Lagrangian planes can be enlarged considerably without altering its topological structure, and in particular its fundamental group.

Salvador Cruz-García (IIMAS, UNAM): : *"Exploring the spectral stability of standing and traveling waves in mesenchymal migration"*

Mesenchymal migration is a proteolytic and path generating strategy of individual cell motion inside the network of collagen fibres that compose the extracellular matrix of tissues. In this talk, Cruz-García analyzed the spectral stability of the families of standing and traveling wave solutions of the one-dimensional version of the M^5-model, which was proposed by T. Hillen [8] to describe mesenchymal cell movement. Regarding the standing waves, they are spectrally stable and the spectrum of the linearized operator around the waves consists solely of essential spectrum. To prove that in the standing case the point spectrum is empty Cruz-García used energy estimates together with the integrated-variable technique of Goodman [7]. The panorama is completely different in the traveling case; the wave profiles are spectrally unstable due to the fact that the essential spectrum reaches the closed right-half complex plane. In our pursuit of spectral stability, we have constructed a weighted Sobolev space where the essential spectrum lies inside the open left-half complex plane.

Bernard Deconinck (University of Washington): *"Nonlinear stability of stationary periodic solutions of the focusing NLS equation"*

The spectral instabilities of the stationary periodic solutions of the focusing NLS equation were completely characterized recently. The crux of this characterization was the analysis of the non-self adjoint Lax pair for the focusing NLS equation. Although all solutions are unstable in the class of bounded perturbations, different solutions were found to be spectrally stable with respect to certain classes of periodic perturbations, with period an integer multiple of the solution period, also called subharmonic perturbations. In this talk, Deconinck examined these ideas and showed that all solutions that are spectrally stable are also (nonlinearly) orbitally stable, using different Krein signature calculations. Similarly, more recent results for the sine-Gordon equation were shown as well. Deconinck showed how to fully and analytically describe the stability spectrum exploiting the Lax par and integrability, and how to use this information to conclude some stability properties with respect to subharmonic perturbations.

Aslihan Demirkaya-Ozkaya (University of Hartford): *"Kink dynamics in a parametric ϕ^6 system: a model with controllably many internal modes"*

In this presentation, Demirkaya-Ozkaya explored a variant of the ϕ^6 model originally proposed in [4] as a prototypical, so-called, "bag" model where domain walls play the role of quarks within hadrons. Demirkaya-Ozkaya examined the prototypical steady state of the model, namely an apparent bound state of two kink structures, and explored its linearization in order to find that, as a function of a prototypical parameter controlling the curvature of the potential, an effectively arbitrary number of internal modes may arise in the point spectrum of the linearized analysis. Demirkaya-Ozkaya proposed an Evans function analysis to predict the bifurcation points of the relevant internal modes and confirm these theoretical predictions numerically. Finally, given the remarkable flexibility of the model in possessing different numbers of internal modes an open problem was proposed, namely, to explore the dynamics of multi-bound-state collisions to identify the role of the additional internal modes in enhancing the complexity of the observed scattering scenarios.

Gianne Derks (University of Surrey): *"Existence and stability of fronts in inhomogeneous wave equations"*

Models describing waves in anisotropic media or media with imperfections usually have inhomogeneous terms. Examples of such models can be found in many applications, for example in nonlinear optical waveguides, water waves moving over a bottom with topology, currents in nonuniform Josephson junctions, DNA-RNAP interactions, etc. Travelling waves in such models tend to interact with the inhomogeneity and get trapped, reflected, or slowed down. In this talk, Gianne Derks considered wave equations with finite length inhomogeneities, assuming that the spatial domain can be written as the union of disjoint intervals, such that on each interval the wave equation is homogeneous. The underlying Hamiltonian structure allows for a rich family of stationary front solutions and the values of the energy (Hamiltonian) in each intermediate interval provide natural parameters for the family of orbits. Derks showed that changes of stability can only occur at critical points of the length of the inhomogeneity as a function of the energy density inside the inhomogeneity, and gave a necessary and sufficient criterion for the change of stability. These results were illustrated with some examples.

Jesús Adrián Espínola-Rocha (Universidad Autonoma Metropolitana, Campus Azcapotzalco): *"Klaus-Shaw potentials for the Ablowitz-Ladik lattice"*

Some PDEs and ODEs admit a Lax pair (a pair of linear operators) to completely solve the equation. One of these operators defines a spectral problem. For some equations (as for the Korteweg-deVries, or KdV, equation) this operator is self-adjoint and, consequently, its discrete spectrum is real. However, for some other equations, this operator is non-selfadjoint, such as for the nonlinear Schrödinger (NLS) equation or the Ablowitz-Ladik equation. In [10], M. Klaus and J. K. Shaw found symmetries and conditions on the potentials for the Zakharov-Shabat system (spectral problem for the NLS equation) for the eigenvalues to lie on the imaginary axis. In this talk, Espínola-Rocha showed which would be an equivalent to the Klaus-Shaw theorem for the Ablowitz-Ladik lattice.

Anna Ghazaryan (Miami University): *"Stability of traveling fronts in a model for porous media combustion"*

In this presentation, Anna Ghazaryan considered a model of combustion in hydraulically resistant porous media. There are several reductions of these systems that can be used to understand the evolution of the combustion fronts. One reduction is based on the assumption that the ratio of pressure and molecular diffusivities is close to zero; a different reduction is obtained when the Lewis number is chosen in a specific way. Fronts exist in both reduced systems. For the stability analysis of the fronts, Ghazaryan considered initial conditions of a specific form, and then showed that the stability results extend to the fronts in the full system with the same Lewis number. The fronts are either absolutely unstable or convectively unstable.

Laura Rocío González-Ramírez (UPI, Campus Hidalgo - Instituto Politécnico Nacional): *"On the study of traveling wave solutions on a cortical wave propagation model including inhibition"*

In this talk, Laura González-Ramírez discussed the influence of inhibition on an activity-based neural field model consisting of an excitatory population with a linear adaptation term that directly regulates the activity of the excitatory population. Such a model has been used to replicate cortical traveling wave data as observed in clinical recordings. González-Ramírez established conditions for the existence of traveling wave solutions with properties observed in *in vivo* data. González-Ramírez also discussed some results concerning the linear stability of traveling wave solutions of this model via the construction of an Evans function. González-Ramírez presented some stability results for non-local models which can actually be validated via comparison with clinical data.

César Adolfo Hernández-Melo (Universidade Estadual de Maringá): *"On stability properties of the cubic-quintic Schrödinger equation with a Dirac potential"*

In this talk, César Hernández presented some results on the existence and orbital stability of the peak-standing-wave solutions for the cubic-quintic nonlinear Schrödinger equation with a point interaction. Via a perturbation method and continuation argument, stability results are obtained, in the case of attractive-attractive and attractive-repulsive nonlinearities. In the case of an attractive-attractive case and an focusing interaction Hernández presented a complete approach for stability based in the extension theory of symmetric operators. When the intensity of the point interaction is zero the equation is Hamiltonian and there exist explicit expressions for the profiles. Hernández' extension explored the possibilities that arise when the point interaction is switched on, with both positive and negative signs.

Vera Mikyoung Hur (University of Illinois at Urbana-Champaign): *"Full-dispersion shallow water models and modulational instability*

In the 1960s, Benjamin and Feir [2], and Whitham [15], discovered that a Stokes wave would be unstable to long wavelength perturbations, provided that the product of the carrier wave number and the undisturbed water depth exceeds ≈ 1.363. In the 1990s, Bridges and Mielke [3] studied the corresponding spectral instability in a rigorous manner. But they left some important issues open, such as the spectrum away from the origin. The governing equations of the water wave problem are complicated. One may resort to simple approximate models to gain insights.

In this talk, Vera Hur discussed Whitham's shallow water equation and the modulational instability index for small amplitude and periodic traveling waves, as well as the effects of surface tension and constant vorticity. In addition, Hur discussed the limitations of Whitham's equation as some of the neglected dispersion effects inhibit wave breaking. Hur also examined higher order corrections, extension to bidirectional propagation, and two-dimensional surfaces. Her presentation was partly based on joint works with Mat Johnson (Kansas) and Ashish Pandey (Illinois).

Michael Jenkinson (Rensselaer Polytechnic Institute): *"On-site and off-site bound states of the discrete nonlinear Schrödinger equation and the Peierls-Nabarro barrier"*

In this talk, Michael Jenkinson presented a procedure to construct several families of symmetric localized standing waves (breathers) to the one-, two-, and three-dimensional discrete nonlinear Schrödinger equation (DNLS) with cubic nonlinearity using bifurcation methods about the continuum limit. Such waves and their energy differences play a role in the propagation of localized states of DNLS across the lattice. The energy differences, which are exponentially small in a natural parameter, are related to the Peierls-Nabarro Barrier in discrete systems, first investigated by M. Peyrard and M. D. Kruskal [12]. These results may be generalized to different lattice geometries and inter-site coupling parameters. Finally, Jenkinson discussed the local stability properties of these bound states.

Mathew Johnson (University of Kansas): *"Nondegeneracy and stability of periodic traveling waves in a fractional NLS equation "*

In the stability and blow-up for traveling or standing waves in nonlinear Hamiltonian dispersive equations, the non-degeneracy of the linearization about such a wave is of paramount importance. That is, one must verify the kernel of the second variation of the Hamiltonian is generated by the continuous symmetries of the PDE. The proof of this property can be far from trivial, especially in cases where the dispersion admits a nonlocal description where shooting arguments, Sturm-Liouville theories, and other ODE methods may not be applicable. In this talk, Mathew Johnson discussed the non-degeneracy and nonlinear orbital stability of antiperiodic traveling wave solutions to a class of defocusing NLS equations with fractional dispersion. Key to the analysis is the development of a ground state theory and oscillation theory for linear periodic, fractional Schrödinger operators with antiperiodic boundary conditions. The talk was based on joint work with a student, Kyle Claassen.

Todd Kapitula (Calvin College): *"Analyzing Hamiltonian spectral problems via the Krein matrix"*

The Krein matrix is a matrix-valued function which can be used to study Hamiltonian spectral problems. Akin to the Evans matrix, it has the property that it is singular when evaluated at an eigenvalue. Unlike the Evans matrix, it is not analytic, but is instead meromorphic. In this presentation, Todd Kapitula explained the construction of the Krein matrix, and then applied it to the study of spectral stability of small periodic waves for a couple of equations.

Richard Kollár (Comenius University): *"Spectral stability in reduced and extended systems*

Spectral stability captures behavior of a solution perturbed by an infinitesimal perturbation. It often determines nonlinear stability but it is limited to the exact form of the dynamics of the system. However, governing equations are often only an approximation of a larger system that models real world situation. In his talk, Richard Kollár showed how the spectral stability of a solution in the reduced and full (extended) system are related, particularly for ODEs in the case of frequently used quasi-steady-state reductions, but also in a general case of reduced/extended systems. A connection is also drawn with the geometric Krein signature that is shown to naturally characterize spectral properties under such extensions.

Stephane Lafortune (College of Charleston): *"Spectral stability of solutions to the vortex filament hierarchy"*

The Vortex Filament Equation (VFE) is part of an integrable hierarchy of filament equations. Several equations in this hierarchy have been derived to describe vortex filaments in various situations. Inspired by these results, Stephane Lafortune presented a general framework for studying the existence and the linear stability of closed solutions of the VFE hierarchy. The framework is based on the correspondence between the VFE and the nonlinear Schrödinger (NLS) hierarchies. These results establish a connection between the AKNS Floquet spectrum and the stability properties of the solutions of the filament equations. Lafortune examined the application of this machinery to solutions of the filament equation associated to the Hirota equation. Lafortune also discussed how this framework applies to soliton solutions. In particular, Lafortune showed a systematic way to study the stability of these vortex solutions based on the Hasimoto map.

Yuri Latushkin (University of Missouri): *"The Maslov index and the spectrum of differential operators"*

In this talk Yuri Latushkin discussed some recent results on connections between the Maslov and the Morse indices for differential operators. The Morse index is a spectral quantity defined as the number of negative eigenvalues counting multiplicities while the Maslov index is a geometric characteristic defined as the signed number of intersections of a path in the space of Lagrangian planes with the train of a given plane. The problem of relating these two quantities is rooted in Sturm's Theory and has a long history going back to the classical work by Arnold, Bott, Duistermaat, Smale, and has attracted recent attention of several groups of mathematicians. Latushkin mentioned how the relation between the two indices helps to prove the conjecture that a pulse in a gradient system of reaction diffusion equations is unstable. Latushkin also discussed a fairly general theorem relating the indices for a broad class of multidimensional elliptic self-adjoint operators.

Gregory Lyng (University of Wyoming): *"Multidimensional stability of large-amplitude Navier-Stokes shocks"*

Extending results of Humpherys-Lyng-Zumbrun [9] in the one-dimensional case, Gregory Lyng presented a combination of asymptotic ODE estimates and numerical Evans-function computations to examine the multidimensional stability of planar Navier-Stokes shocks across the full range of shock amplitudes, including the infinite-amplitude limit, for monatomic or diatomic ideal gas equations of state and viscosity and heat conduction coefficients constant and in the physical ratios predicted by statistical mechanics, with Mach number $M > 1.035$. These results indicate unconditional stability within the parameter range considered, in agreement with the results of Erpenbeck [6] and Majda [11] in the corresponding inviscid case. Notably, this study includes the first successful numerical Evans computation for multi-dimensional stability of a viscous shock wave. At the spectral level, Lyng introduced a properly modified “flux variable” transformation, which removes the eigenvalue zero from the spectrum and for which the Evans function remains analytic.

Robert Marangell (Sydney University): *“An Evans function for 2-D shear flows of the Euler equations on the torus”*

In this talk, Robert Marangell considered the stability of time independent solutions to the incompressible, inviscid Euler equations on the torus whose stream functions have the form $\psi = U(\xi) = U(p_1 x + p_2 y)$ for fixed integers p_1 and p_2. By an appropriate change of coordinates and separation of variables, the linearised spectral problem is reduced to the study of a Hill's equation with a complex potential. By using Hill determinants, an Evans function of the original linearised Euler equation can be constructed. For certain, well-known shear flows, the form of the Hill determinant makes such an Evans function numerically straightforward to compute.

Corrado Mascia (Università di Roma, ‘La Sapienza’): *“Which drift/diffusion formulas for velocity-jump processes?”*

In this presentation, Corrado Mascia examined a class of linear hyperbolic systems which generalizes the Goldstein-Kac model to an arbitrary finite number of speeds with transition rates. Under the basic assumptions that the transition matrix is symmetric and irreducible, and the speed differences generate all the space, the system exhibits a large-time behavior described by a parabolic advection-diffusion equation. The main contribution is to determine explicit formulas for the asymptotic drift speed and diffusion matrix in term of the kinetic parameters, establishing a complete connection between microscopic and macroscopic coefficients. Mascia showed that the drift speed is the arithmetic mean of the velocities. The diffusion matrix has a more complicated representation, based on the graph with vertices the velocities and arcs weighted by the transition rates. The approach is based on an exhaustive analysis of the dispersion relation and on the application of a variant of the Kirchoff's matrix tree theorem from graph theory.

Fábio Natali (Universidade Estadual de Maringá): *“Sufficient conditions for orbital stability of periodic traveling waves”*

In this presentation, Fabio Natali showed sufficient conditions for orbital stability of periodic waves of a general class of evolution equations supporting nonlinear dispersive waves. Firstly, the main result does not depend on the parametrization of the periodic wave itself. Secondly, motivated by the well known orbital stability criterion for solitary waves, Natali showed that the same criterion holds for periodic waves. In addition, the positiveness of the principal entries of the Hessian matrix related to the “energy surface function” is also sufficient to obtain the stability. Consequently, it is possible to establish the orbital stability of periodic waves for several nonlinear dispersive models. Natali discussed the applicability of the method to a wide class of evolution equations; in particular it can be extended to regularized dispersive wave equations.

Pascal Noble (INSA Toulouse): *“Spectral stability of inviscid roll-waves”*

Roll-waves are well known hydrodynamic instabilities appearing in open channel flows driven by gravity. A classical model to describe such flows is given by the shallow water equations with bottom friction. Periodic travelling waves of this system are necessarily discontinuous and shocks are driven by classical Rankine-Hugoniot conditions. In order to regularize these solutions, viscous effects can be taken into account: in this framework, a complete stability theory is established. Less is known in the context of discontinuous periodic waves. The purpose of this talk was to present a framework to study the stability of discontinuous periodic waves. Pascal Noble provided some partial analytical results and numerical results on the stability of discontinuous roll-waves.

Katie Oliveras (Seattle University): *"Instabilities of two-stratified fluids under linear shear"*

In this talk, Katie Oliveras discussed the stability of periodic traveling wave solutions describing the interface between two fluids of varying density and vorticity trapped between two rigid lids. Using a generalization of a non-local formulation of the water wave problem, the spectral stability for the periodic traveling wave solution can be established, by extending Fourier-Floquet analysis to this non-local problem. Oliveras presented a numerical scheme to determine traveling wave solutions by exploiting the bifurcation structure of the non-trivial periodic solutions. It is possible to determine numerically the spectral stability for the periodic traveling wave solution by extending Fourier-Floquet analysis to the non-local problem. The full spectra for all traveling wave solutions can be thus generated. Oliveras discussed Kelvin-Helmholtz and Benjamin-Feir instabilities, as well as the suppression or amplification of such instabilities as a function of shear strength, density stratification, and the ratio of depths between the fluids.

Keith Promislow (Michigan State University): *"Robust pearling inhibition in multi-component bilayers"*

In continuum models bilayers are homoclinic structures that are generically unstable within second-order systems as the associated translational mode has a single zero. Within the single-component, functionalized Cahn-Hillard (FCH) free energy the unstable mode balances against surface diffusion to generate pearling modes: spatially periodic high-frequency lateral variations in the bilayer width that can be weakly stable or weakly unstable. Almost all biologically relevant lipid bilayers are composed of multiple types of lipids. In this talk, Keith Promislow presented a two-component FCH system constructed from a Geirer-Meinhardt (GM) model that possesses one-parameter families of bilayers with adjustable composition. Tuning the composition induces a real-to-complex eigenvalue bifurcation in the underlying GM system yields robust pearling inhibition (stability) in the full system.

Miguel Rodrigues (Université de Rennes 1): *"Linearized space-modulated stability and periodic waves of the Korteweg-de Vries equation"*

Partly motivated by applications to surface waves, recent rapid progresses on the stability theory of periodic waves have been obtained. In particular, for parabolic systems — including those encoding the shallow water description of viscous roll-waves — an essentially complete theory is now available. In this presentation, Miguel Rodrigues presented some first contributions to a dispersive theory, still to come.

Chiara Simeoni (University of Nice, Sophia-Antipolis): *"Analytical and numerical investigation of traveling waves for the Allen-Cahn model with relaxation"*

Chiara Simeoni's talk pertained to a (physically significant) modification of the parabolic Allen-Cahn equation, obtained by substituting the Fick's diffusion law with a relaxation relation of Cattaneo-Maxwell type. The investigation concentrates on existence and stability of traveling fronts connecting the two stable states of the model, and specifically the nonlinear stability as a consequence of detailed spectral and linearized analyses. The outcome of numerical studies are also presented for determining the propagation speed, in comparison with the parabolic case, and for exploring the dynamics of large perturbations of the front. These results ensue from a collaboration with Corrado Lattanzio (University of L'Aquila), Corrado Mascia (Sapienza University of Roma) and Ramon G. Plaza (National Autonomous University of Mexico).

Milena Stanislavova (University of Kansas): *"Stability of vortex solitons for n-dimensional focusing NLS"*

In this presentation, Milena Stanislavova considered the following nonlinear Schrödinger equation in n space dimensions,

$$iu_t + \Delta u + |u|^{p-1}u = 0,\ x \in \mathbb{R}^n,\ t > 0.$$

Stanislavova discussed the existence and stability of standing wave solutions of the form

$$\begin{cases} e^{iwt}e^{i\sum_{j=1}^{k} m_j\theta_j}\phi_w(r_1, r_2, \ldots, r_k), & n = 2k \\ e^{iwt}e^{i\sum_{j=1}^{k} m_j\theta_j}\phi_w(r_1, r_2, \ldots, r_k, z), & n = 2k+1 \end{cases}$$

for $n = 2k$, where (r_j, θ_j) are polar coordinates in $\mathbb{R}^2$, $j = 1, 2, \ldots, k$; for $n = 2k + 1$, (r_j, θ_j) are polar coordinates in $\mathbb{R}^2$, (r_k, θ_k, z) are cylindrical coordinates in $\mathbb{R}^3$, $j = 1, 2, \ldots, k - 1$. Stanislavova showed the existence of such solutions as minimizers of a constrained functional and conclude from there that such standing waves are stable if $1 < p < 1 + 4/n$.

Atanas Stefanov (University of Kansas): *"Solitary waves for the Whitham equation on the whole line"*

If we consider the Whitham equation on the whole line then due to the smoothing nature of the linear operator, the question for existence of traveling wave solutions had been open till recently. In [5], Ehrnström, Groves and Wahlen constructed such waves, but only for values of c slightly bigger than one, even though the admissible range of wave speeds is $c \in (1, 2)$. The approach in [5] consists of a tour de force calculus of variations, supplemented by a bifurcation argument from the small KdV waves. Note that the waves by Ehrnström, Groves and Wahlen are of small amplitude. In this talk, Atanas Stefanov presented some joint work with M. Ehrnström, in which they construct a one parameter family of such waves, with wave speeds $c \in (1, c_0)$ for some limiting value c_0, not necessarily close to 1. Stefanov conjectures that the wave with speed c_0 is the maximal amplitude wave (i.e. the highest wave, with amplitude $c/2$) and that there are no waves with wave speeds $c \in (c_0, 2)$. However, some interesting objects in the interval $(c_0, 2)$ can be found. The argument uses calculus of variation construction, very different than the one employed in [5]. It is based on constraints on appropriately selected Orlicz spaces. Finally, all the traveling waves are shown to be bell-shaped, confirming the available numerical evidence. Stefanov also especulated about their stability as they are constructed as ground states of the appropriate constrained maximization problems.

Olga Trichtchenko (University of Washington): *"Stability of periodic travelling wave solutions to Korteweg-de Vries and related equations"*

In this talk, Olga Trichtchenko explored the simplest equation that exhibits high frequency instabilities: the fifth-order Korteweg-de Vries equation. Trichtchenko showed how to derive the necessary condition for an instability of a perturbation of a small-amplitude, periodic travelling wave solution. This is done by examining how these unstable perturbations change and grow in time as the underlying solution changes. Trichtenko commented on what happens when a different nonlinearity is considered in the underlying equation.

J. Douglas Wright (Drexel University): *"Traveling waves in diatomic Fermi-Pasta-Ulam-Tsingou lattices"*

Consider an infinite chain of masses, each connected to its nearest neighbors by a (nonlinear) spring. This is an FPUT lattice. In the instance where the masses are identical, there is a well-developed theory on the existence, dynamics and stability of solitary waves and the system has come to be one of the paradigmatic examples of a dispersive nonlinear equation. In this talk, J. Douglas Wright discussed recent rigorous results (together with T. Faver, A. Hoffman, R. Perline, A. Vainchstein and Y. Starosvetsky) on the existence of traveling waves in the setting where the masses alternate in size. In particular, Wright examined the limit where the mass ratio tends to zero. The problem is inherently singular and as such the existence theory becomes rather complicated. In particular, one finds that the traveling waves are not true solitary waves but rather "nanopterons", which is to say, waves which asymptotic at spatial infinity to very small amplitude periodic waves. Moreover, one can only find solutions when the mass ratio lies in a certain open set. The difficulties in the problem all revolve around understanding Jost solutions of a nonlocal Schrödinger operator in its semi-classical limit.

3 Scientific Progress Made

After the workshop we solicited from the participants some statements about what they felt were important developments either from their personal scientific perspectives. Here are some highlights of scientific progress that occurred as a result of the workshop. (Participant names are in boldface.)

- **Olga Trichtchenko**: I definitely benefited from attending the workshop because I had the chance to meet with my two collaborators on the project that I presented on (**Bernard Deconinck** and **Richard Kollár**, who are in different parts of the world from me). It allowed us to see where exactly to place this work because similar work was presented as well, such as knowing who else had stability results similar to ours (for example **Todd Kapitula's** work) and other people who were aware of related work. As I have started a postdoc in a new field but I am still wrapping up the project related to the workshop, it was very important for me to hear about new developments since I'm not around the community as much anymore. Since it was a small workshop and everyone was in the same place most of the time, this allowed **Bernard**, **Richard** and I to make progress on the project and come up with a better criteria for stability, inspired by what **Todd Kapitula** said in his talk and based on the criticism he offered, thus uniting a two step process we had into a single step. We would talk at dinner and do more derivations and I believe this workshop was very important to me personally as a younger researcher, in getting more attention and enthusiasm for this work I've made progress on, from my more experienced and busier collaborators as well as other people in the field. In a way, it allows me to get comments before, for example, submitting a paper on a certain topic, allowing me to make the work better. Otherwise, I would be a lot slower in accomplishing my tasks if I didn't get to attend these smaller workshops since people are much slower at answering e-mails and collaborating remotely takes a lot longer, especially if everyone is in different time zones. Ever since the workshop, **Bernard**, **Richard** and I have been sending a lot of e-mails back and forth on the project I gave a talk on which wouldn't have happened if we didn't all meet in Oaxaca and have the idea of how to complete the work.

  Bernard Deconinck: **Olga Trichtchenko**, **Richard Kollár** and I continued talking about an ongoing project about the stability of KdV-like equations where the dispersion relation allows for resonance. This was not a new development, but the three of us being in the same location was highly beneficial.

  Richard Kollár: During the workshop I had very fruitful discussions with **Olga Trichtchenko** and **Bernard Deconinck** about our project on stability of small-amplitude waves for the Kawahara equation. The meeting, including a talk by **Todd Kapitula** on a related topic, then led to fast development that significantly improved our results.

- **Corrado Mascia**: It has been a great opportunity for meeting some people that I did not the opportunity to meet in other occasions. Of course, it was also very nice to meet again some others that I have met in previous conferences/workshops. Many of the talks have been very stimulating and determined some direct implication on my current research projects. Among others, let me quote the possibility of incorporating a multi-component analysis in the velocity-jump process, as suggested by **Keith Promislow**, which is currently part of one of my future works. In addition, together with **Chiara Simeoni**, we started a promising and fruitful collaboration with **Salvador Cruz-Garcia** on the subject of cell mesenchymal motion.

  Salvador Cruz-Garcia: I started a collaboration with **Corrado Mascia** and **Chiara Simeoni**. Our purpose is to study, through suitable mathematical modeling, the effect of Myo-Inositol in reversing the epithelial-to-mesenchymal transition during the earliest manifestations of cancer. A promising approach is to extend the M^5-model for mesenchymal motion in the extracellular matrix.

- **Keith Promislow**: I had some detailed conversations with **Graham Cox** about the use of Maslov index techniques for the identification of connection/non-connection between critical points of a 4N-order dynamical system. The non-connection establishes a finite energy barrier for heteroclinic connections in an amphiphilic polymer system. My hope is that these discussions can turn into a collaboration.

- **Yuri Latushkin**: Here are the two most important projects that I discussed during the workshop:

  - A joint project with **Gianne Derks** on Grillakis-Shatah-Strauss theory for the case when the Poisson operator has kernel scanned by a finite number of gradients of Casimirs. There were several talks at the workshop that were instrumental in our understanding of the problem.
  - A joint project with **Robert Marangell** on instability of Kolmogorov-type unidirectional flows of the two-dimensional Euler equations. The talk given by **Robby** at the workshop triggered our discussions and we now have a plan on how to proceed with a particular case of the flow.

  I do not know if one can say that I made a breakthrough during the workshop but in my opinion the scientific program was superb: I would rank this workshop as the most useful for me personally since the Oberwolfach meeting on stability of traveling waves in 2012.

- **Atanas Stefanov**: I and **Stephane Lafortune** began a new project during the workshop. It is related to the stability of harmonic map type solutions in the context of Schrödinger maps.

- **Robert Marangell**: **Graham Cox** and I did make a nice breakthrough on a problem that we've been working on for a while, and **Ramón Plaza** and I started a collaboration that potentially might involve a student. I plan on offering an Honours project on it for the coming year, so we will see if anyone is interested.

- **Cesar Adolfo Hernandez Melo**: The conference talk given by **Rocio Gonzales** encouraged me to effectively study the stability of travelling wave solutions of the generalized Huxley equation perturbed by a family of Dirac interactions:

  $$u_t = u_{xx} + Z\delta(x)u + \beta(1 - u^{p-1})(u^{p-1} - \gamma).$$

  In a recent work developed with a collaborator, we studied the stability of equilibrium solutions of a more general model that the previous one. However, we did not study the stability of travelling waves, basically, because we did not know anything about possible applications of the peak travelling wave solutions associated to the model mentioned above. After the talk of **Rocio**, I realized the peak travelling wave solutions of the perturbed Huxley equation could be a good approximation for the propagation of nerve impulses. So, I expect to understand the stability of those travelling waves in the near future.

- **Aslihan Demirkaya**: I am collaborating with **Fabio Natali** on the stability of periodic snoidal waves for the ϕ^4 equation.

- **Stephane Lafortune**:

  - I discussed a research project with **Anna Ghazaryan** concerning the stability of solutions to a model of surfactants.
  - I also discussed another project with **Anna Ghazaryan**. This one concerns the stability of 2d pulses for a model of combustion.
  - After my talk on vortex filaments, **Atanas Stefanov** and I discussed a project on higher dimensional versions of the vortex filament equations and stability in that context.

- **Martin Molina-Fructuoso**: I would like to say that **Fabio Natali's** talk was very interesting to **Ramón Plaza** and I and might offer a different perspective on a project we have been working on.

  All the talks and speakers were very inspiring. I found the stability problem that **Antonio Capella-Kort** presented very interesting, as it related the study of waves to a variational problem in material science. I was also surprised by the unexpected appearance of the fractional Laplacian.

  Both **Keith Promislow** and **Rocio Gonzalez** talked about applying mathematics to tackle problems in exciting and new application fields.

4 Outcome of the Meeting

The first goal of the workshop was to bring together experts working on various aspects of self-adjoint and non-self-adjoint spectral problems arising in applied mathematics, with an emphasis on geometric, topological, numerical and analytical methods being developed in other contexts by (possibly) different researchers. We believe that we succeeded in this endeavor. By bringing together researchers who work on related problems in different fields we provided a forum for the exchange of innovative ideas, and set the groundwork for unifying the different methodologies into a cohesive whole.

The second goal of the workshop was to provide younger researchers, at the doctoral and postdoctoral levels, with the opportunity to expand their knowledge and expertise in the subject, and to collaborate with experts. The workshop served as a forum where researchers at all levels of experience in related fields could get together to exchange ideas, to foster collaborations, and to accelerate the development of new theories.

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