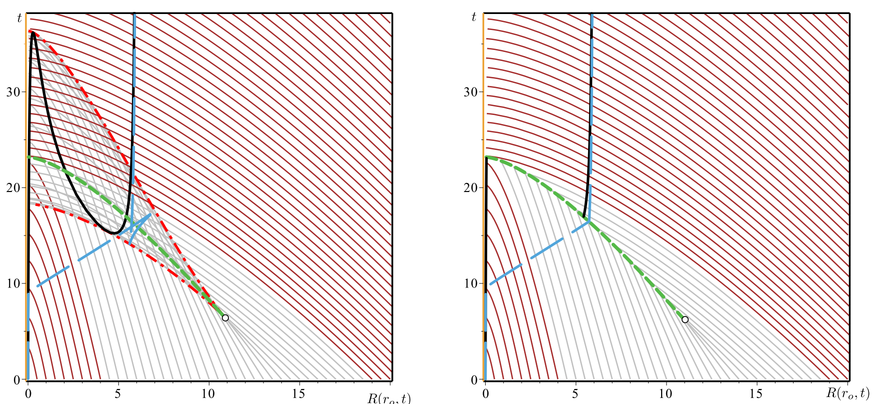
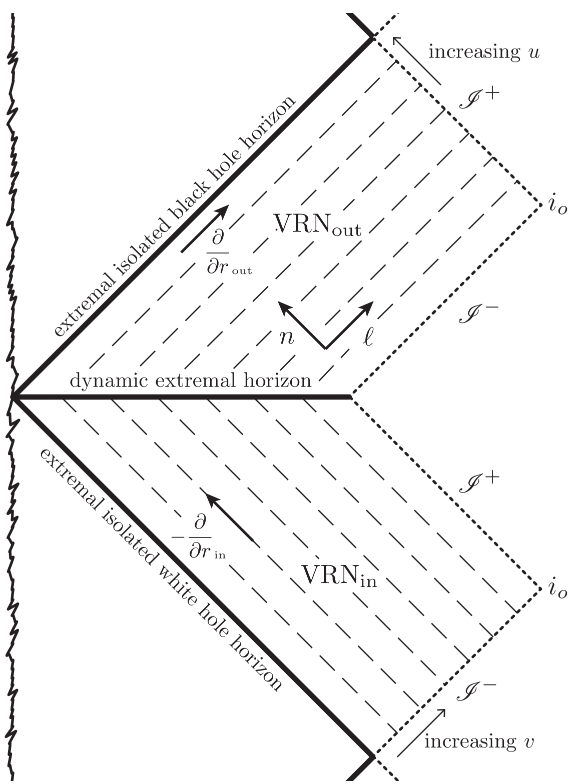
Range of Horizon Dynamics



(a) Pre-excision (b) Post-excision



Black Holes: New Horizons
BIRS Oaxaca, May 17, 2016

Ivan Booth
(Memorial University of Nfld)

Some collaborators: L. Brits, J. Martin, B. Tippett, C. Van Den Broeck

Related papers: gr-qc/0506119 (CQG)
arXiv:1406.4039 (PRD)
arXiv:1510.01759 (PRD)

NSERC CRSNG

Black Holes: Two ways

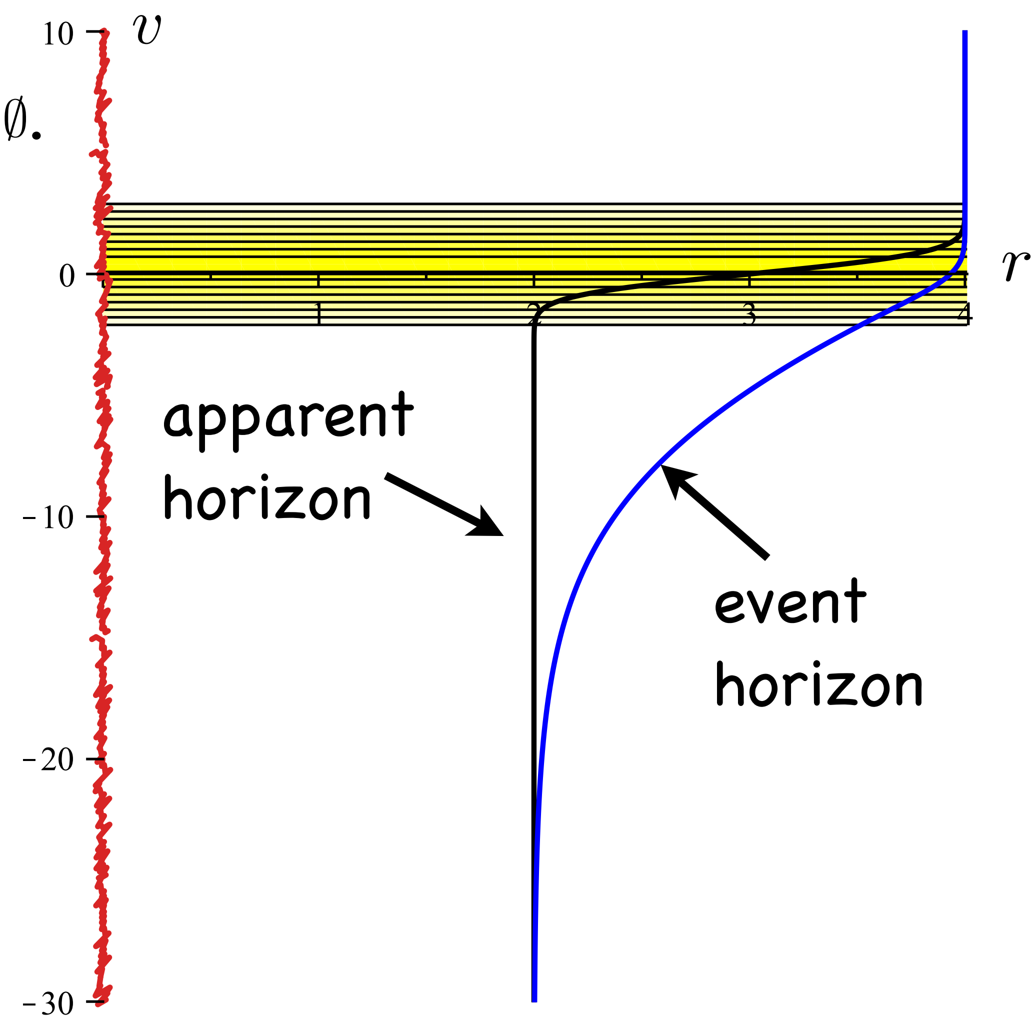
BH I: Causal black holes

- A black hole is a region of spacetime from which nothing ever escapes: $B = [M - J^{-}(\mathscr{I}^{+})] \neq \emptyset$. Boundary is the event horizon.

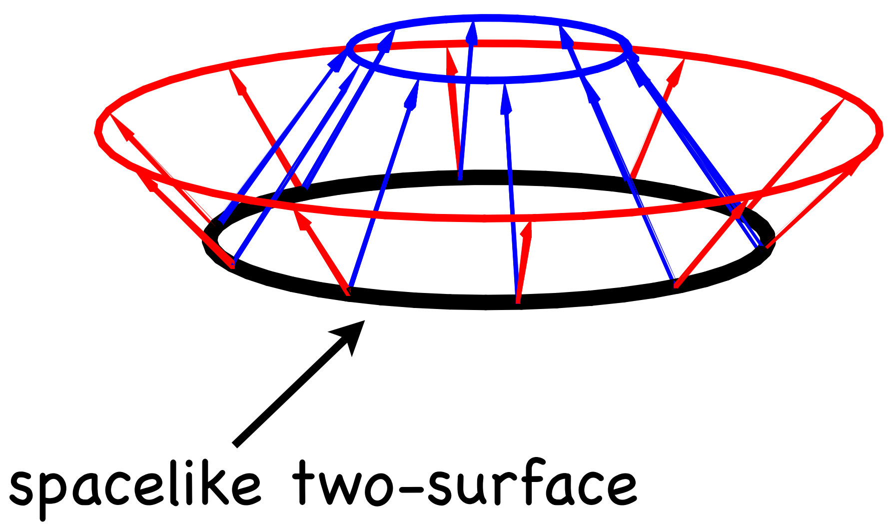
BH II: Geometric black holes

(apparent, isolated, trapping, dynamical horizons, holographic screen)

- Interior is made up of trapped surfaces
- Boundary is (intuitively) a marginally outer trapped surface (MOTS)



Trapped Surfaces

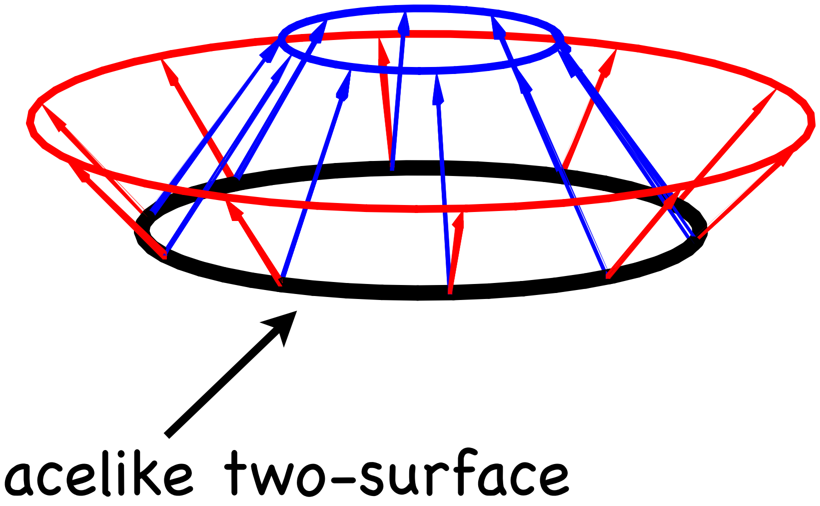


ℓ^a - outward null normal

n^a - inward null normal

- "Regular" convex surface (ie sphere):
- Trapped surface:
- Interior of stationary holes is trapped
- Trapped surfaces imply the existence of singularities and event horizons (Penrose 65)

Trapped Surfaces

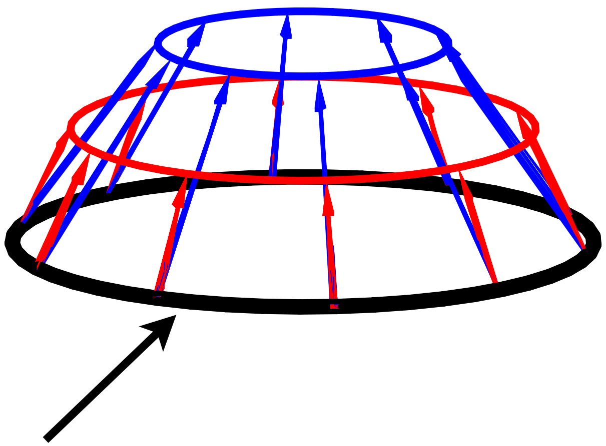


ℓ^a - outward null normal

n^a - inward null normal

- "Regular" convex surface (ie sphere): $\theta_{(\ell)} > 0$, $\theta_{(n)} < 0$
- Trapped surface:
- Interior of stationary holes is trapped
- Trapped surfaces imply the existence of singularities and event horizons (Penrose 65)

Trapped Surfaces



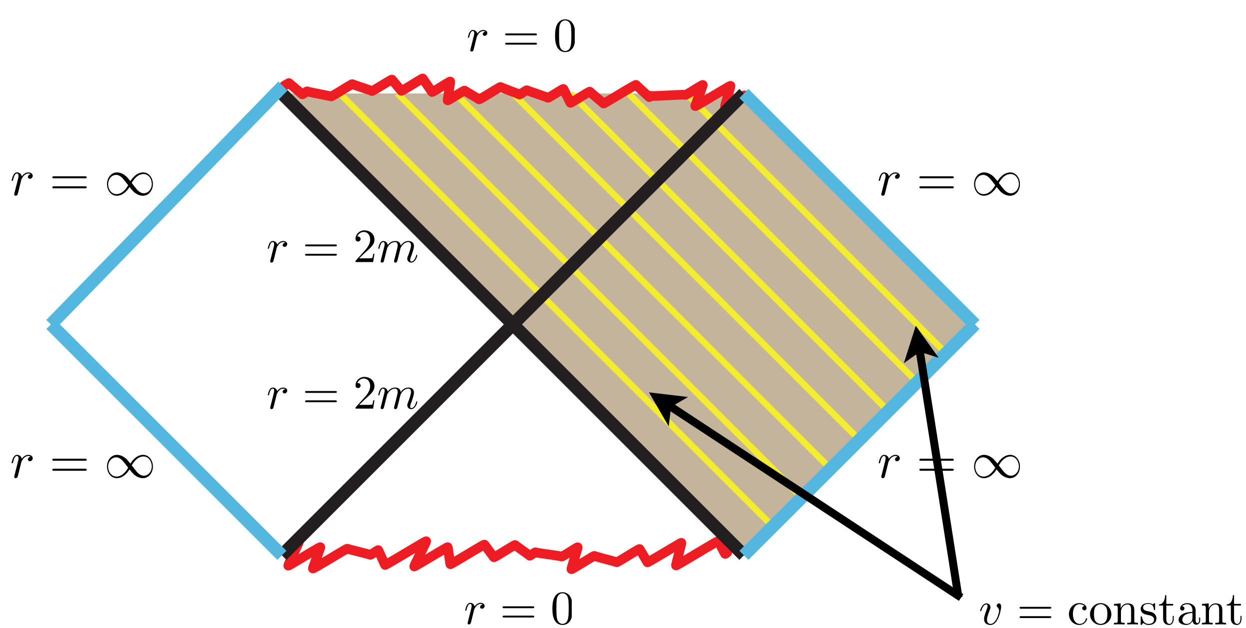
ℓ^a - outward null normal

n^a - inward null normal

- "Regular" convex surface (ie sphere): $\theta_{(\ell)} > 0\ ,\ \theta_{(n)} < 0$
- Trapped surface: $\theta_{(\ell)} < 0\ ,\ \theta_{(n)} < 0$
- Interior of stationary holes is trapped
- Trapped surfaces imply the existence of singularities and event horizons (Penrose 65)

Schwarzschild Black Holes

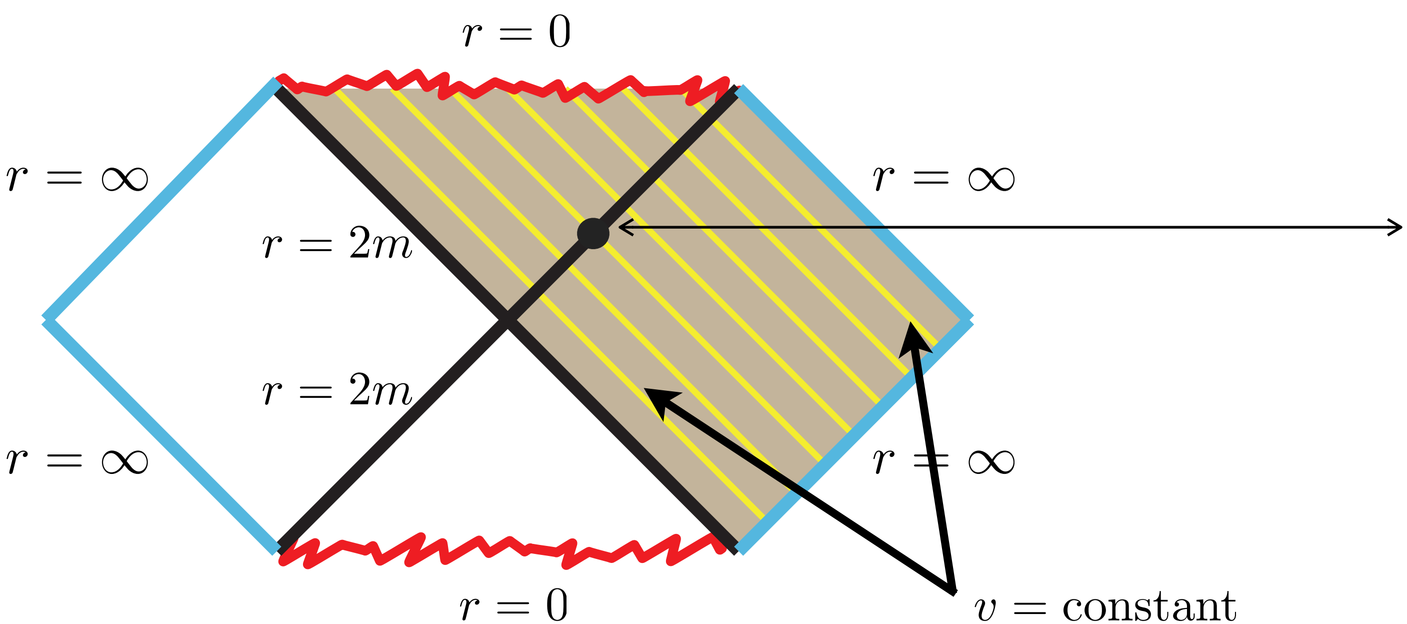
$$ds^2 = -\left(1 - \frac{2m}{r}\right) dv^2 + 2dvdr + r^2 d\Omega^2$$

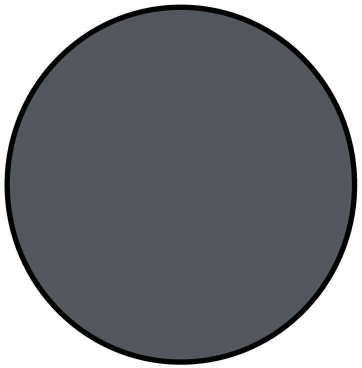


$$\left.\begin{array}{r} \theta_{(n)} < 0 \\ \mathcal{L}_n \theta_{(\ell)} < 0 \end{array}\right\} \text{for all } r = \text{constant surfaces}$$

Schwarzschild Black Holes

$$ds^2 = -\left(1 - \frac{2m}{r}\right) dv^2 + 2dvdr + r^2 d\Omega^2$$



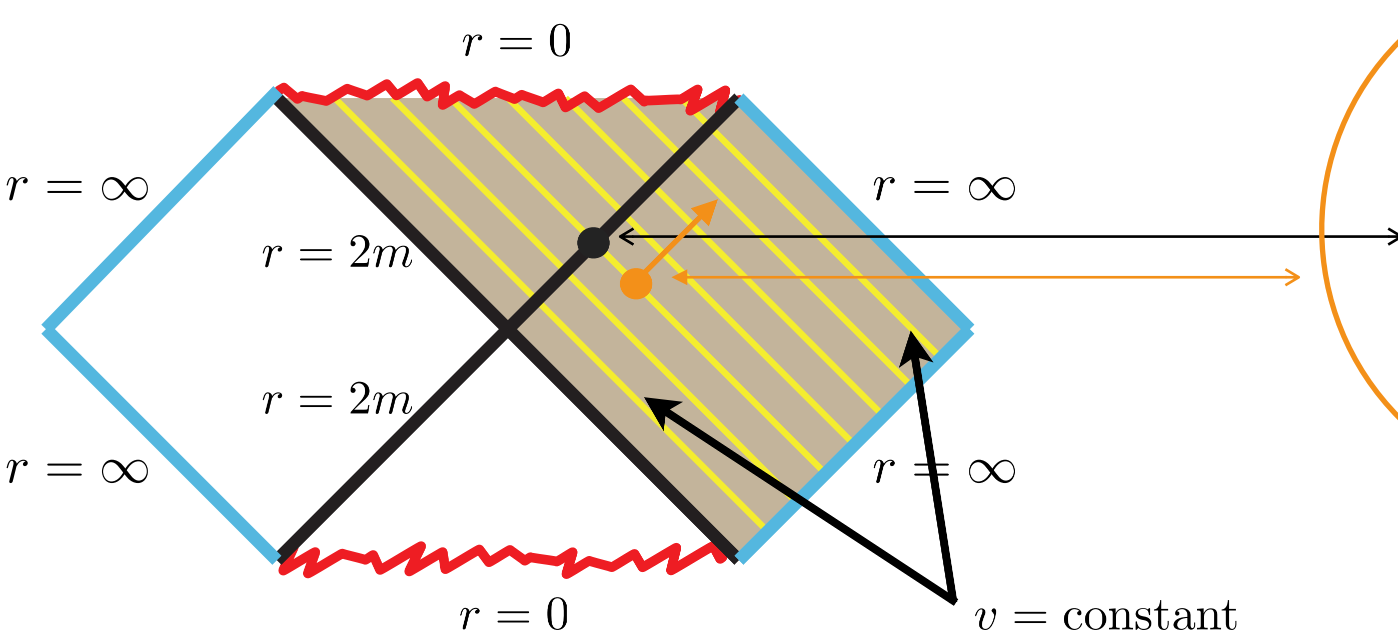


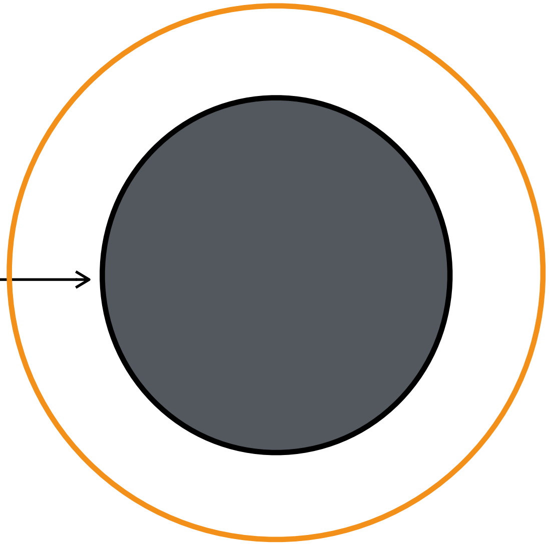
$\theta_{(\ell)} = 0$

$$\left.\begin{aligned} \theta_{(n)} &< 0 \\ \mathcal{L}_n \theta_{(\ell)} &< 0 \end{aligned}\right\} \text{for all } r = \text{constant surfaces}$$

Schwarzschild Black Holes

$$ds^2 = -\left(1 - \frac{2m}{r}\right) dv^2 + 2dvdr + r^2 d\Omega^2$$





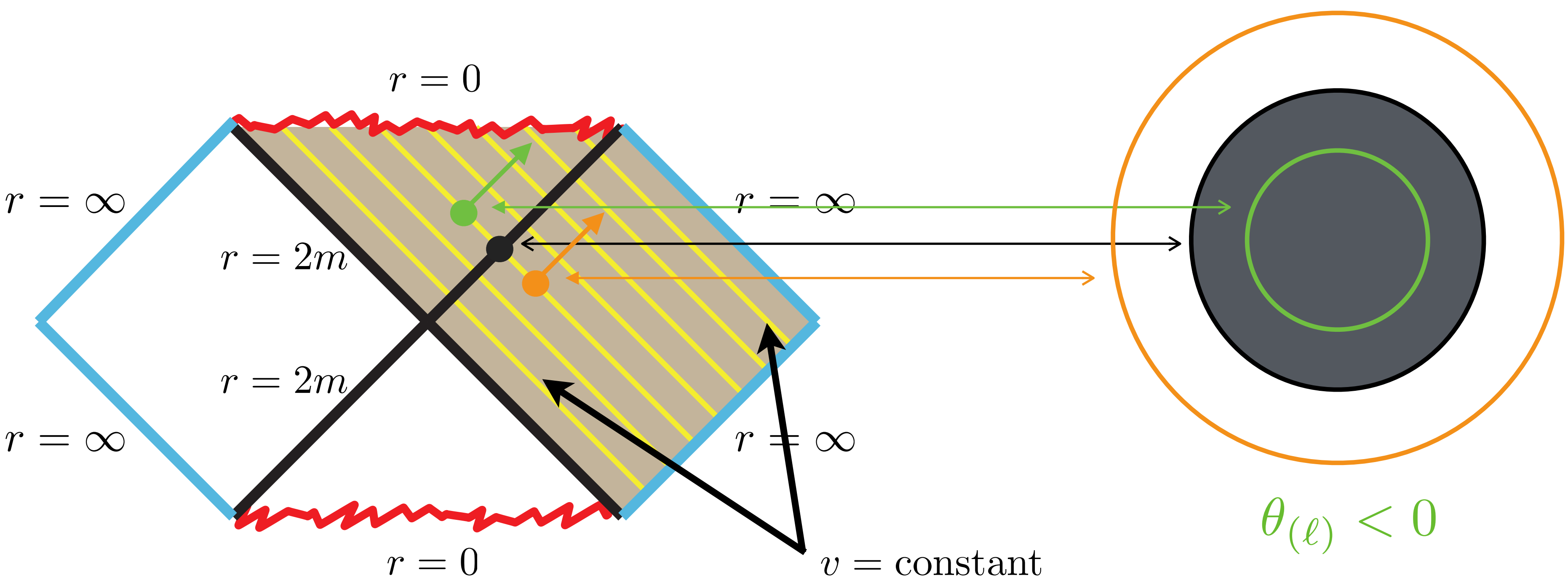
$\theta_{(\ell)} = 0$

$\theta_{(\ell)} > 0$

$$\left.\begin{aligned} \theta_{(n)} &< 0 \\ \mathcal{L}_n \theta_{(\ell)} &< 0 \end{aligned}\right\} \text{for all } r = \text{constant surfaces}$$

Schwarzschild Black Holes

$$ds^2 = -\left(1 - \frac{2m}{r}\right) dv^2 + 2dvdr + r^2 d\Omega^2$$



$\theta_{(\ell)} < 0$

$\theta_{(\ell)} = 0$

$\theta_{(\ell)} > 0$

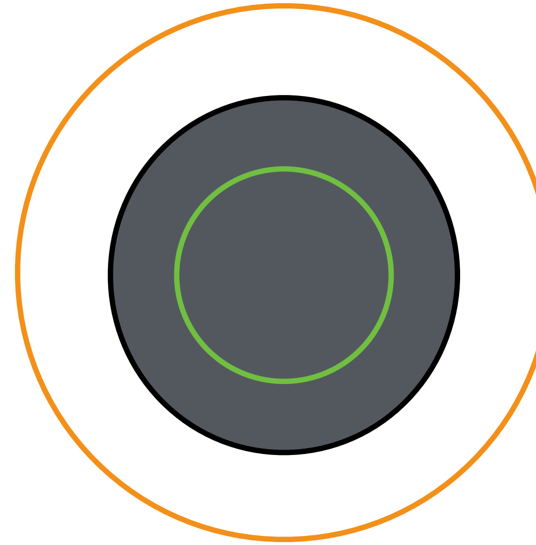
$$\left.\begin{array}{r} \theta_{(n)} < 0 \\ \mathcal{L}_n \theta_{(\ell)} < 0 \end{array}\right\} \text{for all } r = \text{constant surfaces}$$

Geometric Horizons

$$\theta_{(\ell)} = 0 \quad , \qquad \theta_{(n)} < 0 \quad , \qquad \mathcal{L}_n \theta_{(\ell)} < 0$$

marginally outer trapped (MOTS)

fully trapped surfaces inside (FOTH)



$\theta_{(\ell)} < 0$

$\theta_{(\ell)} = 0$

$\theta_{(\ell)} > 0$

Many well-known geometric horizon theorems from mathematical relativity depend on these properties:

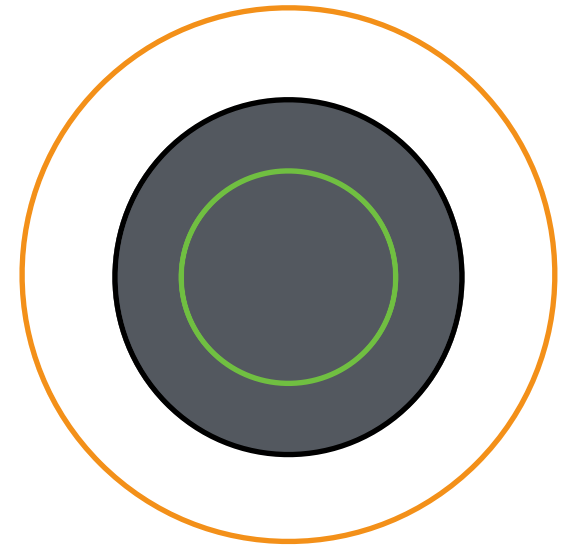
- Persistence of horizon and uniqueness of time-evolution Andersson, Mars, Simon (05)
- Area increase theorems: $\dot{A} \geq 0$ Hayward (93), Ashtekar-Krishnan (01), Bousso (15)
- Area bounds on charge and angular momentum: $Q^4 + 4J^2 \leq R_H^4$ Dain, Reiris, Jaramillo, Khuri et al (06+)

apparent horizon, FOTH, MTT, dynamical horizon, isolated horizon, strictly stably outermost MOTS, holographic screens

Geometric Horizons

$$\theta_{(\ell)} = 0 \;, \qquad \theta_{(n)} < 0 \;, \qquad \mathcal{L}_n \theta_{(\ell)} < 0$$

marginally outer trapped (MOTS) — fully trapped surfaces inside (FOTH)



$\theta_{(\ell)} < 0$

$\theta_{(\ell)} = 0$

$\theta_{(\ell)} > 0$

Many well-known geometric horizon theorems from mathematical relativity depend on these properties:

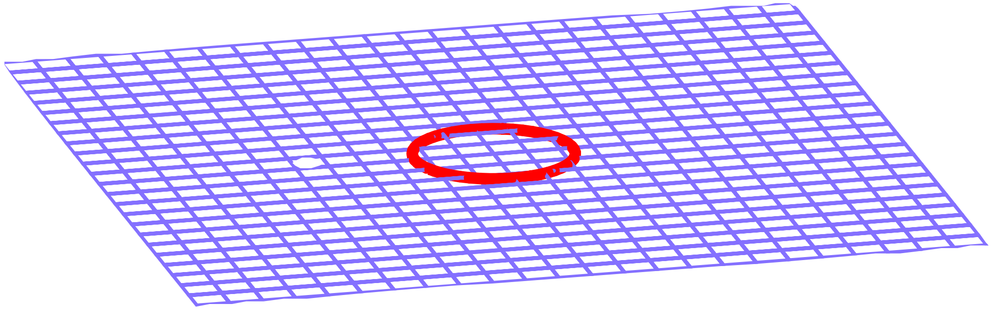
- Persistence of horizon and uniqueness of time-evolution Andersson, Mars, Simon (05)
- Area increase theorems: $\dot{A} \geq 0$
  Hayward (93), Ashtekar-Krishnan (01), Bousso (15)
- Area bounds on charge and angular momentum:
  $Q^4 + 4J^2 \leq R_H^4$ Dain, Reiris, Jaramillo, Khuri et al (06+)

apparent horizon, FOTH, MTT, dynamical horizon, isolated horizon, strictly stably outermost MOTS, holographic screens

The Andersson-Mars-Simon Theorem

$\mathcal{L}_n \theta_{(\ell)} < 0 \implies \exists$ spacelike $\hat{r}$ st $\mathcal{L}_{\hat{r}} \theta_{(\ell)} < 0$ = strictly stably outermost

IF a strictly stably outermost MOTS exists on one leaf of a smooth foliation of a spacetime

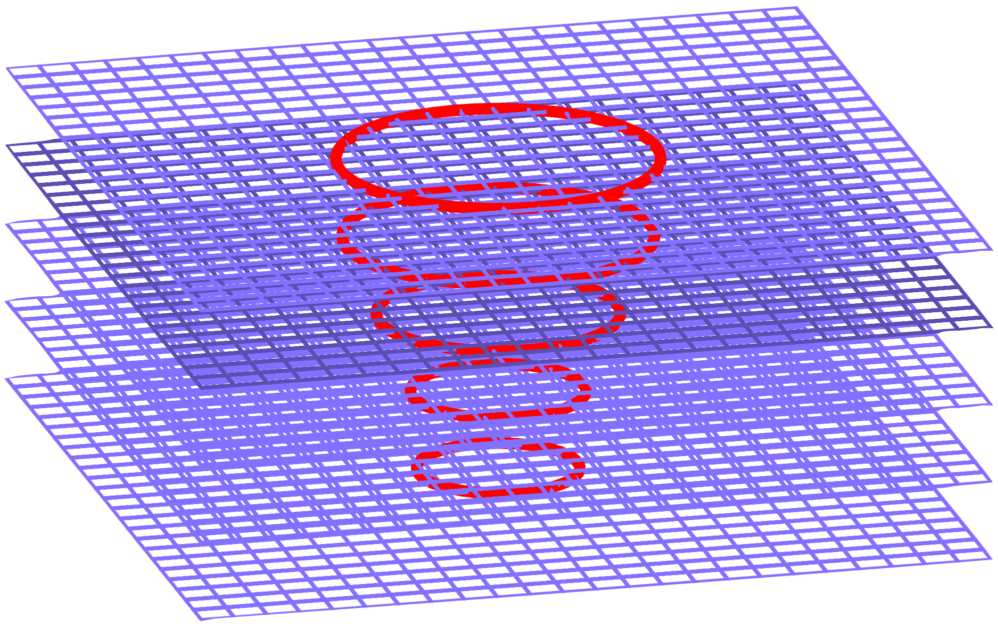


The Andersson-Mars-Simon Theorem

$\mathcal{L}_n \theta_{(\ell)} < 0 \implies \exists$ spacelike $\hat{r}$ st $\mathcal{L}_{\hat{r}} \theta_{(\ell)} < 0$ = strictly stably outermost

IF a strictly stably outermost MOTS exists on one leaf of a smooth foliation of a spacetime

THEN MOTS exist on future slices



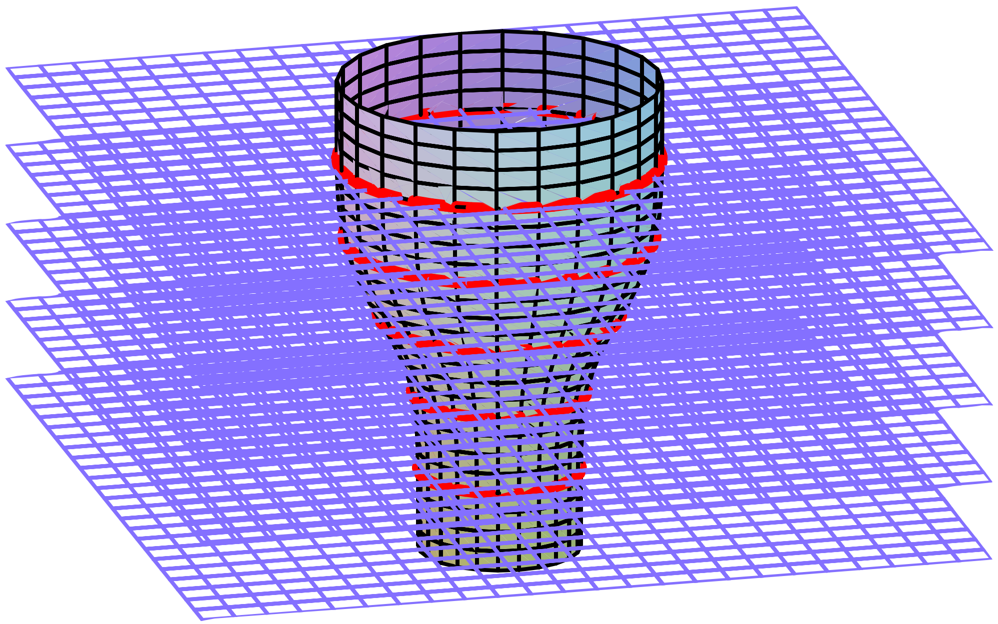
The Andersson-Mars-Simon Theorem

$\mathcal{L}_n \theta_{(\ell)} < 0 \Longrightarrow \exists$ spacelike $\hat{r}$ st $\mathcal{L}_{\hat{r}} \theta_{(\ell)} < 0$ = strictly stably outermost

IF a strictly stably outermost MOTS exists on one leaf of a smooth foliation of a spacetime

THEN MOTS exist on future slices

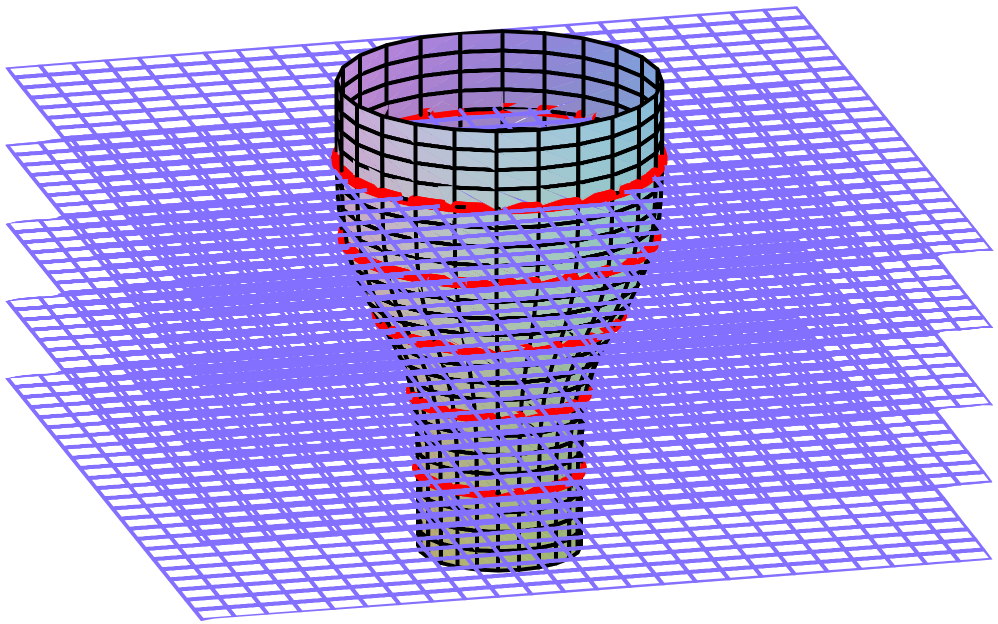
AND form a smooth 3D geometric horizon which exists at least as long as the MOTS remain strictly stably outermost.



The Andersson-Mars-Simon Theorem

$\mathcal{L}_n \theta_{(\ell)} < 0 \Longrightarrow \exists$ spacelike $\hat{r}$ st $\mathcal{L}_{\hat{r}} \theta_{(\ell)} < 0$ = strictly stably outermost

IF a strictly stably outermost MOTS exists on one leaf of a smooth foliation of a spacetime



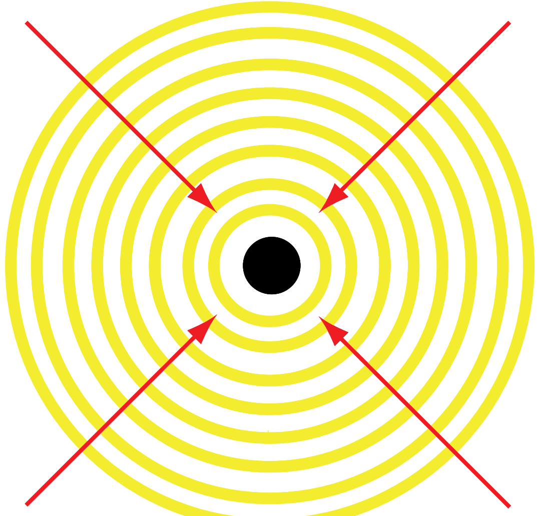
THEN MOTS exist on future slices

AND form a smooth 3D geometric horizon which exists at least as long as the MOTS remain strictly stably outermost.

GIVEN the null energy condition it is: i) null if isolated
(Hayward 93) ii) spacelike if dynamical

(Basic) Example #1: Vaidya

$$ds^2 = -\left(1 - \frac{2m(v)}{r}\right) dv^2 + 2dvdr + r^2 d\Omega^2$$



Describes infalling shells of null dust with density: $\mu = \frac{dm}{dv}$

falling along v = constant curves with tangent vector: $n = -\frac{\partial}{\partial r}$

Outward null $\ell = \frac{\partial}{\partial v} + \frac{1}{2}\left(1 - \frac{2m}{r}\right)\frac{\partial}{\partial r}$

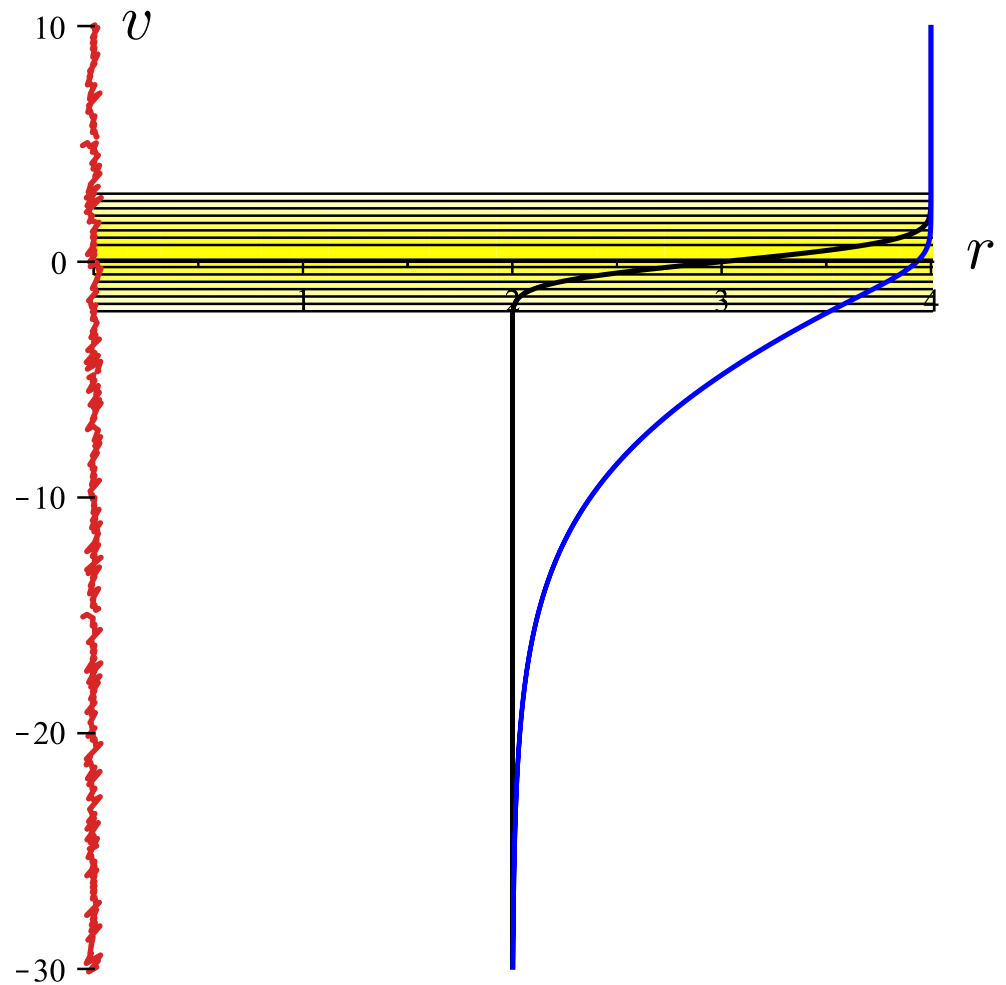
Then $\theta_{(\ell)} = 0 \Leftrightarrow r = 2m(v)$, $\theta_{(n)} < 0$ and $\mathcal{L}_n \theta_{(\ell)} = -\frac{1}{r^2} < 0$

All are strictly stably outermost

(Basic) Example #1: Vaidya

So with $m = \frac{1}{2}\left(3 + \mathrm{erf}(v/L)\right)$:

$$L = 1$$

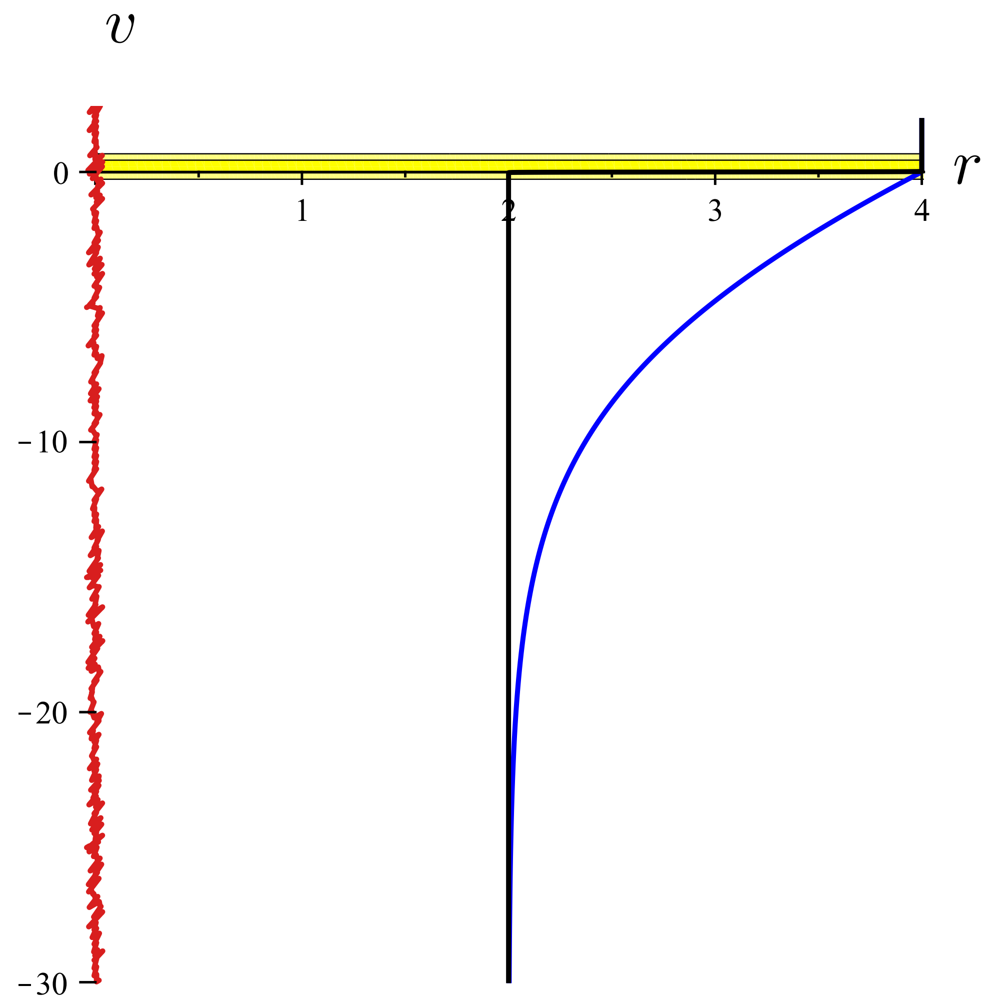


All are space-like and expanding

(Basic) Example #1: Vaidya

So with $m = \frac{1}{2}\left(3 + \mathrm{erf}(v/L)\right)$:

$$L = 1/100$$

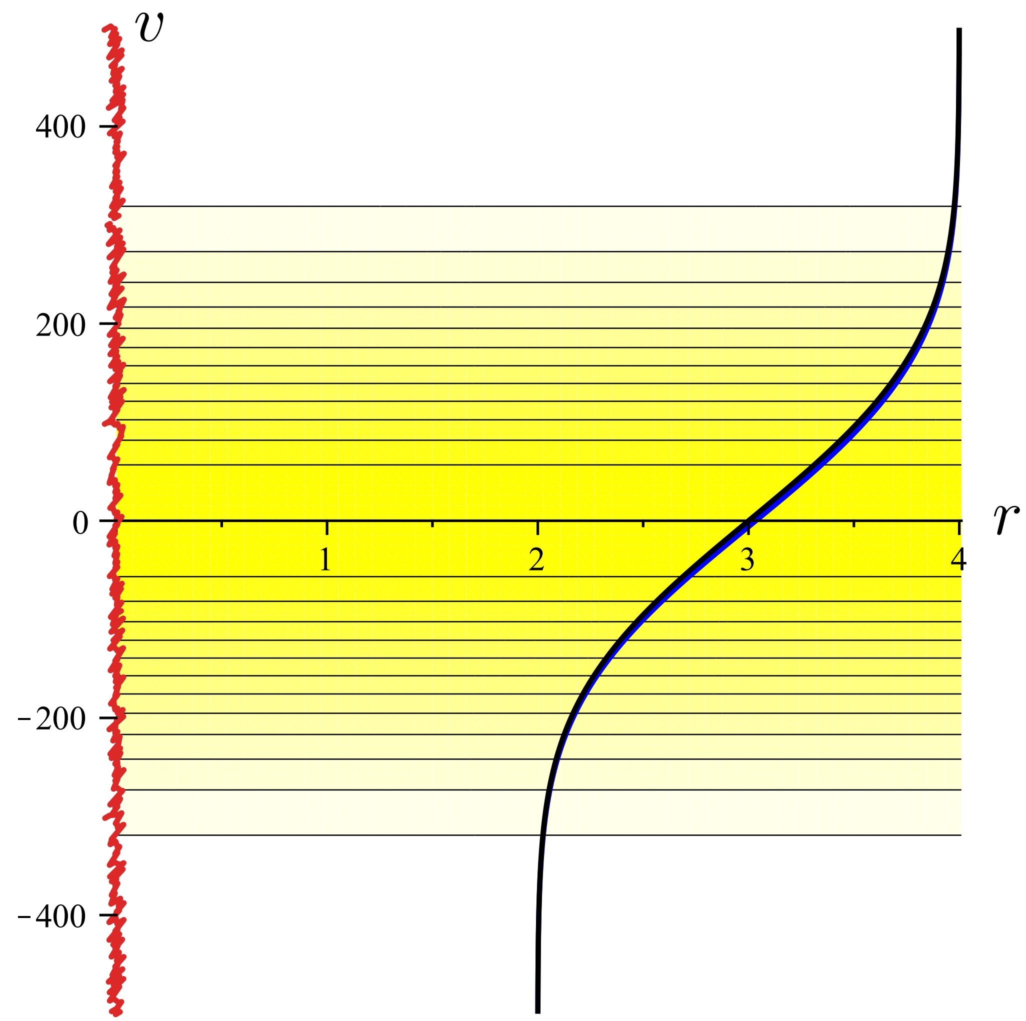


All are space-like and expanding

(Basic) Example #1: Vaidya

So with $m = \frac{1}{2}(3 + \mathrm{erf}(v/L))$:

$$L = 200$$

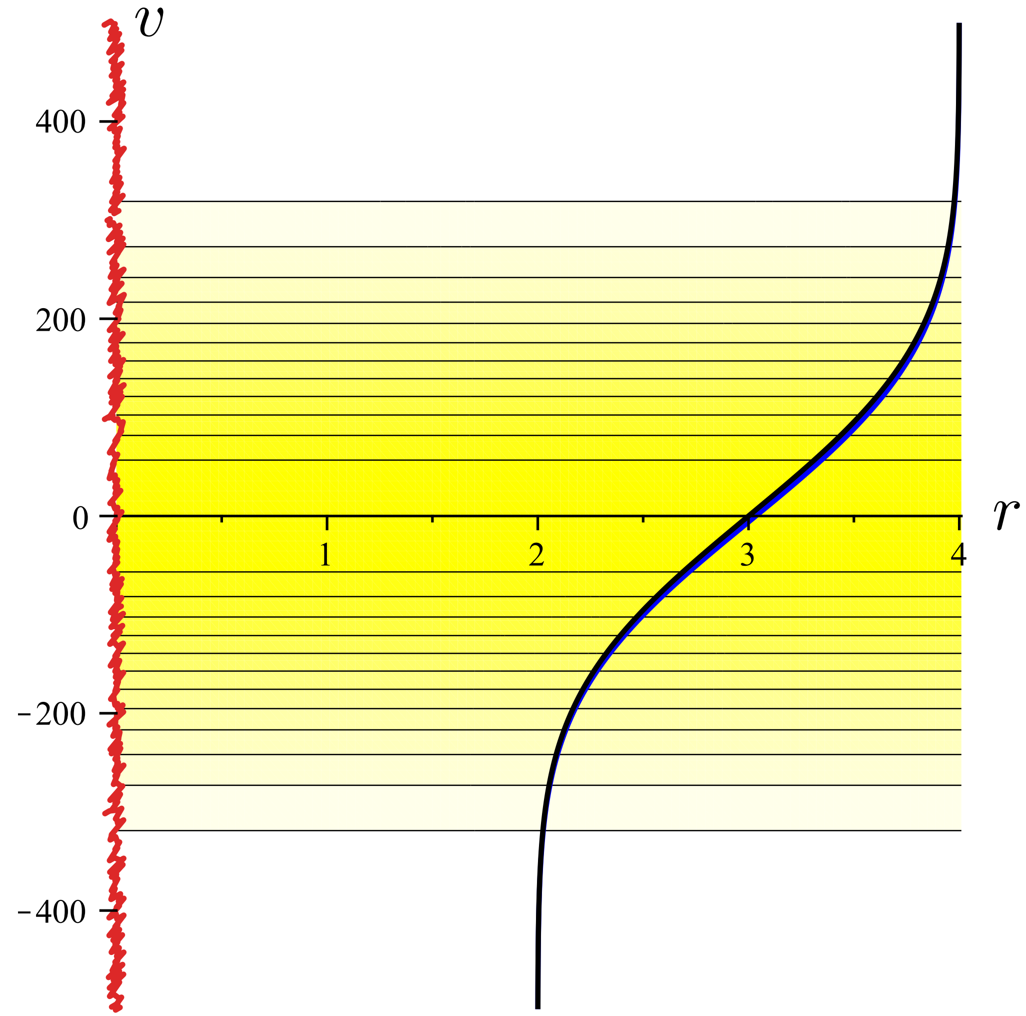


All are space-like and expanding

(Basic) Example #1: Vaidya

So with $m = \frac{1}{2}\left(3 + \text{erf}(v/L)\right)$:

$$L = 200$$

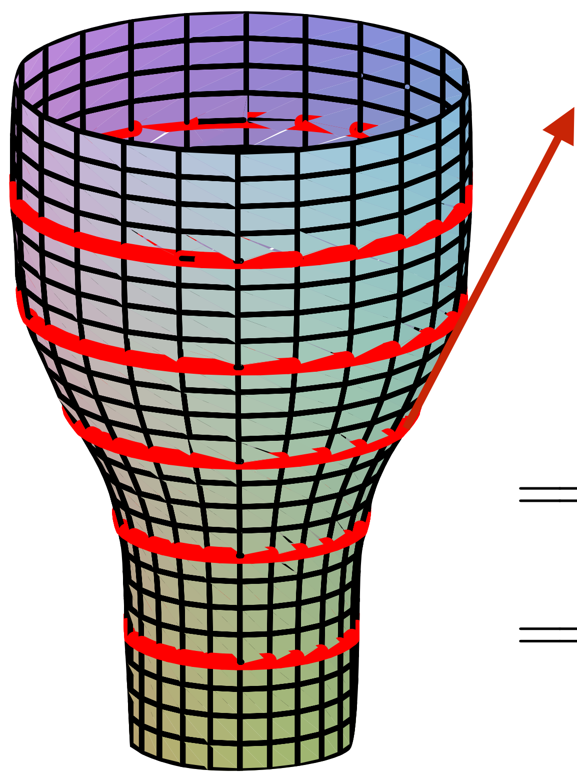


All are space-like and expanding

Can we get more interesting evolutions?

(Hayward 93)

Spherical horizon dynamics I



For a tangent/evolution vector:

$$\mathcal{V}^a = \ell^a - C n^a$$

evolution parameter

we have

$$\mathcal{L}_{\mathcal{V}} \theta_{(\ell)} = 0$$

$$\implies \mathcal{L}_\ell \theta_{(\ell)} - C \mathcal{L}_n \theta_{(\ell)} = 0$$

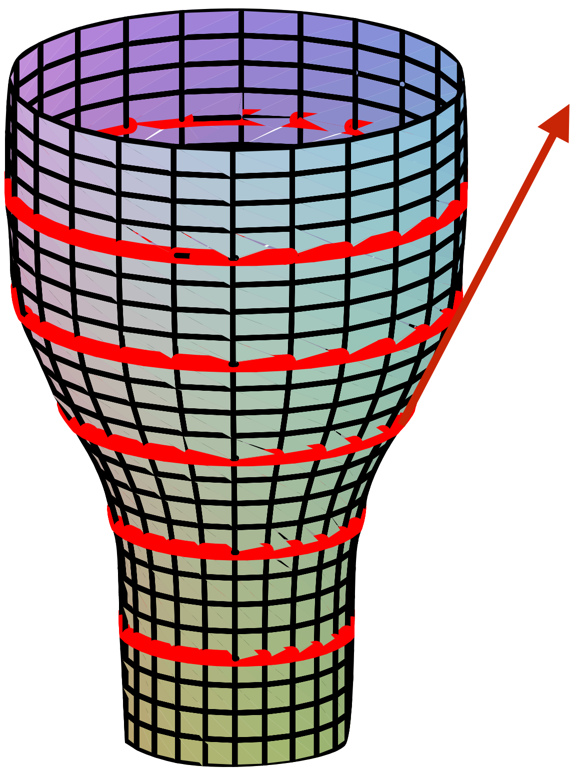
$$\implies C = \frac{\mathcal{L}_\ell \theta_{(\ell)}}{\mathcal{L}_n \theta_{(\ell)}}$$

$$\implies C = \frac{G_{ab}\ell^a \ell^b}{1/r^2 - G_{ab}\ell^a n^b}$$

by the null energy condition this is positive unless $G_{ab}\ell^a n^b > 1/r^2$

ie $\mathcal{L}_n \theta_{(\ell)} > 0 \implies$ not strictly stably outermost

Spherical horizon dynamics II



For a tangent vector:

$$\mathcal{V}^a = \ell^a - C n^a$$

Then C determines the geometry

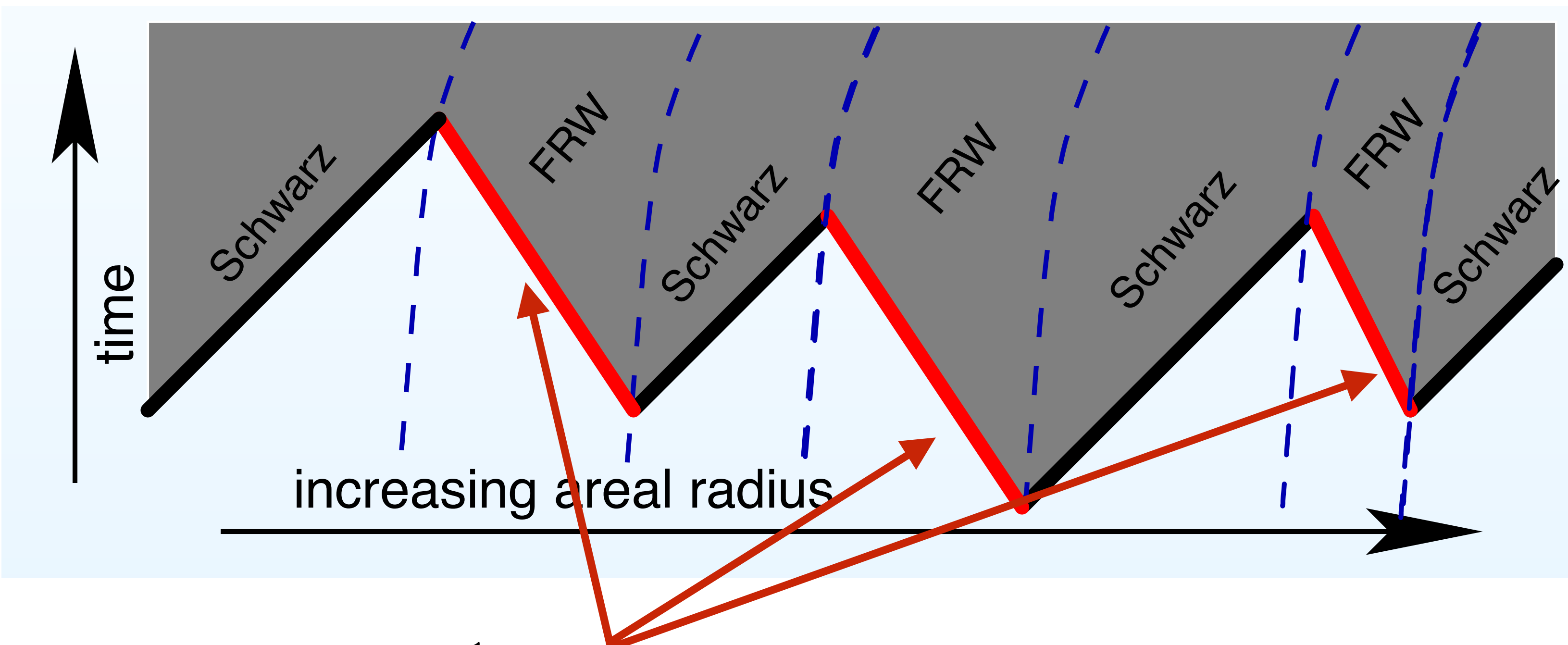
- signature: $\mathcal{V} \cdot \mathcal{V} = 2C$
- expansion: $\mathcal{L}_{\mathcal{V}} \sqrt{\tilde{q}} = -\sqrt{\tilde{q}} C \theta_{(n)}$

$$C = \frac{G_{ab}\ell^a \ell^b}{1/r^2 - G_{ab}\ell^a n^b}$$

$C = 0$	null	not-expanding
$C > 0$	spacelike	expanding
$C < 0$	timelike	contracting

Example 2: Schwarz-FRW

- Ben-dov (2004) demonstrated a very different behaviour by cutting and pasting Schwarzschild and (collapsing) FRW



$$G_{ab}\ell^a n^b > \frac{1}{r^2} \implies \mathcal{L}_n \theta_{(\ell)} > 0$$

- Are there smooth spacetimes exhibiting timelike sections?

Example 3: Tolman-Bondi (smooth)

- Lemaitre-Tolman-Bondi describes the collapse of timelike dust

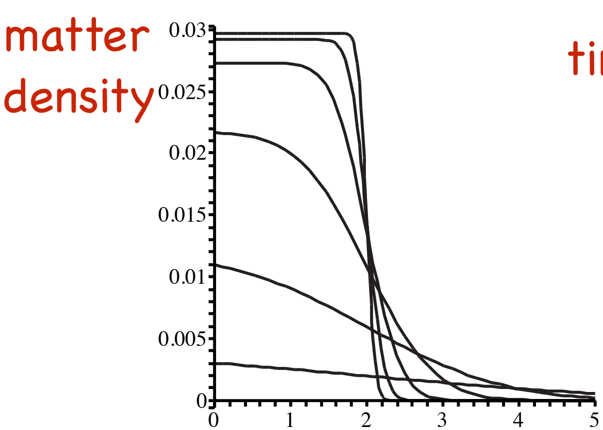
$$ds^2 = -dt^2 + \left(\frac{B(r_o,t)}{A(r_o,t)^{1/3}}\right)^2 dr_o^2 + R(r_o,t)^2 d\Omega^2$$ (Yodzis 70s)

where $A(r_o,t)$, $B(r_o,t)$ and $R(r_o,t)$ are explicit functions of (r_o,t)

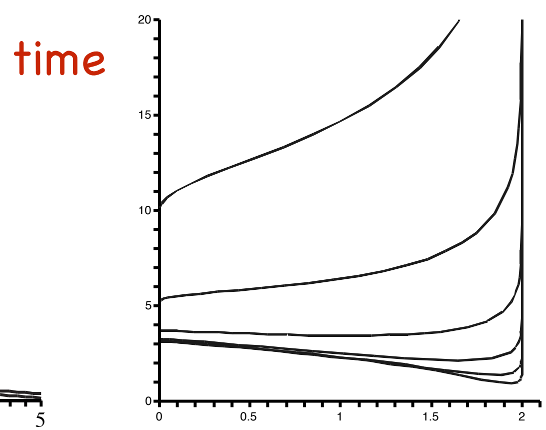
and $m(r_o) = \int_0^{r_o}\int_0^{\pi}\int_0^{2\pi} \sqrt{h}\rho \, dr \, d\theta \, d\phi$ (initial mass distribution)

- MOTS at $\theta_{(\ell)} = 0 \iff R(r_o,t) = 2m(r_o)$

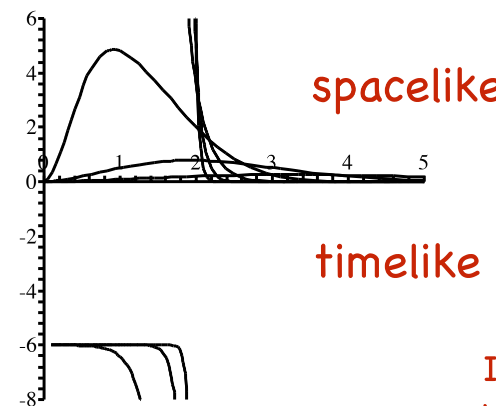
- Allows us to evolve dust/spacetime and track geometric horizons



Mass distribution



MTT evolution



MTT signature

IB, Brits, Gonzalez, Van Den Broeck CQG 2006

Example 3: Tolman-Bondi (smooth)

- Lemaitre-Tolman-Bondi describes the collapse of timelike dust

$$ds^2 = -dt^2 + \left(\frac{B(r_o,t)}{A(r_o,t)^{1/3}}\right)^2 dr_o^2 + R(r_o,t)^2 d\Omega^2$$ (Yodzis 70s)

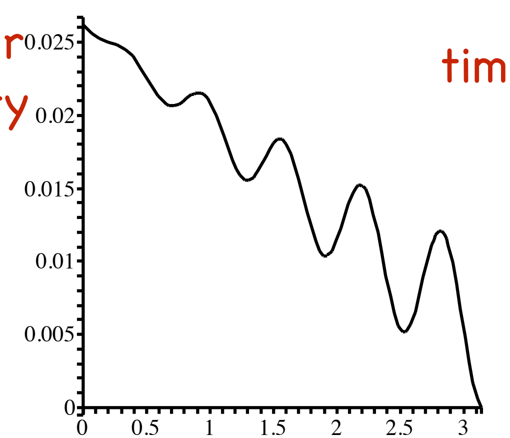
where $A(r_o,t)$, $B(r_o,t)$ and $R(r_o,t)$ are explicit functions of (r_o,t)

and $m(r_o) = \int_0^{r_o}\int_0^{\pi}\int_0^{2\pi} \sqrt{h}\rho\, dr\, d\theta\, d\phi$ (initial mass distribution)

- MOTS at $\theta_{(\ell)} = 0 \iff R(r_o,t) = 2m(r_o)$

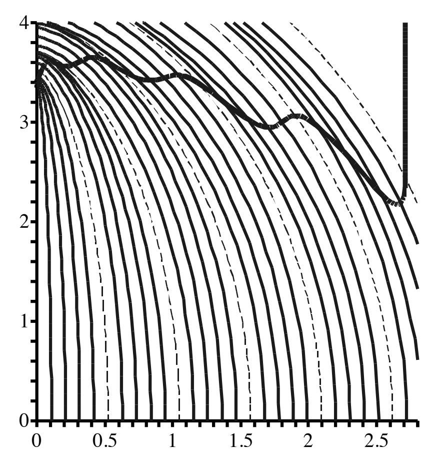
- Allows us to evolve dust/spacetime and track geometric horizons

matter density



Mass distribution

time



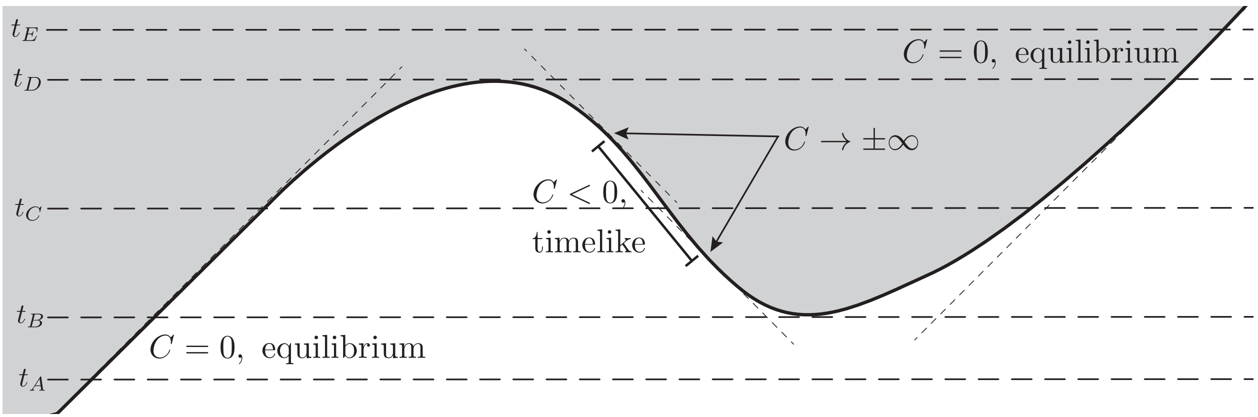
MTT evolution

MTT signature

IB, Brits, Gonzalez, Van Den Broeck CQG 2006

Collapse of timelike dust - schematic

- Timelike sections = either horizon "jumps" or creations/annihilations



$$G_{ab} = 8\pi \rho u_a u_b$$

$$G_{ab}\ell^a \ell^b = \frac{4\pi}{\xi^2}\rho$$

$$G_{ab}\ell^a n^b = 4\pi\rho$$

$$C = \frac{G_{ab}\ell^a\ell^b}{1/r^2 - G_{ab}\ell^a n^b}$$

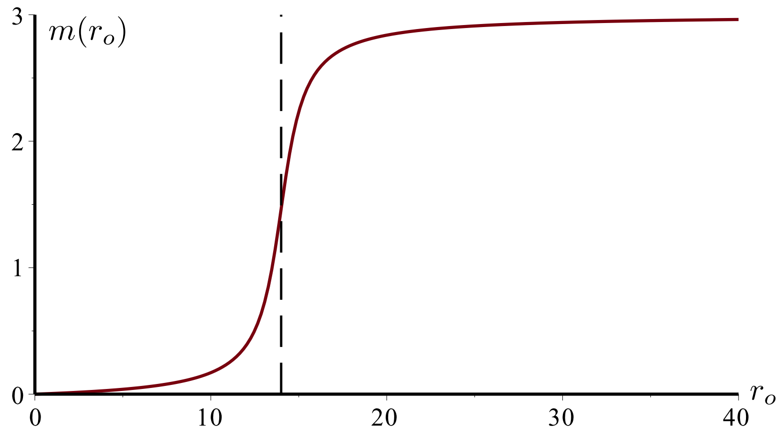
$$\rho > \frac{1}{A} \quad \Rightarrow \text{ spacelike}$$

- Doesn't violate AMS existence theorem as it isn't strictly stably outermost in interesting sections
- There can be shell-crossing singularities

Aside: There may be critical exponents associated with jumps... Cao, Cai, Yang arXiv:1604.03363

Example 4: Tolman-Bondi (shockwaves)

(B.Tippett and IB, 2014)



$$m(r_o) \equiv m_o + \mu \cdot \left[\frac{\arctan(\Gamma(r_o - \bar{r}_o))}{\pi/2 + \arctan(\Gamma \bar{r}_o)} + \frac{\arctan(\Gamma \bar{r}_o)}{\pi/2 + \arctan(\Gamma \bar{r}_o)} \right]$$

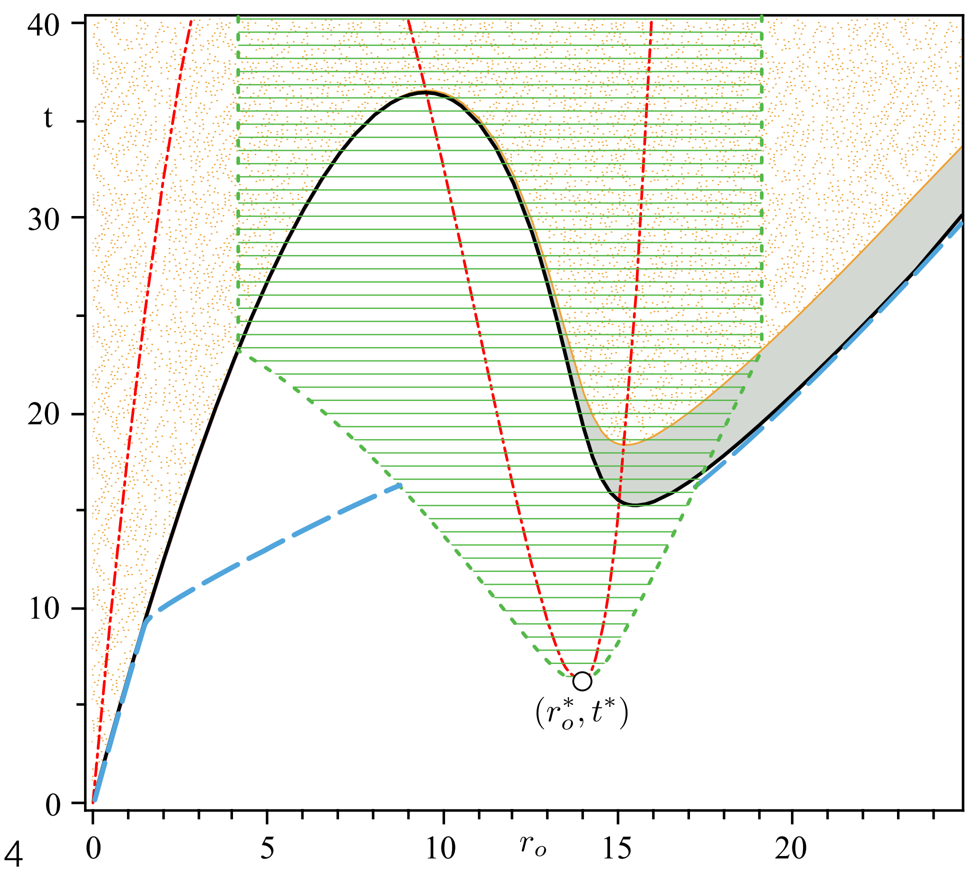
$$m_o = M_\odot \qquad \mu = 3M_\odot$$
$$\bar{r}_o = 14M_\odot \qquad \Gamma = 1$$

- It is straightforward to identify shell-focussing and shell-crossing singularities:

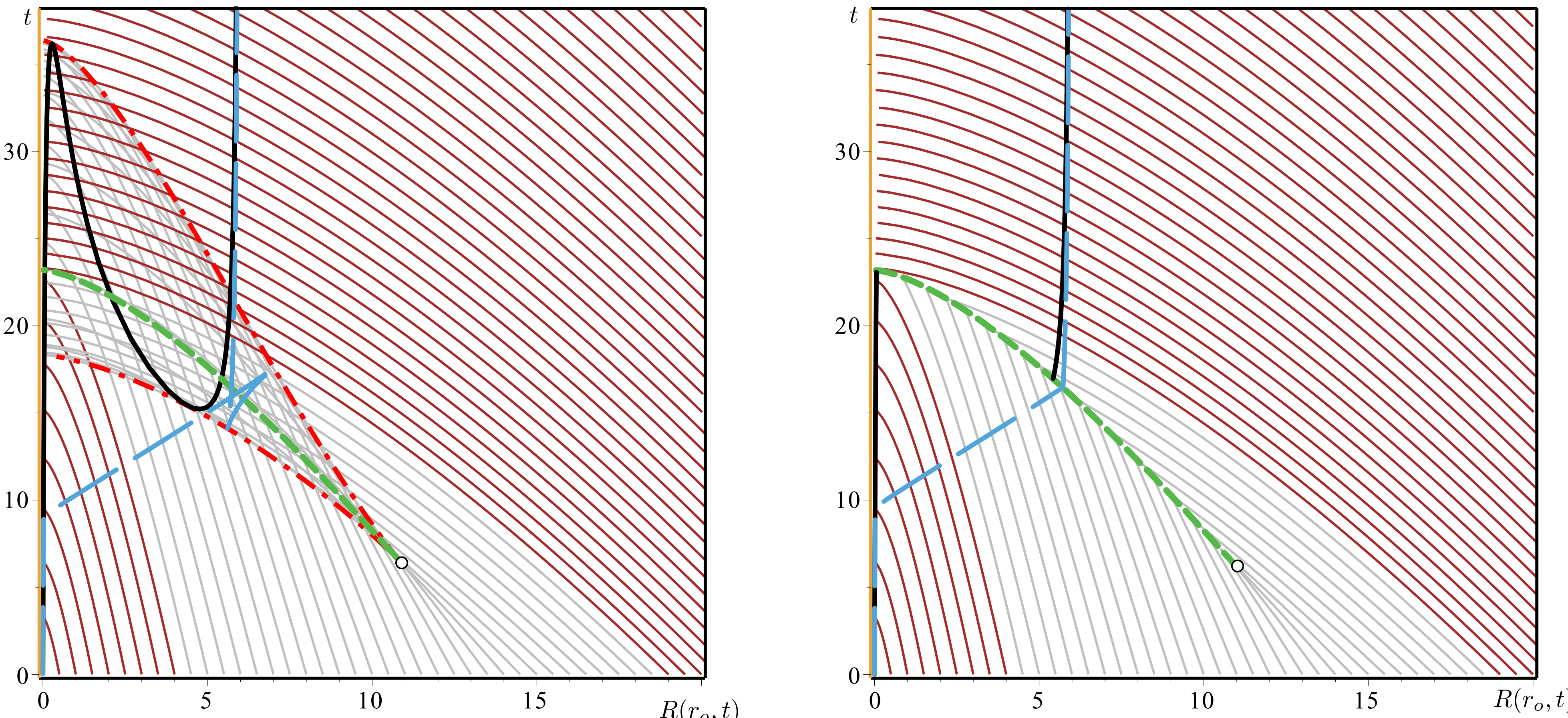
$$R_{SFS} = 0 \qquad R'_{SCS} = 0$$

- These mark out coordinate regions where the spacetime is ill-defined

- These singularities can be replaced by shockwaves



Shockwaves II



(a) Pre-excision (b) Post-excision

- Remove by identifying R=constant points (motivated by shockwave physics) (Nolan 2003)
- Darmois-Israel junction conditions => thin shell of matter remains
- Geometric horizon "jumps" - it can disappear into and reappear out of singularity

Example 5: Evolutions from extremality

Vaidya RN: $$ds^2 = -\left(1 - \frac{2m(v)}{r} + \frac{q(v)^2}{r^2}\right) dv^2 + 2dvdr + r^2 d\Omega^2$$

$$A = \frac{q(v)}{r} dv$$

Then the stress-energy tensor is:

$$T_{ab} = \mu [dv]_a \otimes [dv]_b + T^{\text{EM}}_{ab} \quad \text{for} \quad \mu = \frac{1}{4\pi r^3}(\dot{m}r - q\dot{q})$$

energy conditions

$$\mu \geq 0$$

at extremality

$$\frac{d}{dv}\left(q^2\right) \leq \frac{d}{dv}\left(m^2\right)$$

Null vectors: $$\ell = \frac{\partial}{\partial v} + \frac{1}{2}\left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right)\frac{\partial}{\partial r} \text{ and}$$

$$n = -\frac{\partial}{\partial r}\,.$$

MOTS location: $\theta_{(\ell)} = 0 \implies r^2 - 2m(v)r + q(v)^2 = 0$

Example 5: Evolutions from extremality

Vaidya RN: $$ds^2 = -\left(1 - \frac{2m(v)}{r} + \frac{q(v)^2}{r^2}\right) dv^2 + 2dvdr + r^2 d\Omega^2$$
$$A = \frac{q(v)}{r} dv$$

Then the stress-energy tensor is:

$$T_{ab} = \mu [dv]_a \otimes [dv]_b + T^{\text{EM}}_{ab}$$ for $$\mu = \frac{1}{4\pi r^3}(\dot{m} r - q\dot{q})$$

energy conditions

$$\mu \geq 0$$

at extremality

$$\frac{d}{dv}(q^2) \leq \frac{d}{dv}(m^2)$$

Null vectors: $$\ell = \frac{\partial}{\partial v} + \frac{1}{2}\left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right)\frac{\partial}{\partial r} \text{ and}$$
$$n = -\frac{\partial}{\partial r} .$$

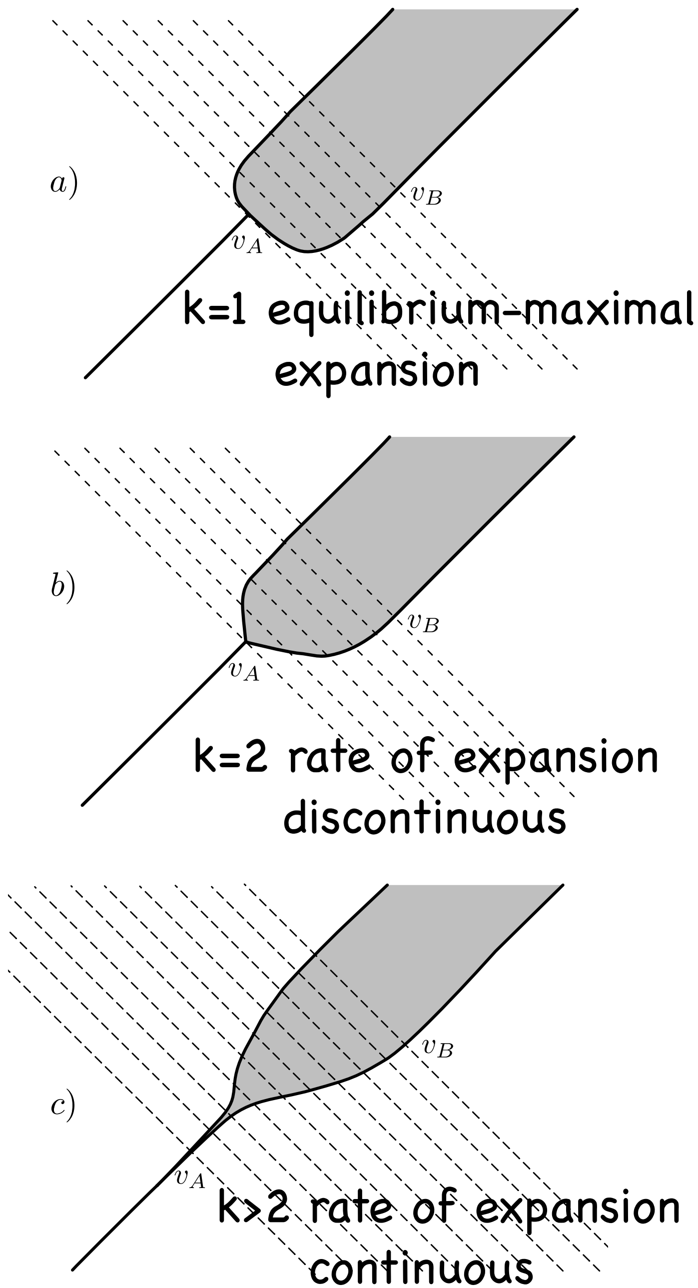
MOTS location: $\theta_{(\ell)} = 0 \implies r^2 - 2m(v)r + q(v)^2 = 0$

$$\mathcal{L}_n \theta_{(\ell)} = 0$$

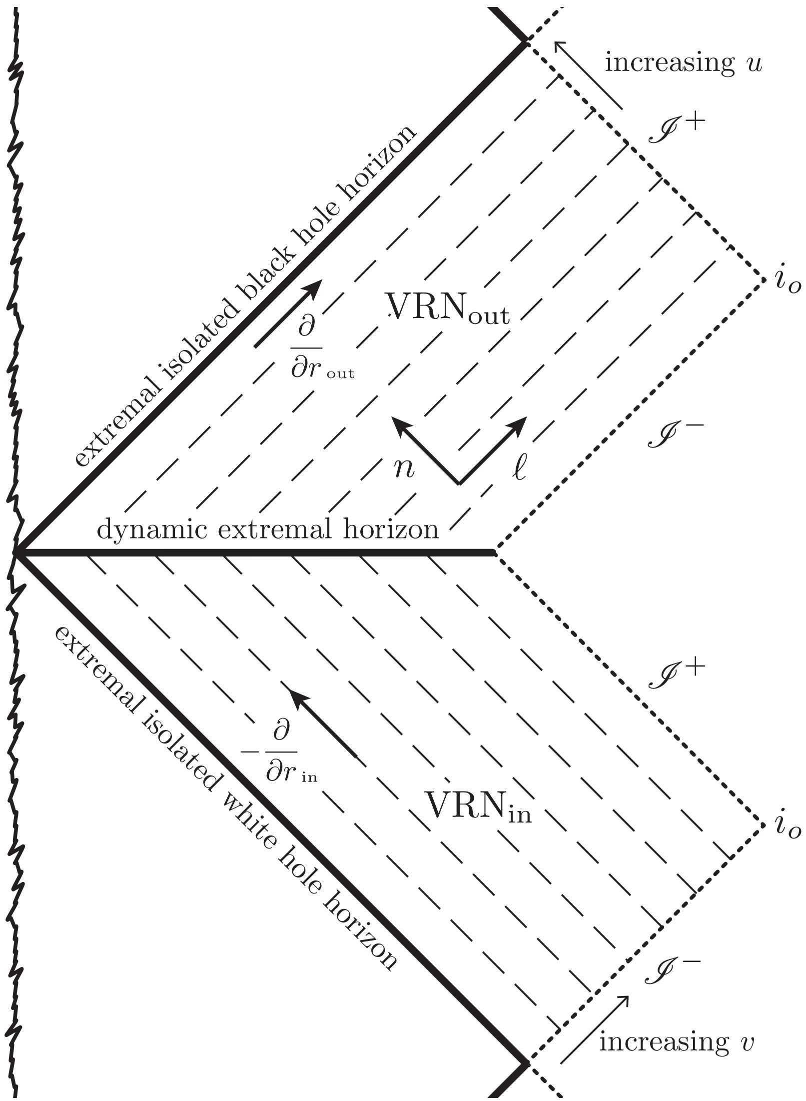
Exits from extremality

- Take limits of $C_\pm = \pm \left(\frac{r_\pm \dot{m} - q\dot{q}}{\sqrt{m^2 - q^2}} \right)$
- Horizons bifurcate
- Doesn't violate unique evolution theorems due to extremality

$$m(v) = m_o \left(1 + (v/v_o)^k + O\left(\frac{v}{v_o}\right)^{k+1} \right)$$



Evolutions at extremality

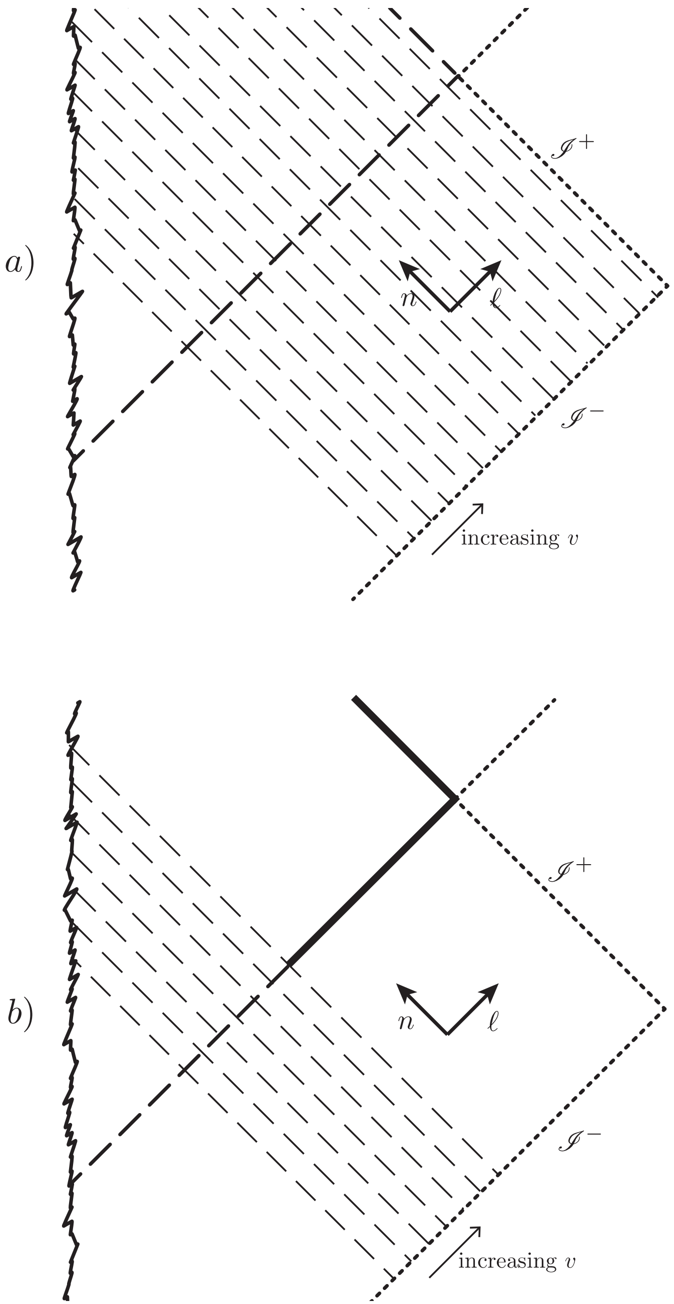


- When extremal matter falls onto an extremal horizon a dynamical extremal horizon results
- Some care needs to be taken in constructing and interpreting these solutions. Recall:

$$\mu = \frac{1}{4\pi r^3}(\dot{m}r - q\dot{q})$$

- Full solution pastes together ingoing and outgoing Vaidya RN
- There is a thin shell along the horizon... Ori, CQG 1991

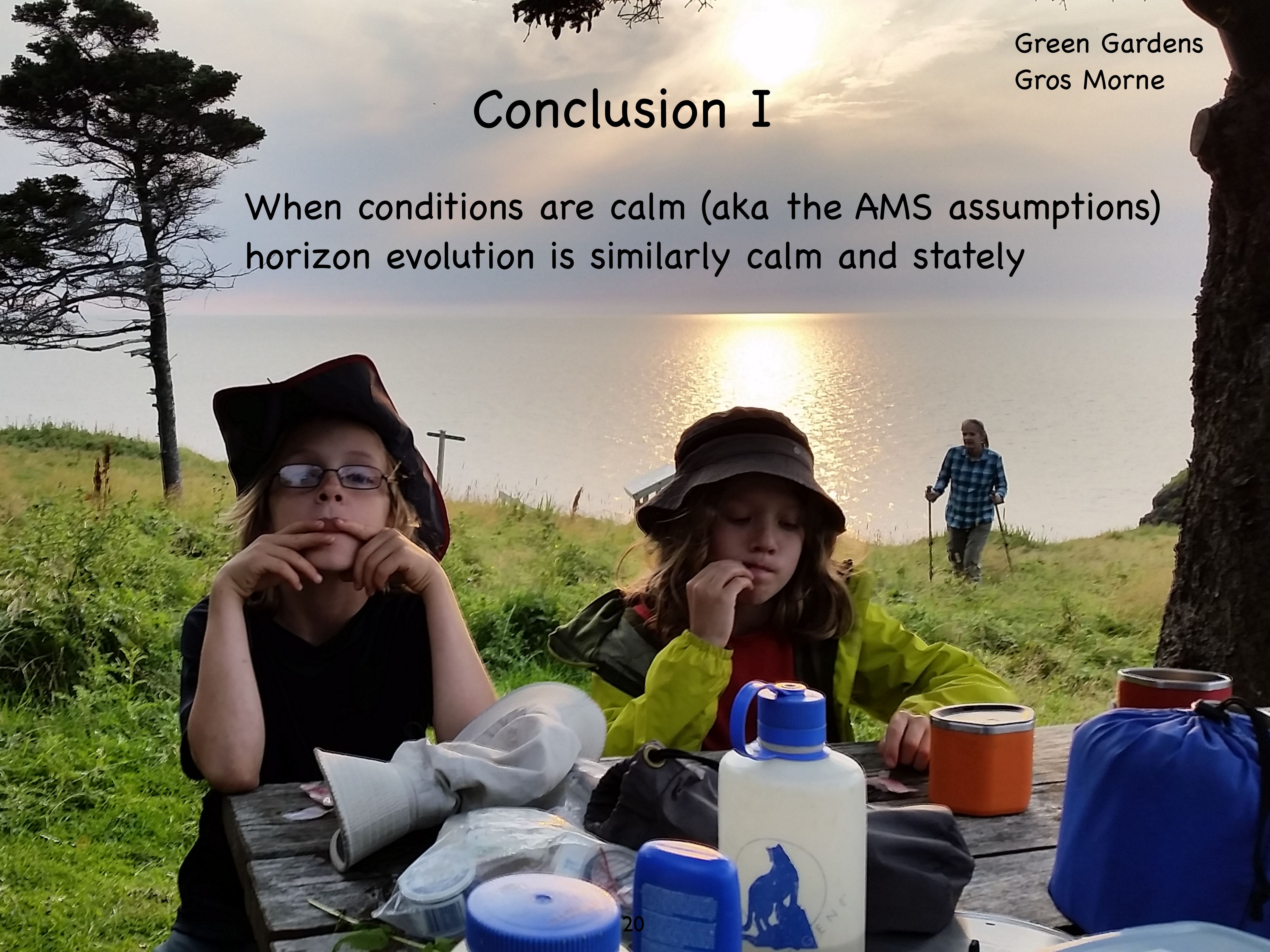
Extremal formations



- Accrete matter onto a charged naked singularity
- Then there can be event horizons without a geometric horizon (a)
- OR a geometric horizon can form instantaneously (b)

Conclusion I

When conditions are calm (aka the AMS assumptions) horizon evolution is similarly calm and stately



Green Gardens
Gros Morne

Conclusion II

Blow-Me-Down Mountains
West Coast, Newfoundland

However those conditions don't always hold and so the evolution can be more interesting.