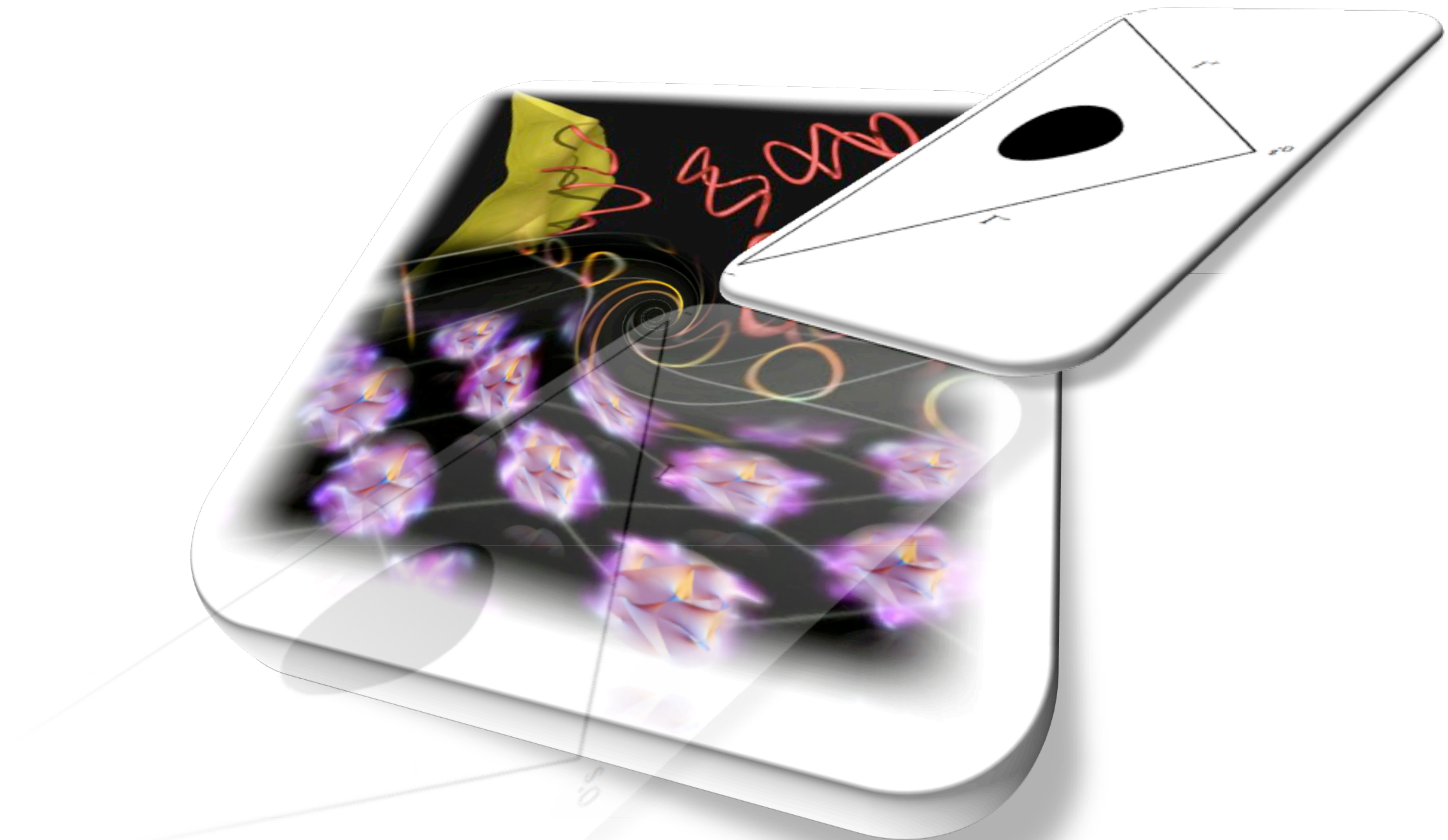
Finite Quantum Gravity, Conformal Invariance & Nonsingular Spacetimes

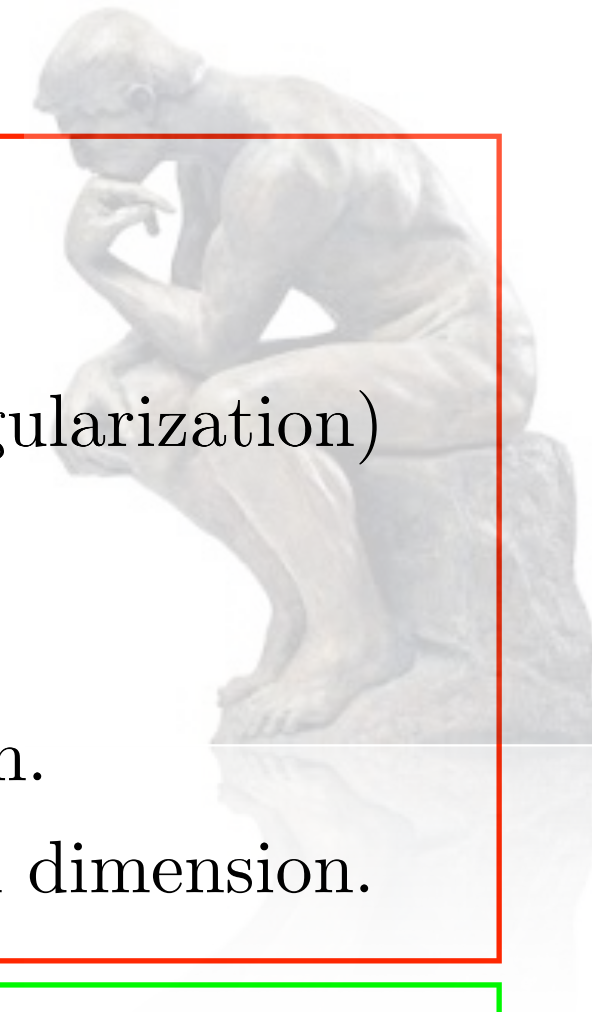


Leonardo Modesto

Center for Quantum Field Theory & Particle Physics

Fudan University (Shanghai)

Outline



- Super-renormalizable Gravitational Theories.
    - Unitarity (no ghosts).
    - Renormalizability (Dimensional Regularization)

- Finite Gravitational Theories :
    - Quantum Gravity in Odd Dimension.
    - Local Terminating Potential in even dimension.

- Exact solutions and spacetime singularities :
    - Ricci flat, (A)dS, FRW spacetimes.
    - Singularity theorem in nonlocal gravity.

- Scattering amplitudes.

- Conformal invariant quantum gravity.
- Nonsingular spacetimes in Conformal gravity.

Multidimensional Renormalizable Gravity

Renormalizable $\Longrightarrow$ Super-renormalizable

$D = 4$ Stelle Theory (Renormalizable & Asymptotically Free)

$$\mathcal{L}_4 = \left[R + aR_{\mu\nu}R^{\mu\nu} + bR^2\right] \quad \rightarrow \quad \mathcal{L}_4^{\mathrm{NL}} = R + R_{\mu\nu}\, a(\Box) R^{\mu\nu} + R\, b(\Box) R\,.$$

Stelle Theory

Krasnikov, Kuzmin, Tomboulis, Khoury, LM, Biswas, Gerwick, Koivisto, Mazumdar.

$$\mathcal{L}_{D-\mathrm{Ren}} = a_1 R + a_2 R^2 + b_2 R_{\mu\nu}^2 + \cdots + a_X R^{X/2} + b_x R_{\mu\nu}^{X/2} + c_X R_{\mu\nu\rho\sigma}^{X/2} + d_X R \Box^{\frac{X}{2}-2} R \ldots$$

For $X = D$ this theory is renormalizable but not unitary.

L.M. (2012).

$$D = 4$$

N. V. Krasnikov (1988), A.T. Tomboulis (1997), L.M. (2011)

$$\mathcal{L}_{4D} = \frac{2}{\kappa_4^2}\left(R + G_{\mu\nu}\frac{e^{H(-\Box_\Lambda)} - 1}{\Box}R^{\mu\nu}\right) .$$

The Lagrangian

$$\mathcal{L}_{\rm g} = -2\kappa_D^{-2}\sqrt{-g}\Big(R + R\gamma_0(\Box)R + R_{\mu\nu}\gamma_2(\Box)R^{\mu\nu} + R_{\mu\nu\rho\sigma}\gamma_4(\Box)R^{\mu\nu\rho\sigma} + \mathcal{V}(\mathcal{R})\Big),$$

$$\gamma_0(\Box) = -\frac{(D-2)(e^{H_0}-1) + D(e^{H_2}-1)}{4(D-1)\Box} + \gamma_4(\Box)\,,$$

$$\gamma_2(\Box) = \frac{e^{H_2}-1}{\Box} - 4\gamma_4(\Box)\,.$$

$$\mathcal{V}(R) = s_1\,\mathcal{R}^3 + \cdots + s_k\,\mathcal{R}^{\frac{D}{2}} + \cdots + s_n\mathcal{R}^{\frac{D}{2}}\nabla^{2\gamma-4}\mathcal{R}^2\,.$$

L.M., L. Rachwal

Nonlocal Gravity in Einstein (Ricci) Basis

L.M., L. Rachwal

$$\mathcal{L}_{\rm E} = -2\kappa_D^{-2}\sqrt{|g|}\Big[\mathbf{R} + \mathbf{G}\,\gamma_{\rm G}(-\Box_\Lambda)\mathbf{Ric} + \mathbf{R}\gamma_{\rm S'}(-\Box_\Lambda)\mathbf{R} + \mathbf{V_g}\Big]\,,$$

$$\gamma_{\rm G} = \gamma_2 = \frac{e^{H_2}-1}{\Box}\,, \quad \gamma_{\rm S'} = \frac{1}{2}\gamma_2 + \gamma_0 = \frac{D-2}{4(D-1)}\,\frac{e^{H_2}-e^{H_0}}{\Box}\,.$$

Unitarity: Anselmi, Piva.
F. Briscese, LM, S. Tsujikawa

Nonlocal Gravity in Weyl Basis

Y. D. Li, M., Rachwal.

$$\mathcal{L}_{\mathrm{C}} = -2\kappa_D^{-2}\sqrt{|g|}\Big[\mathbf{R} + \mathbf{C}\gamma_{\mathrm{C}}(\Box)\mathbf{C} + \mathbf{R}\gamma_{\mathrm{S}}(\Box)\mathbf{R} + \mathbf{V}(\mathbf{C})\Big]\,,$$

$$\gamma_{\mathrm{C}} = \frac{D-2}{4(D-3)}\frac{e^{H_2}-1}{\Box}\,,\ \gamma_{\mathrm{S}} = -\frac{D-2}{4(D-1)}\frac{e^{H_0}-1}{\Box}.$$

Unitarity: Anselmi, Piva.

Nonlocal Gravity in Bach Basis

L.M., L. Rachwal

$$\mathcal{L}_{\rm B} = -2\kappa_D^{-2}\sqrt{|g|}\Big[R + B_{ab}\gamma_{\rm B}(\Box)B^{ab} + R\gamma_{\rm S}(\Box)R + \mathbf{V}(\mathbf{B})\Big]\,,$$

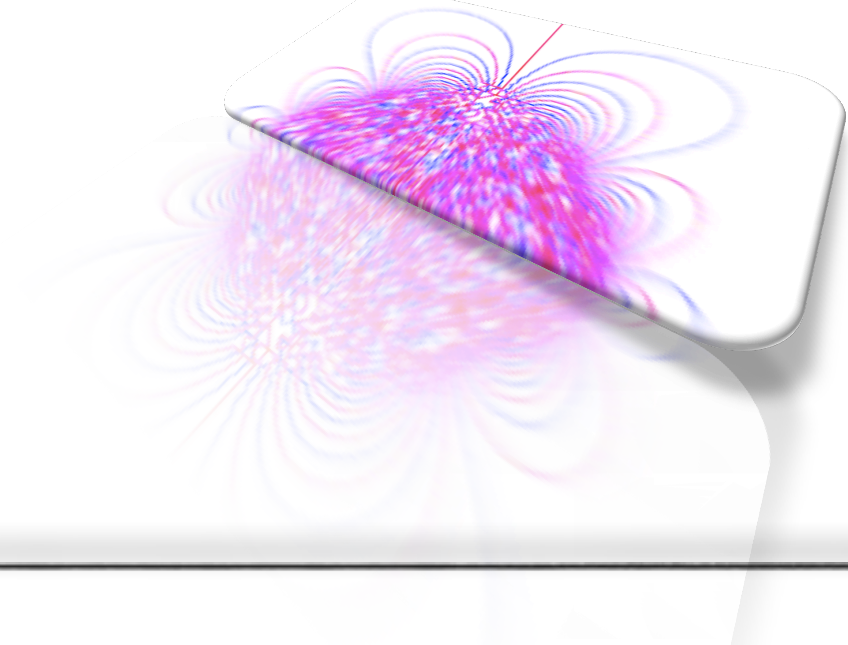
$$\gamma_{\rm B} = \left(\frac{D-2}{D-3}\right)^2 \frac{e^{H_2}-1}{\Box^3}\,,$$

$$\gamma_{\rm S} = -\frac{(D-2)(D-3)\left(e^{H_0}-1\right) + D(D-4)\left(e^{H_2}-1\right)}{4(D-1)(D-3)\Box}\,.$$

$$\begin{aligned}B_{ac} &= \nabla^b\nabla^d C_{abcd} - R^{bd}C_{abcd} = \\ &= \frac{D-3}{D-2}\Box R_{ac} - \frac{D-3}{2(D-1)}R_{;ac} + \frac{D(D-3)}{(D-2)^2}R_{ab}R^b_c - \frac{D-3}{D-2}R^{bd}C_{abcd} - \frac{D-3}{(D-2)^2}g_{ac}R_{bd}R^{bd} \\ &\quad - \frac{D(D-3)}{(D-1)(D-2)^2}RR_{ac} - \frac{D-3}{2(D-1)(D-2)}g_{ac}\Box R + \frac{D-3}{(D-1)(D-2)^2}g_{ac}R^2 - R^{bd}C_{abcd}\,.\end{aligned}$$

The Form Factor (explicit example)

Tomboulis, M., Rachwal.



$$V^{-1}(z) \equiv e^{H(z)} = e^{\frac{1}{2}\left[\Gamma\left(0,p^2(z)\right)+\gamma_E+\log(p^2(z))\right]}$$

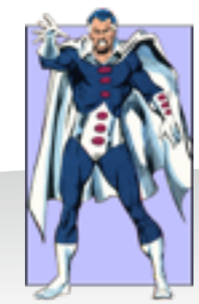
$$\stackrel{\mathbb{R}}{=} e^{\frac{1}{2}\left[\Gamma\left(0,p^2(z)\right)+\gamma_E\right]}\ |p(z)|$$

$$\approx e^{-p^2(z)}\left(\frac{1}{2p^2(z)} - \frac{1}{2p^4(z)} + O\left(\frac{1}{p^6(z)}\right) + \dots\right) e^{\frac{\gamma_E}{2}}\ |p(z)| + \boxed{e^{\frac{\gamma_E}{2}}\ |p(z)|\ ,}$$

$$p_{\gamma+\mathrm{N}+1}(z) = a_{\gamma+\mathrm{N}+1}\, z^{\gamma+\mathrm{N}+1} + \dots + a_{\gamma+\mathrm{N}+1-\frac{D}{2}}\, z^{\gamma+\mathrm{N}+1-\frac{D}{2}} + \dots .$$

Propagator and Unitarity

F. Briscese, LM, S. Tsujikawa



$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

For $\quad H_2(\Box) = H_0(\Box)$:

$$\mathcal{O}^{-1}(k) = \frac{V(k^2/\Lambda^2)}{k^2}\left(P^{(2)} - \frac{P^{(0)}}{D-2}\right).$$

For $\quad H_2(\Box) \neq H_0(\Box)$:

$$\mathcal{O}^{-1}(k) = \frac{P^{(2)}}{k^2 e^{H_2(-k^2)}} - \frac{P^{(0)}}{(D-2)\, k^2 e^{H_0(k^2)}\left(1 + \frac{k^2}{m^2}\right)}\,.$$

$$= \frac{P^{(2)}}{k^2 e^{H_2}} - \frac{P^{(0)}}{(D-2) k^2 e^{H_0}} + \frac{P^{(0)}}{(D-2) e^{H_0}(k^2 + m^2)}\,.$$

A. koshelev, LM, L. Rachwal, A. Starobinsky

Simplified Power Counting

Rigorous power counting: T. Tomboulis.

Propagator (UV)

$$G(k) \sim \frac{1}{k^{2\gamma+2N+4}}\,.$$

Interactions (UV)

$$\mathcal{R}\,\frac{\exp H(-\Box_\Lambda)}{\Box}\,\mathcal{R} \quad \Longrightarrow \quad h^m\,(\partial^2 h)\,\frac{p(-\Box_\Lambda)}{\Box}\,(\partial^2 h)\;\sim h^m\,k^{2\gamma+2\mathrm{N}+4}\,h^2.$$

Loop Amplitudes

$$\mathcal{A}^{(L)} \sim \int (d^D k)^L \left(\frac{1}{k^{2\gamma+2\mathrm{N}+4}}\right)^I \left(k^{2\gamma+2\mathrm{N}+4}\right)^V$$

$$= \int (dk)^{DL} \left(\frac{1}{k^{2\gamma+2\mathrm{N}+4}}\right)^{L-1}\,.$$

$$\omega(G)_{\text{even}} = D_{\text{even}} - 2\gamma(L-1)\,, \quad \gamma > \frac{D_{\text{even}}}{2}\,.$$

$$\omega(G)_{\text{odd}} = D_{\text{odd}} - (2\gamma+1)(L-1)\,, \quad \gamma > \frac{D_{\text{odd}}-1}{2}\,.$$

Renormalization & Asymptotic Freedom

LM, L. Rachwal

$$\mathcal{L} = \frac{2}{\kappa_D^2} R - 2\, G_{\mu\nu} \frac{e^{H(-\Box_\Lambda)} - 1}{\tilde{\kappa}_D^2 \Box} R^{\mu\nu} + c_1^{(3)} R^3 + \cdots + c_1^{(\mathrm{N}+2)} R^{\mathrm{N}+2}$$
$$+ \sum_{n=0}^{\mathrm{N}} \left[(a_n - \tilde{a}_n) R\, (-\Box_\Lambda)^n\, R + (b_n - \tilde{b}_n) R_{\mu\nu}\, (\Box_\Lambda)^n\, R^{\mu\nu} \right].$$

Vertexes

$$\text{set 1}: \quad \mathcal{R},\, \mathcal{R}^2,\, \mathcal{R}^3, \mathcal{R}\,\Box\mathcal{R}, \ldots,\, \mathcal{R}^{\mathrm{N}+2}, \mathcal{R}\,\Box^{\mathrm{N}}\mathcal{R}$$
$$\Longrightarrow \quad h^m(\partial^2 h),\, h^m(\partial^2 h)^2,\, h^m(\partial^2 h)^3, \ldots, h^m(\partial^2 h)^{\mathrm{N}+2},$$

$$\text{set 2}: \quad \mathcal{R}\, \frac{\exp H(-\Box_\Lambda)}{\Box}\, \mathcal{R} \quad \Longrightarrow \quad h^m\, (\partial^2 h)\, \frac{p(-\Box_\Lambda)}{\Box}\, (\partial^2 h) \;\sim\; (\nabla^{2\gamma+2\mathrm{N}+4}\, h^{m+2}).$$

$$\mathcal{L}_{\mathrm{ren}} = \mathcal{L} + \mathcal{L}_{\mathrm{ct}}$$
$$= \mathcal{L} + 2(Z_\kappa - 1)\kappa^{-2}\, R + (Z_{\bar{\lambda}} - 1)\bar{\lambda} + (Z_{c_1^{(1)}} - 1)c_1^{(1)} R^3 + \cdots + (Z_{c_1^{(\mathrm{N})}} - 1)c_1^{(\mathrm{N})} R^{\mathrm{N}+2}$$
$$+ \sum_{n=0}^{\mathrm{N}} \Big((Z_{a_n} - 1) a_n\, R\, (-\Box_\Lambda)^n\, R + (Z_{b_n} - 1) b_n\, R_{MN}\, (-\Box_\Lambda)^n\, R^{MN} \Big),$$
$$\mathcal{L}_{\mathrm{ct}} = \frac{1}{\epsilon} \left[\beta_\kappa R + \beta_{\bar{\lambda}} + \sum_{n=0}^{\mathrm{N}} \Big(\beta_{a_n} R\, (-\Box_\Lambda)^n\, R + \beta_{b_n}\, R_{MN}\, (-\Box_\Lambda)^n\, R^{MN} \Big) + \beta_{c_1^{(1)}}\, R^3 + \cdots + \beta_{c_1^{(\mathrm{N})}}\, R^{\mathrm{N}+2} \right],$$

$\alpha_i \in \{\kappa_D^{-2}, \bar{\lambda}, a_n, b_n, c_1^{(1)}, \cdots, c_1^{(\mathrm{N})}\} \equiv \{\kappa_D^{-2}, \bar{\lambda}, \tilde{\alpha}_n\}.$ Beta functions : $\beta_\kappa, \beta_{\bar{\lambda}}, \beta_{a_n}, \beta_{b_n}, \beta_{c_1^{(1)}}, \cdots, \beta_{c_1^{(\mathrm{N})}}$,

$$(Z_{\alpha_i} - 1)\alpha_i = \frac{1}{\epsilon}\beta_{\alpha_i} \quad \Longrightarrow \quad Z_{\alpha_i} = 1 + \frac{1}{\epsilon}\beta_{\alpha_i}\frac{1}{\alpha_i},$$

$$\alpha_i(\mu) \sim \alpha_i(\mu_0) + \beta_i \log\left(\frac{\mu}{\mu_0}\right).$$

Yang-Mills & Gravity

Asymptotic Freedom

$$\mathcal{L}_{\rm YM} = \frac{1}{2g^2(t)} {\rm Tr} F^2 \sim dA\, dA + g(t) A^2 dA + g^2(t) A^4 \,,$$

$$g(t)^{-2} = g_o^{-2} + \beta_g t \,,$$

$$\mathcal{L}_{\rm G} = \frac{2}{\kappa_D^2(t)} \left(R - G_{\mu\nu} \frac{e^{H(-\Box_\Lambda)} - 1}{\Box} R^{\mu\nu} \right) \sim \partial h\, e^{H(-\Box_\Lambda)}\, \partial h + \kappa_D\, h \partial h\, \partial h + O(\kappa_D^2) \,,$$

$$\kappa_D^{-2}(t) = \kappa_{D\,o}^{-2} + \beta_{\kappa_D} t \,.$$

All the beta-functions do not depend on the gauge fixing condition.

Beyond Renormalizability

$$\int < \infty$$

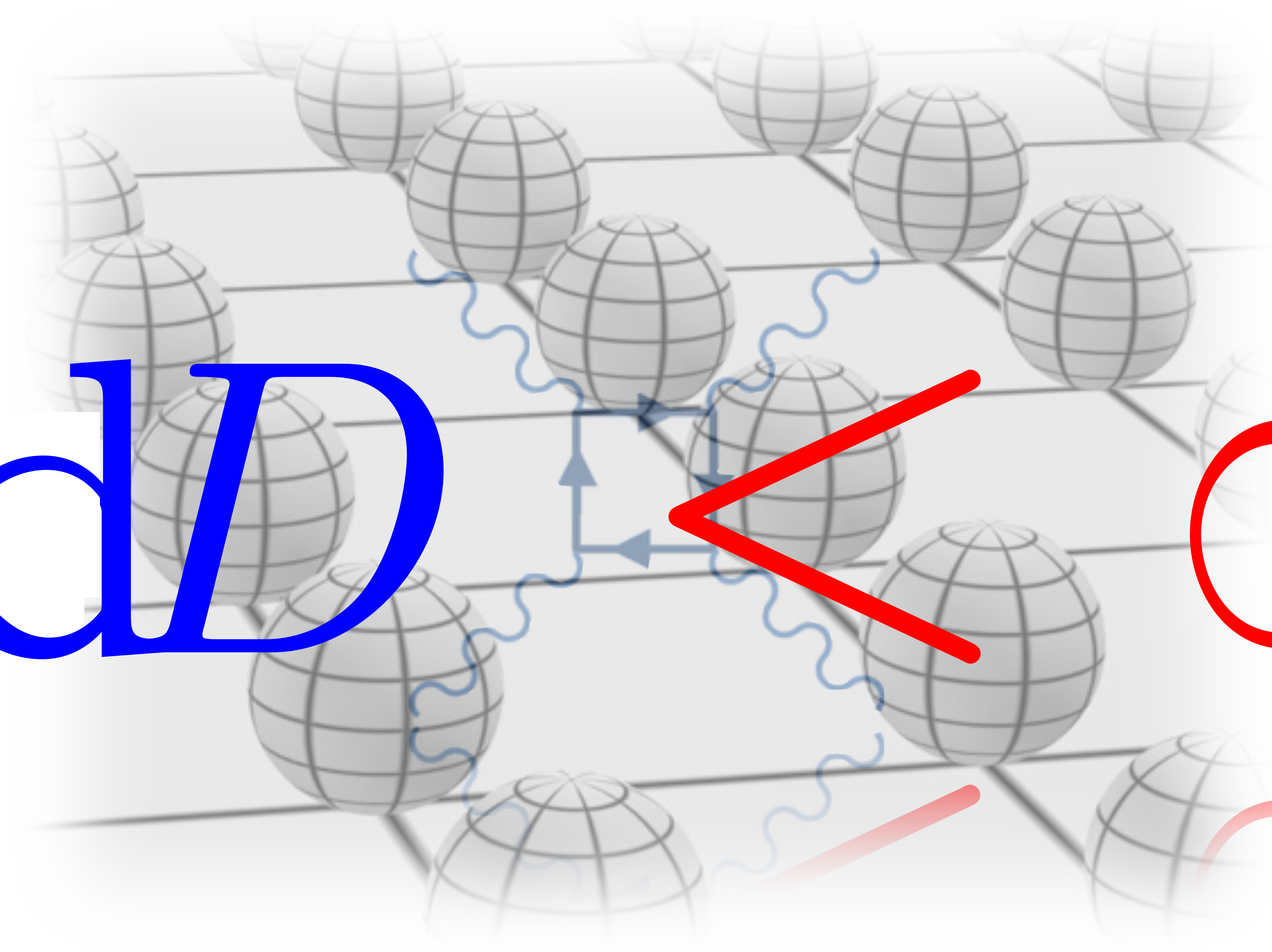
Could we make quantum gravity finite?

S. Giaccari, L.M., L. Rachwal

$$\beta_{\alpha_i} \equiv 0$$

- Multidimensional Finite Quantum Gravity;
- Terminating Potential $V(\mathcal{R})$.

$i.$ Multidimensional Gravity



$$\infty dD < \infty$$

$1 - $ Loop

No counter terms in Odd dimension

↓

Finite Quantum Gravity in Odd dimension

$$\beta_i = 0$$

One loop effective action:

$$W_{\rm div} = -\frac{1}{\epsilon}\left[c_D\, A_D(\Delta) + c_{D-2}\, A_{D-2}(\Delta) + c_{D-4}\, A_{D-4}(\Delta) + \cdots + c_0\, A_0(\Delta)\right] = 0\,,$$

$$A_l(\Delta) \propto \int d^D x R^{l/2}_{\mu\nu\rho\sigma}.$$

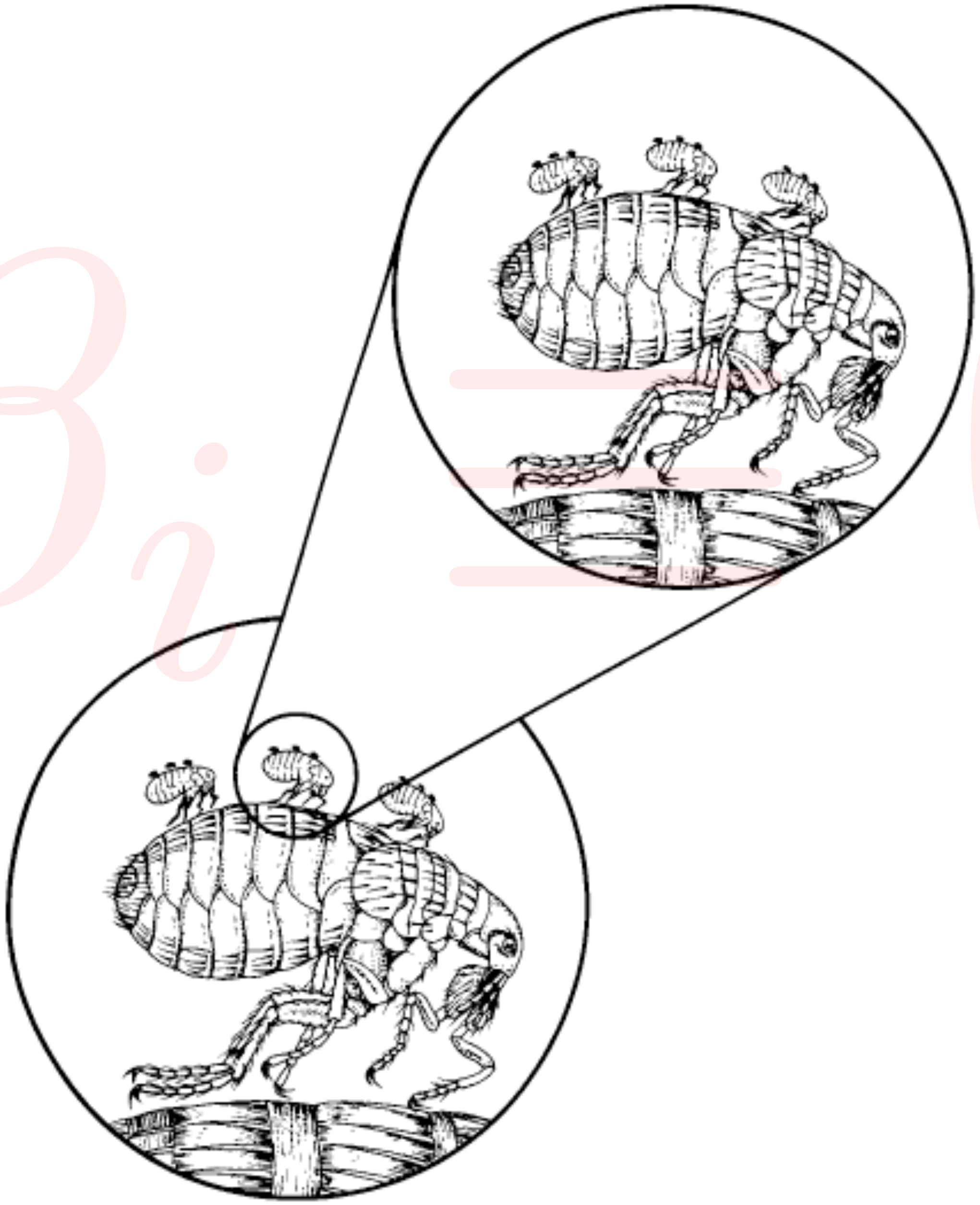
$$\beta_{a_n} = \beta_{b_n} = \beta_{c_i^{(n)}} = 0\,, \quad i \in 1, \dots, (\text{number} \quad \text{of} \quad \text{invariants of order } N)\,, \quad n = 1, \dots, N.$$

$$\mathcal{I}_{k,n} = \int d^D p \frac{(p^2)^k}{(p^2+C)^n} = i \frac{C^{\frac{D}{2}-(n-k)}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(n-k-D/2)\Gamma(k+D/2)}{\Gamma(D/2)\Gamma(n)}\,,$$

$$\mathcal{I}_{\rm null} = \int d^D p \frac{1}{p^{2N}} \equiv 0 \ \text{ for } \ N < D/2\,.$$

ii. Scale Invariant Quantum Gravity

$$\beta_i = 0$$



Theory in $D = 4$

L.M. , Leslaw Rachwal

$$\mathcal{L}_{\rm g} = -2\kappa_D^{-2}\sqrt{-g}\left(R + G_{\mu\nu}\,\frac{e^{H(-\Box_\Lambda)}-1}{\Box}R^{\mu\nu} + s_1 R^2\,\Box^{\gamma-2}R^2 + s_2 R_{\mu\nu}R^{\mu\nu}\,\Box^{\gamma-2}R_{\rho\sigma}R^{\rho\sigma}\right).$$

$$\Longrightarrow \quad -2\kappa_4^{-2}\sqrt{|g|}\left[\omega_1\, R\,\Box^\gamma\, R + \omega_2\, R_{\mu\nu}\,\Box^\gamma\, R^{\mu\nu} + s_1\, R^2\,\Box^{\gamma-2}\, R^2 + s_2\, R_{\mu\nu}R^{\mu\nu}\,\Box^{\gamma-2}\, R_{\rho\sigma}R^{\rho\sigma}\right],$$

$$\omega_2 = -2\,\omega_1 = e^{\gamma_E/2}/\Lambda^{2\gamma+2}.$$

$$\Gamma^{(1)}_{\rm div} = \frac{1}{\epsilon}\left(\beta_{R^2_{\mu\nu}}\, R_{\mu\nu}R^{\mu\nu} + \beta_{R^2}\, R^2\right)\,,$$

$$\beta_{R^2} := a_1 s_1 + a_2 s_2 + c_1 = 0$$

$$\beta_{R^2_{\mu\nu}} := b_2 s_2 + c_2 = 0$$

$\Rightarrow$

$$s_1 = \frac{-(3\omega_1+\omega_2)(40c_1\omega_1 + 10c_2\omega_1 + 14c_1\omega_2 - c_2\omega_2)}{48(20\omega_1 + 7\omega_2)}\,,$$

$$s_2 = \frac{-3c_2\omega_2(3\omega_1+\omega_2)}{2(20\omega_1+7\omega_2)}.$$

Scattering Amplitudes

P. Donà, S. Giaccary, L.M., L. Rachwal, Yiwei-Zhu.

$$\mathcal{L}_{\rm g} = -2\kappa_D^{-2}\sqrt{g}\,[R + R\,\gamma_0(\Box)R + R_{\mu\nu}\gamma_2(\Box)R^{\mu\nu} + R_{\mu\nu\rho\sigma}\,\gamma_4(\Box)R^{\mu\nu\rho\sigma}]\,.$$

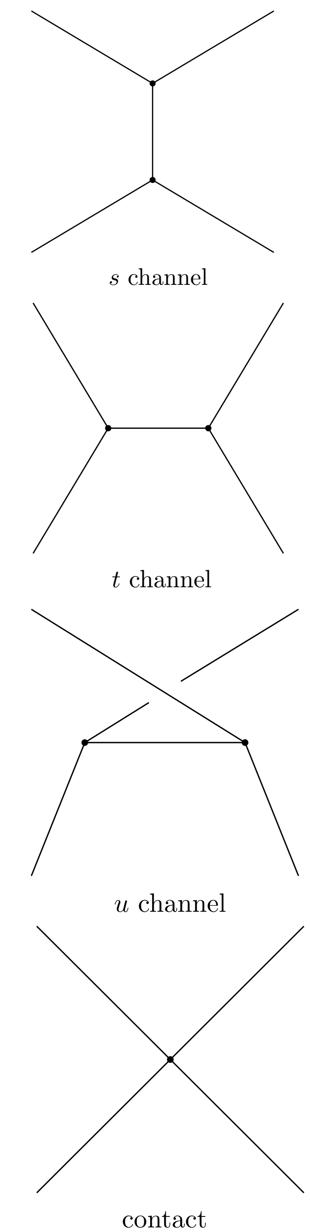
$$\gamma_4(\Box) = 0,$$

$$\mathcal{A}_s(++,++) = -g\frac{9}{8}\frac{t(s+t)}{s} + \frac{9}{32}\gamma_2(s)\left(s^2+(s+2t)^2\right) + \frac{9}{8}s^2\gamma_0(s)\,,$$

$$\mathcal{A}_t(++,++) = -\frac{g}{8}\frac{\left(s^3-5s^2t-st^2+t^3\right)(s+t)^2}{s^3t} + \frac{1}{16}\gamma_2(t)\frac{\left(2s^4-10s^3t-s^2t^2+4st^3+t^4\right)(s+t)^2}{s^4} + \frac{1}{8}\gamma_0(t)\frac{t^2(s+t)^4}{s^4}\,,$$

$$\mathcal{A}_u(++,++) = -\frac{g}{8}\frac{\left(s^3-5s^2u-su^2+u^3\right)(s+u)^2}{s^3u} + \frac{1}{16}\gamma_2(u)\frac{\left(2s^4-10s^3u-s^2u^2+4su^3+u^4\right)(s+u)^2}{s^4} + \frac{1}{8}\gamma_0(u)\frac{u^2(s+u)^4}{s^4}\,,$$

$$\mathcal{A}_{\rm contact}(++,++) = -\frac{g}{4}\frac{s^4+s^3t-2st^3-t^4}{s^3} + -\frac{9}{32}\gamma_2(s)\left(s^2+(s+2t)^2\right) - \frac{9}{8}s^2\gamma_0(s) - \frac{1}{16}\gamma_2(t)\frac{\left(2s^4-10s^3t-s^2t^2+4st^3+t^4\right)(s+t)^2}{s^4} - \frac{1}{8}\gamma_0(t)\frac{t^2(s+t)^4}{s^4} - \frac{1}{16}\gamma_2(u)\frac{\left(2s^4-10s^3u-s^2u^2+4su^3+u^4\right)(s+u)^2}{s^4} - \frac{1}{8}\gamma_0(u)\frac{u^2(s+u)^4}{s^4}\,.$$



$$\mathcal{A}(++,++) = \mathcal{A}_s(++,++) + \mathcal{A}_t(++,++) + \mathcal{A}_u(++,++) + \mathcal{A}_{\rm contact}(++,++) = \mathcal{A}(++,++)_{\rm EH}$$

Stelle, Weyl gravity, etc.

Theorem. All the n-point functions in any gravitational theory (in particular super-renormalizable or finite) with an action

$$\mathcal{L}_{\rm gr} = -2\kappa_D^{-2}\sqrt{-g}\,[\mathbf{R} + \mathbf{R}\,\gamma_0(\Box)\mathbf{R} + \mathbf{Ric}\,\gamma_2(\Box)\mathbf{Ric} + \mathbf{V}(\mathbf{R},\mathbf{Ric},\mathbf{Riem},\nabla)\,]\,,$$

give the same on-shell tree-level amplitudes as the Einstein-Hilbert theory, $\mathcal{L}_{\rm EH} = -2\kappa_D^{-2}\sqrt{-g}\,\mathbf{R}$, provided that the potential $\mathbf{V}$ is at least quadratic in $\mathbf{Ric}$ and/or $\mathbf{R}$. In particular for any theory in which we can recast the potential in the following form

$$\mathbf{V} = \mathbf{Ric}\cdot\tilde{\mathbf{V}}\cdot\mathbf{Ric} \equiv R_{\mu\nu}[\tilde{\mathbf{V}}(\mathbf{R},\mathbf{Ric},\mathbf{Riem},\nabla)]^{\mu\nu\rho\sigma}R_{\rho\sigma}\,,$$

the theorem is valid.

Proof. Based on the Anselmi's field redefinition theorem.

$$S(g') \equiv S_{\rm EH}(g') \equiv S_{\rm EH}(g) + R_{\mu\nu}(g)F^{\mu\nu,\rho\sigma}(g)R_{\rho\sigma}(g) = S'(g) \equiv S_{\rm gr}(g).$$

Non-perturbative Spectrum

Calcagni, Montobbio, Nardelli, Phys. Lett. B662 (2008)

Under investigation : G. Calcagni, LM, G. Nardelli.

Localization of the theory : 3 fields, only second order EOM.

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Physics Letters B 662 (2008) 285–289

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Localization of nonlocal theories

Gianluca Calcagni [a,*], Michele Montobbio [b], Giuseppe Nardelli [c,d]

[a] *Department of Physics, University of Sussex, Brighton BN1 9QH, United Kingdom*
[b] *Dipartimento di Fisica and INFN Gruppo Collegato di Trento, Università di Trento, via Sommarive 14, 38100 Povo (Trento), Italy*
[c] *Dipartimento di Matematica e Fisica, Università Cattolica, via Musei 41, 25121 Brescia, Italy*
[d] *INFN Gruppo Collegato di Trento, Università di Trento, 38100 Povo (Trento), Italy*

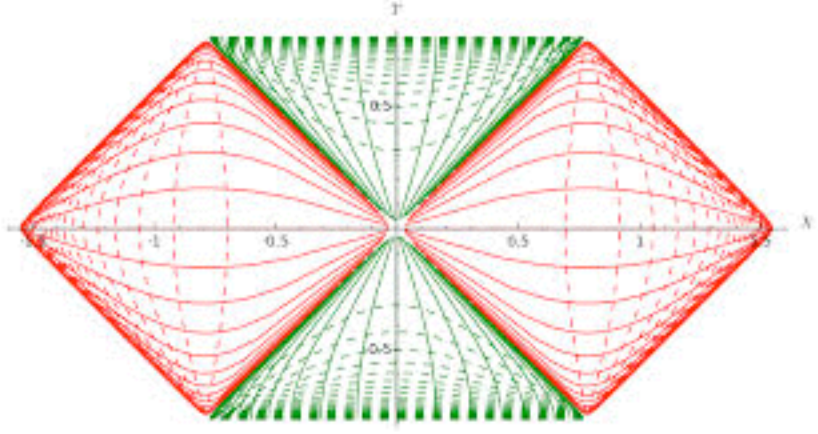
Received 13 December 2007; received in revised form 28 January 2008; accepted 5 March 2008

Available online 14 March 2008

Editor: M. Cvetič

Exact Solutions, Spacetime Singularities, Conformal Invariance

L.M., L. Rachwal (2014)



Singularity Theorem in non-local gravity

L.M. , Leslaw Rachwal

Given the EOM : $\mathbf{E} = 8\pi G_N \mathbf{T}$,

the following implication turns out to be true,

- $R_{\mu\nu} = 0 \quad \& \quad T_{\mu\nu} = 0 \quad \Longrightarrow \quad E_{\mu\nu} = 0\,;$
- $R_{\mu\nu} = 8\pi G_N T_{\mu\nu} \quad \& \quad T^{\mu}_{\mu} = 0 \quad \Longrightarrow \quad E_{\mu\nu} = 8\pi G_N T_{\mu\nu}\,.$

Therefore,

- all **Ricci flat** spacetimes are exact solutions;
- all the **FRW** spacetime sourced by conformally coupled matter are exact solutions.

The theorem works for any potential at least quadratic in the Bach tensor.

EOM

$$E_{\mu\nu} = \frac{\delta\left[\sqrt{|g|}\left(R + R\gamma_{\rm S}(\Box)R + B_{\alpha\beta}\gamma_{\rm B}(\Box)B^{\alpha\beta} + V(B)\right)\right]}{\sqrt{|g|}\delta g^{\mu\nu}} = 8\pi G_N T_{\mu\nu}\,,$$

$$G_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\left(R\gamma_{\rm S}(\Box)R\right) - \frac{1}{2}g_{\mu\nu}\left(B_{\alpha\beta}\gamma_{\rm B}(\Box)B^{\alpha\beta}\right)$$
$$+2\frac{\delta R}{\delta g^{\mu\nu}}\left(\gamma_{\rm S}(\Box)R\right) + \frac{\delta B_{\alpha\beta}}{\delta g^{\mu\nu}}\left(\gamma_{\rm B}(\Box)B^{\alpha\beta}\right) + \frac{\delta B^{\alpha\beta}}{\delta g^{\mu\nu}}\left(\gamma_{\rm B}(\Box)B_{\alpha\beta}\right)$$
$$+\frac{\delta\Box^r}{\delta g^{\mu\nu}}\left(\frac{\gamma_{\rm S}(\Box^l) - \gamma_{\rm S}(\Box^r)}{\Box^l - \Box^r}RR\right) + \frac{\delta\Box^r}{\delta g^{\mu\nu}}\left(\frac{\gamma_{\rm B}(\Box^l) - \gamma_{\rm B}(\Box^r)}{\Box^l - \Box^r}B_{\alpha\beta}B^{\alpha\beta}\right) + \frac{1}{\sqrt{|g|}}\frac{\delta V(B)}{\delta g^{\mu\nu}} = 8\pi G_N T_{\mu\nu}.$$

Proof.

- Ricci flat spacetimes :

$$R_{\mu\nu} = 0 \quad \& \quad T_{\mu\nu} = 0 \quad \Longrightarrow B_{\mu\nu} = 0 \quad \Longrightarrow \quad E_{\mu\nu} = 0\,.$$

Schwarzschild, Kerr, Kasner, etc. are **EXACT SOLUTIONS**.

EOM

$$G_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \left(R \gamma_{\rm S}(\Box) R\right) - \frac{1}{2} g_{\mu\nu} \left(B_{\alpha\beta} \gamma_{\rm B}(\Box) B^{\alpha\beta}\right)$$
$$+ 2 \frac{\delta R}{\delta g^{\mu\nu}} \left(\gamma_{\rm S}(\Box) R\right) + \frac{\delta B_{\alpha\beta}}{\delta g^{\mu\nu}} \left(\gamma_{\rm B}(\Box) B^{\alpha\beta}\right) + \frac{\delta B^{\alpha\beta}}{\delta g^{\mu\nu}} \left(\gamma_{\rm B}(\Box) B_{\alpha\beta}\right)$$
$$+ \frac{\delta \Box^r}{\delta g^{\mu\nu}} \left(\frac{\gamma_{\rm S}(\Box^l) - \gamma_{\rm S}(\Box^r)}{\Box^l - \Box^r} R R \right) + \frac{\delta \Box^r}{\delta g^{\mu\nu}} \left(\frac{\gamma_{\rm B}(\Box^l) - \gamma_{\rm B}(\Box^r)}{\Box^l - \Box^r} B_{\alpha\beta} B^{\alpha\beta} \right) + \frac{1}{\sqrt{|g|}} \frac{\delta V(B)}{\delta g^{\mu\nu}} = 8\pi G_N T_{\mu\nu}.$$

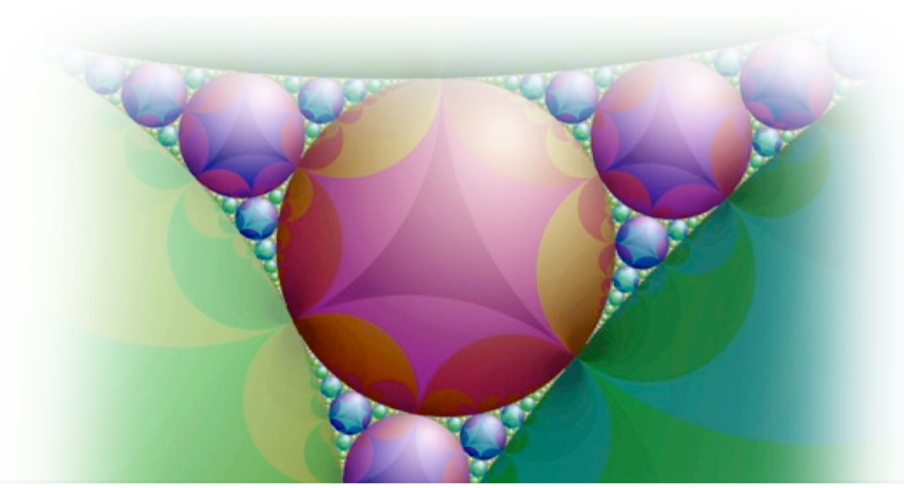
Proof.

- FRW spacetimes :

$$\text{FRW} \quad \Longrightarrow \quad G_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \left(R \gamma_{\rm S}(\Box) R\right) + 2 \frac{\delta R}{\delta g^{\mu\nu}} \left(\gamma_{\rm S}(\Box) R\right) + \frac{\delta \Box^r}{\delta g^{\mu\nu}} \left(\frac{\gamma_{\rm S}(\Box^l) - \gamma_{\rm S}(\Box^r)}{\Box^l - \Box^r} R R \right) = T_{\mu\nu} \, .$$

$$\mathrm{tr}\, \mathbf{T} = 0 \implies R = 0 \implies \mathbf{G} = 8\pi G_N \, \mathbf{T} \implies \boxed{\text{Big Bang Singularity}} .$$

FRW for Radiation is an **EXACT SOLUTION** .



Conformal Invariant Gravity

$$g_{\mu\nu} = (\phi^2\,\kappa_D^2)^{\frac{2}{D-2}}\,\hat{g}_{\mu\nu}\,, \qquad \hat{g}_{\mu\nu} \to \Omega^2(x)\,\hat{g}_{\mu\nu}\,, \quad \phi \to \Omega^{\frac{2-D}{2}}(x)\,\phi\,.$$

Einstein conformal gravity

$$\mathcal{L}_4 = -2\kappa_4^{-2}\sqrt{|g|}R(g) = -12\phi\left(-\Box + \frac{1}{6}R\right)\phi.$$

Nonlocal conformal gravity

$$\mathcal{L}_{\rm g} = -2\sqrt{\hat{g}}\left[\phi^2 R(\hat{g}) + \frac{4(D-1)}{D-2}\hat{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right] - \frac{2}{\kappa_D^2}\sqrt{g}\left[\mathbf{R}(g)\,\gamma_0(\Box)\mathbf{R}(g) + \mathbf{Ric}(g)\,\gamma_2(\Box)\mathbf{Ric}(g) + \mathbf{V}(g)\,\right]\Big|_{\phi\hat{g}}\,.$$

Quantum nonlocal conformal gravity

(Fradkin, Tsytlin, Percacci.)

$$g_{\mu\nu} = \overbrace{(\bar{\phi} + \varphi)^2}^{\phi^2} \, \overbrace{(\,\bar{\hat{g}}_{\mu\nu} + \hat{h}_{\mu\nu}\,)}^{\hat{g}_{\mu\nu}} \, ,$$

$$Z\left(\bar{g}_{\mu\nu} = \bar{\phi}^2 \bar{\hat{g}}_{\mu\nu}\right) = \int \mathcal{D}\phi \, \mathcal{D}\hat{g}_{\mu\nu} \, \Delta_{\rm FP}^{\rm diff}(g) \, \delta(\chi(g)^\alpha - \ell^\alpha) \, \Delta_{\rm FP}^{\rm conf}(\phi) \, \delta(\chi(\phi) - \ell) \, e^{iS(g)} \Big|_{g_{\mu\nu} = \phi^2 \hat{g}_{\mu\nu}}$$

$$= \int \mathcal{D}\varphi \, \mathcal{D}\hat{g}_{\mu\nu} \det(C_{\alpha\beta}(\bar{g}))^{\frac{1}{2}} \, e^{\frac{1}{2}\chi^\alpha(g) C_{\alpha\beta}(\bar{g}) \chi^\beta(g)} \, \det(M^{\rm diff}_{\alpha\beta}(\bar{g})) \det(\bar{\phi} + \varphi) \, \delta(\varphi - \ell) e^{iS(g)} \Big|_{\phi \hat{g}},$$

$$Z\left(\bar{g}_{\mu\nu} = \bar{\phi}^2 \bar{\hat{g}}_{\mu\nu}\right) = \int \mathcal{D}\left[\hat{g}_{\mu\nu} (-\hat{g})^{-\frac{1}{4}}\right] \det(C_{\alpha\beta}(\bar{g}))^{\frac{1}{2}} \, \det(M^{\rm diff}_{\alpha\beta}(\bar{g})) \, e^{i[S(g) + S_{\rm gf}(g)]} \Big|_{g_{\mu\nu} = \bar{\phi}^2 \hat{g}_{\mu\nu}}$$

$$= \int \mathcal{D}\left[g_{\mu\nu} (-g)^{-\frac{1}{4}}\right] \det(C_{\alpha\beta}(\bar{g}))^{\frac{1}{2}} \, \det(M^{\rm diff}_{\alpha\beta}(\bar{g})) \; e^{i[S(g) + S_{\rm gf}(g)]} \Big|_{g_{\mu\nu} = \bar{\phi}^2 \hat{g}_{\mu\nu}}$$

$$= \left(\int e^{-\frac{1}{2}\delta^4(0) \int d^4x \log\sqrt{-g}} \, \mathcal{D}h_{\mu\nu} \det(C_{\alpha\beta}(\bar{g}))^{\frac{1}{2}} \, \det(M^{\rm diff}_{\alpha\beta}(\bar{g})) \; e^{i[S(g) + S_{\rm gf}(g)]} \Big|_{g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}} \right) \Bigg|_{\bar{g}_{\mu\nu} = \bar{\phi}^2 \bar{\hat{g}}_{\mu\nu}}$$

$$= \left(\int \mathcal{D}h_{\mu\nu} \det(C_{\alpha\beta}(\bar{g}))^{\frac{1}{2}} \, \det(M^{\rm diff}_{\alpha\beta}(\bar{g})) \; e^{i[S(g) + S_{\rm gf}(g)]} \Big|_{g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}} \right) \Bigg|_{\bar{g}_{\mu\nu} = \bar{\phi}^2 \bar{\hat{g}}_{\mu\nu}}$$

Counterterms

$$\sqrt{|g|}R^2\,,\quad \sqrt{|g|}R^2_{\mu\nu}\,,\quad \sqrt{|g|}R\,,\quad \sqrt{|g|}=\sqrt{|\hat{g}|}\phi^4\,,\qquad g_{\mu\nu}=\phi^2\kappa_4^2\,\hat{g}_{\mu\nu}\,.$$

. . . all conformal invariant.

In DIMREG :

$$\frac{1}{\epsilon}\phi^{-\epsilon}\mathcal{O}_i(\phi^2\hat{g};\epsilon)\approx\frac{1}{\epsilon}(1-\epsilon\log\phi)\,\mathcal{O}_i(\phi^2\hat{g};\epsilon)=\frac{1}{\epsilon}\,\mathcal{O}_i(\phi^2\hat{g})+\tilde{\mathcal{O}}_i(\phi^2\hat{g})-\log(\phi)\,\mathcal{O}_i(\phi^2\hat{g}),$$

$$\sqrt{|g|}=\phi^{D-\epsilon}\sqrt{|\hat{g}|}=\overbrace{\phi^D\sqrt{|\hat{g}|}}^{\text{conf. inv.}}\,\phi^{-\epsilon}=\sqrt{|g|}\,\phi^{-\epsilon}\,.$$

Gauge Fixing or Spontaneous Symmetry Breaking

$$\hat{g}_{\mu\nu} \to \Omega^2(x)\,\hat{g}_{\mu\nu}\,, \quad \phi \to \Omega^{\frac{2-D}{2}}(x)\,\phi\,.$$

$$\underbrace{g_{\mu\nu}}_{10} = \underbrace{\phi^2}_{1\ \text{comp.}} \times \underbrace{\hat{g}_{\mu\nu}}_{10\ \text{comp.}} \qquad \text{or} \qquad \underbrace{g_{\mu\nu}}_{10} = \underbrace{\phi^2}_{1\ \text{comp.}} \times \underbrace{\hat{g}_{\mu\nu}}_{9\ \text{comp.}}\,.$$

Splitting : $g_{\mu\nu} := e^{2\omega(x)}\hat{g}_{\mu\nu}\,.$

U(1) Toy Analog Model

$$\mathcal{L} = \frac{1}{4}\mathbf{F}(\mathbf{A})^2 - |D_\mu\phi|^2 \,, \quad 2\ (A) + 1\ (\phi)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \,, \quad D_\mu = \partial_\mu - igA_\mu \,, \quad \phi(x) = \frac{1}{\sqrt{2}} v\, e^{i\frac{\theta(x)}{v}} \quad (v = \text{const. and } [v] = 1) \,,$$

$$\phi'(x) = e^{ig\alpha(x)}\phi(x) \quad \text{or} \quad \theta'(x) = \theta(x) + b\, v\, \alpha(x) \quad \text{and} \,, \quad A'_\mu = A_\mu + \partial_\mu\alpha(x) \,,$$

$$D_\mu\phi = -\frac{1}{\sqrt{2}} ig\, v\, e^{i\frac{\theta(x)}{v}} \left(A_\mu - \frac{1}{vg}\partial_\mu\theta(x) \right) .$$

$$B_\mu = A_\mu - \partial_\mu\theta(x)/vg \quad \Longrightarrow \quad \mathcal{L} = \frac{1}{4}\mathbf{F}(\mathbf{B})^2 - \frac{1}{2}g^2v^2\mathbf{B}^2 \,. \quad 3\ (A) + 0\ (\phi)$$

Or imposing $\theta(x) = \text{const.}$,

$$|D_\mu\phi|^2 = \frac{1}{2}g^2v^2A_\mu A^\mu \quad \Longrightarrow \quad \mathcal{L} = \frac{1}{4}\mathbf{F}(\mathbf{A})^2 - \frac{1}{2}g^2v^2\mathbf{A}^2 \,.$$

Spacetime Singularities and Conformal Invariance

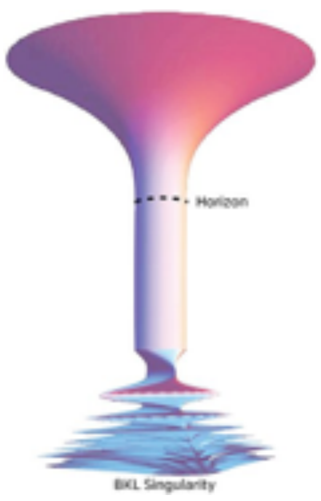
Narlikar, Kembhavi (1977).

If $(\hat{g}_{\mu\nu}, \phi)$ is a solution $\Longrightarrow$ $(\hat{g}^*_{\mu\nu}, \phi^*)$ is a solution, where $\hat{g}^*_{\mu\nu} = \Omega^2\, \hat{g}_{\mu\nu}\,,\quad \phi^* = \Omega^{\frac{2-D}{2}}\,\phi\,.$

FRW :
$$ds^{*2} = S(t) ds^2\,,\quad ds^2 = a^2(t)(-dt^2 + d\vec{x}^2)\,;$$
$$S(t)^{-1} = a^2(t)\,.$$

Belinski, Khalatnikov, Lifshitz

BKL :
$$ds^2 = -dt^2 + a_1(t)dx_1^2 + a_2(t)dx_2^2 + a_3(t)dx_3^2\,,$$



$$a_i(t) = t^{2p_i} \quad (i = 1, 2, 3)\,,$$
$$\sum_i^3 p_i = \sum_i^3 p_i^2 = 1;$$

$$\hat{\mathbf{R}}\mathbf{iem^2}(\hat{\mathbf{g}}^*_{\mu\nu}) = \frac{4\left(\frac{4\left(L^2+t^2\right)^4}{u^2+u+1} - \frac{8\left(L^2+t^2\right)^4}{(u^2+u+1)^2} + \frac{4\left(L^2+t^2\right)^4}{(u^2+u+1)^3} + 3L^4\left(L^4 + 4L^2t^2 + 8t^4\right)\right)}{\left(L^2+t^2\right)^6}\,.$$

$$ds^{*2} = S(t)\, ds^2 = \frac{t^2 + L^2}{t^2}\left[-dt^2 + a_1(t)dx_1^2 + a_2(t)dx_2^2 + a_3(t)dx_3^2\right].$$

Schwarzschid Singularity

$$g_{\mu\nu}^{\rm Sch} = (\phi\,\kappa_D)^{\frac{4}{D-2}}\,\hat{g}_{\mu\nu} = (\phi^*\,\kappa_D)^{\frac{4}{D-2}}\,\hat{g}^*_{\mu\nu} \quad \Longleftrightarrow \quad \hat{g}^*_{\mu\nu} = \Omega^2\,\hat{g}_{\mu\nu}\,, \quad \phi^* = \Omega^{\frac{2-D}{2}}\,\phi.$$

$$ds^{*2} \equiv \hat{g}^*_{\mu\nu} dx^\mu dx^\nu = S(r)\hat{g}_{\mu\nu} dx^\mu dx^\nu = S(r)\left[\left(1 - \frac{2m}{r}\right)dt^2 + \frac{dr^2}{1-\frac{2m}{r}} + r^2 d\Omega^{(2)}\right]\,,$$

$$\phi^* = S(r)^{-1/2}\kappa_4^{-1}\,.$$

Schwarzschild Singularity = one element of the gauge orbit

$$g^{\rm Sch}_{\mu\nu} = (\phi\,\kappa_D)^{\frac{4}{D-2}}\,\hat{g}_{\mu\nu} = (\phi^*\,\kappa_D)^{\frac{4}{D-2}}\,\hat{g}^*_{\mu\nu} \quad \Longleftrightarrow \quad \hat{g}^*_{\mu\nu} = \Omega^2\,\hat{g}_{\mu\nu}\,, \quad \phi^* = \Omega^{\frac{2-D}{2}}\,\phi.$$

$$S(r) = \frac{1}{r^2}\left(\frac{L^4}{r^2} + r^2\right),$$

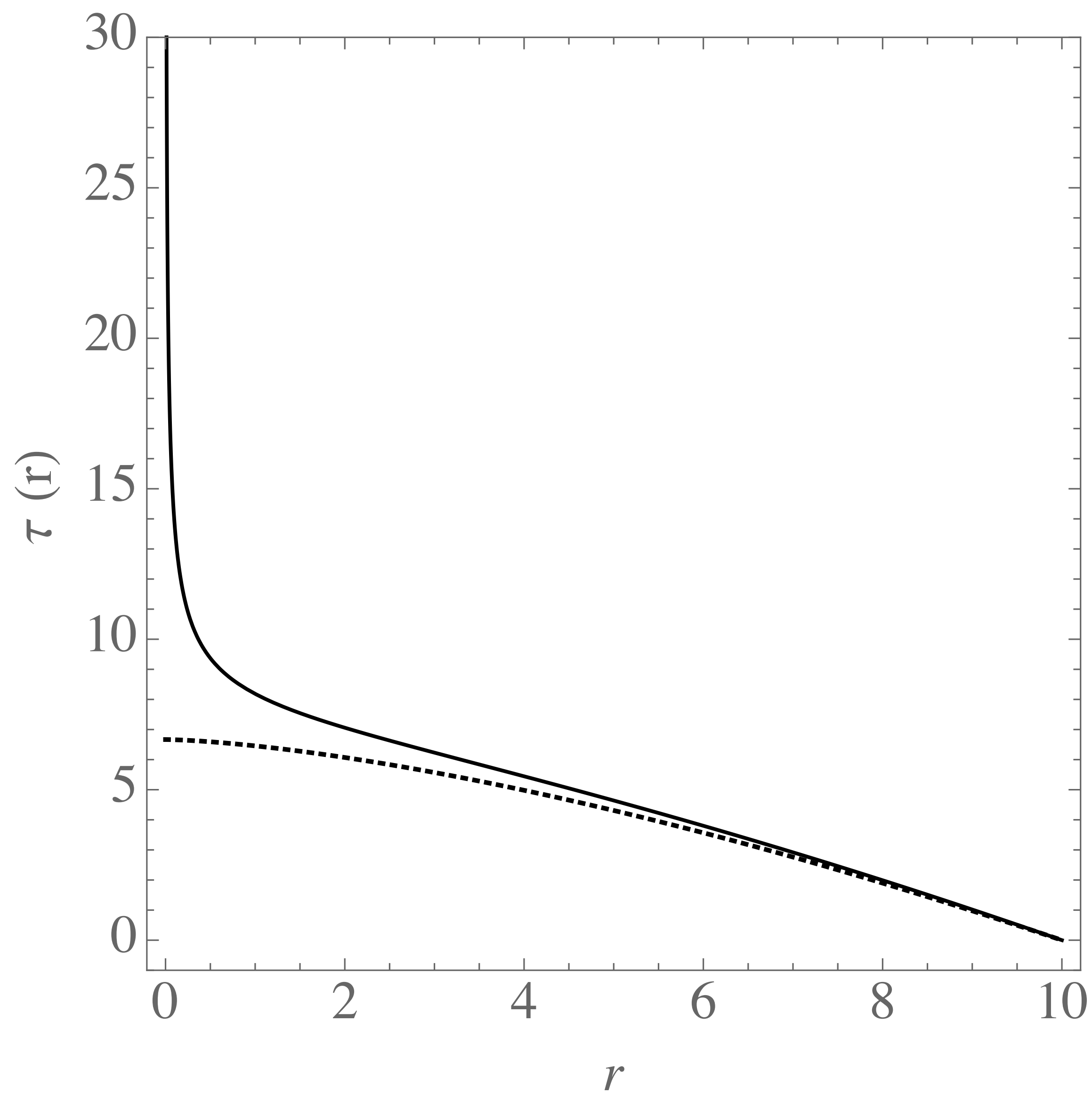
$$ds^{*2} = -\frac{1}{r^2}\left(\frac{L^4}{r^2} + r^2\right)\left(1 - \frac{2m}{r}\right)dt^2 + \frac{1}{r^2}\left(\frac{L^4}{r^2} + r^2\right)\frac{dr^2}{1 - \frac{2m}{r}} + \left(\frac{L^4}{r^2} + r^2\right)d\Omega^{(2)}\,.$$

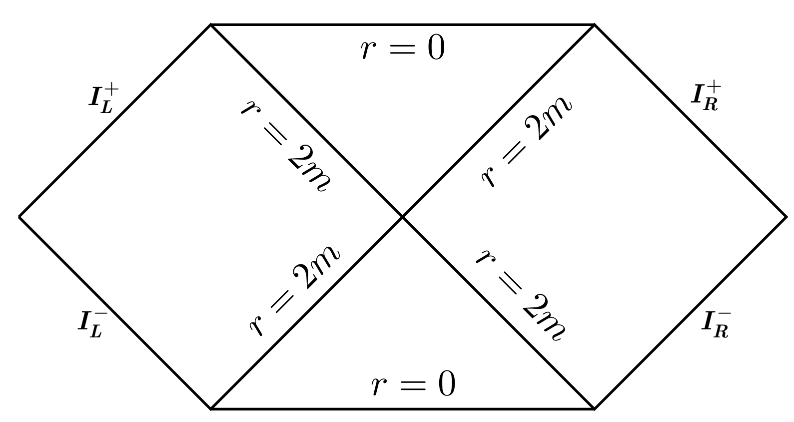
$$\hat{\mathbf{K}} = \frac{1}{(L^4 + r^4)^6}[16r^2(L^{16}(39m^2 - 20mr + 3r^2) + 2L^{12}r^4(66m^2 - 32mr + 3r^2)$$
$$+L^8r^8(342m^2 - 284mr + 63r^2) + 12L^4m^2r^{12} + 3m^2r^{16})].$$

$$\text{Hawking Temperature}: \quad T_H = \frac{1}{8\pi m} \quad \forall\ S(r)\,.$$

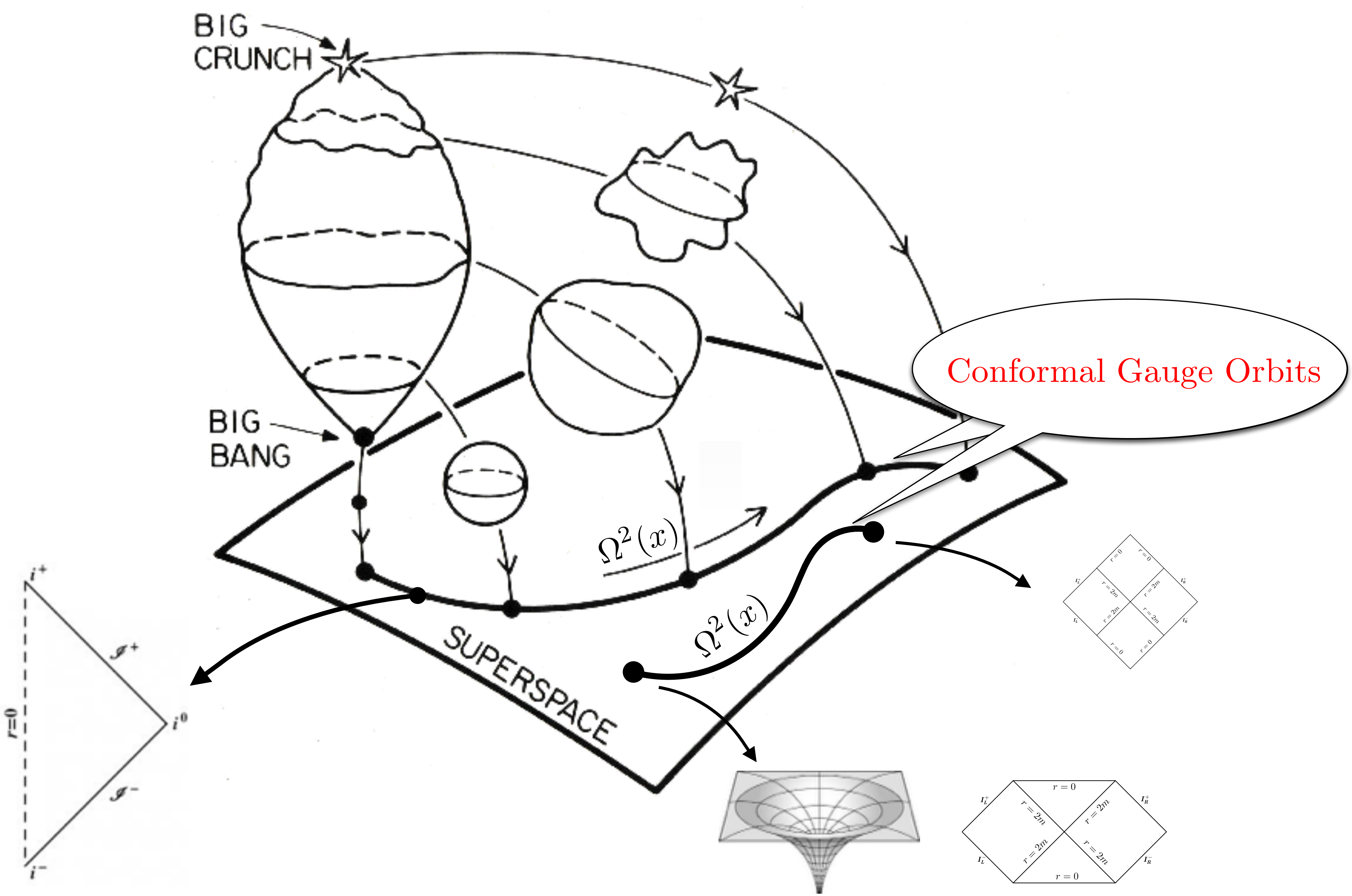
$\boxed{r = 0 \text{ Singularity in Conformal Gravity}} \equiv \boxed{r = 2m \text{ Singularity in GR}}.$

Geodesic Completion

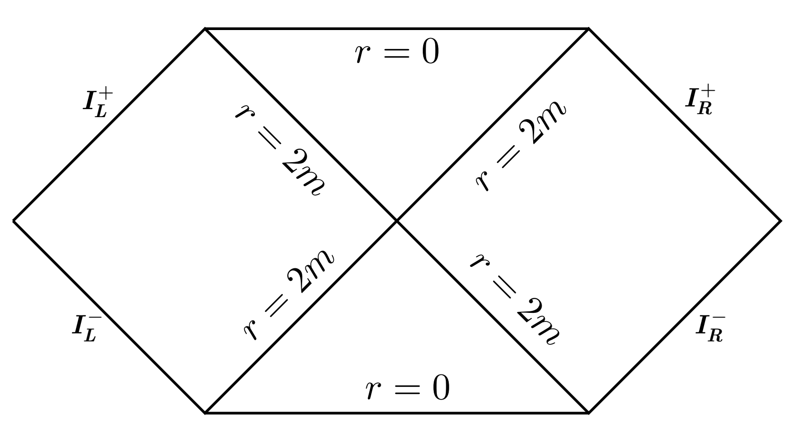


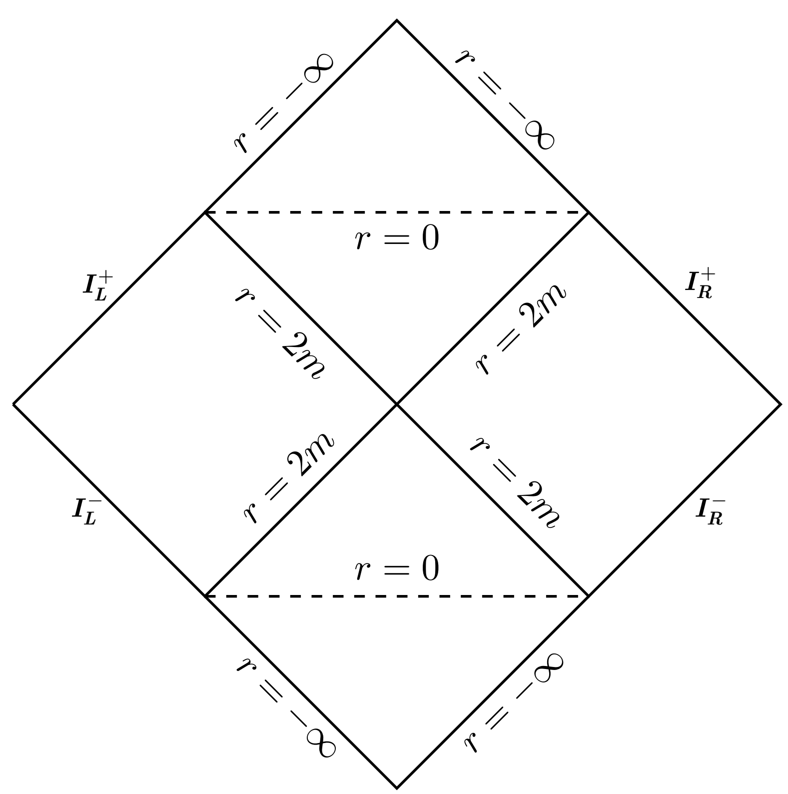


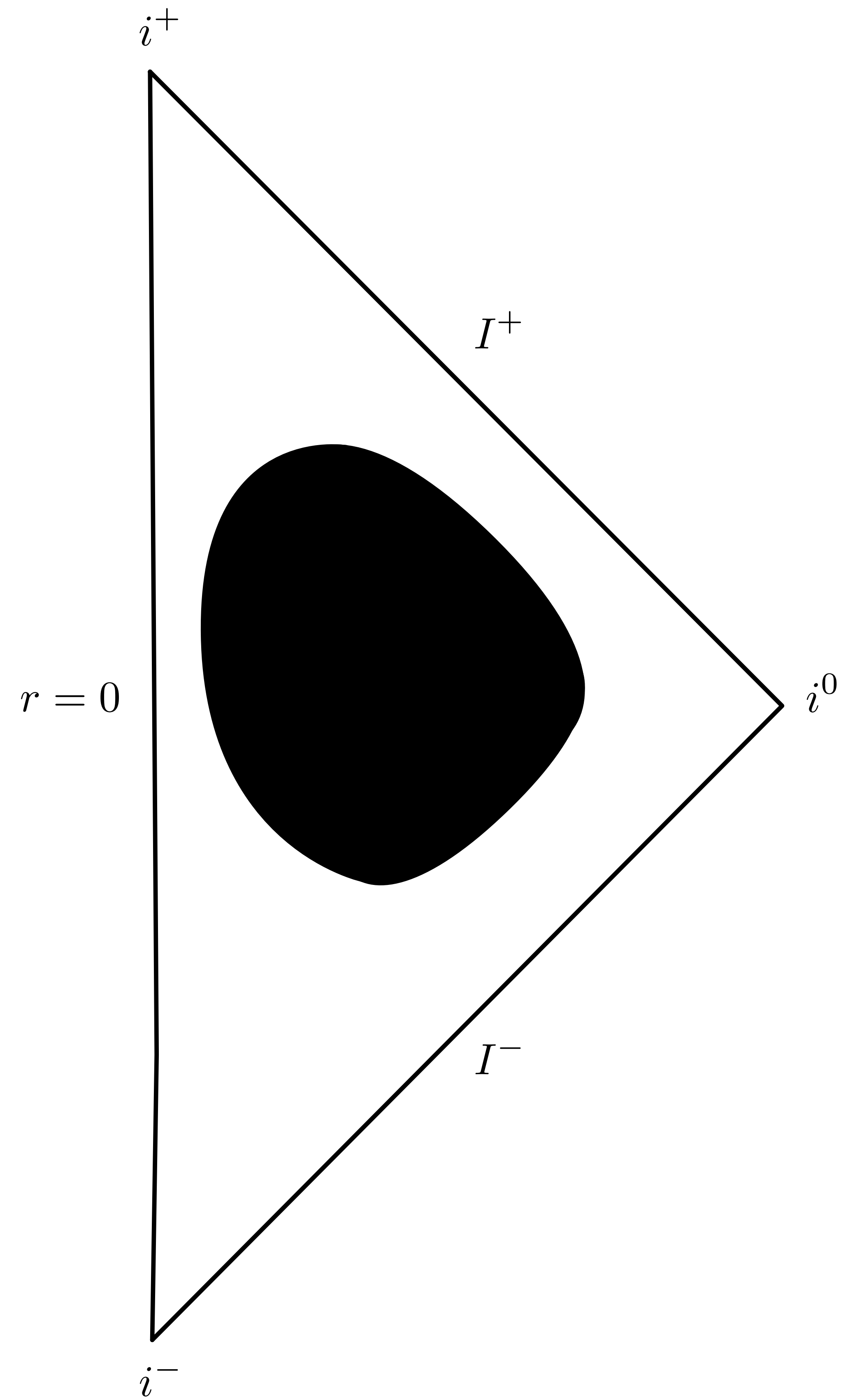
Metric Superspace



V. Frolov, C. Bambi, D. Malafarina, LM







Quantum Gravity in Cut-Off Regularization Scheme

Stefano Giaccari, LM, Lesław Rachwał, Yiwei Zhu.

$$\mathcal{L}_{\rm g} = -\frac{2}{\kappa_4^2}\left(R - 2\Lambda_{\rm cc} + G_{\mu\nu}\,\frac{e^{H(\Box)}-1}{\Box}R^{\mu\nu} + s_1 R^2\,\Box^{\gamma-2}R^2 + s_2\mathbf{Ric}^2\,\Box^{\gamma-2}\mathbf{Ric}^2 + s_\kappa R^2\,\Box^{\gamma-1}R\right).$$

$$\beta_\kappa = 0\,,\quad \beta_{R^2} = 0\,,\quad \beta_{\mathbf{Ric^2}} = 0\,,$$

$$\beta_{\Lambda_{\rm cc}} \neq 0\,.$$

Finite Entanglement Entropy (replica trick)

$$S_{\rm ent} = (\alpha\,\partial_\alpha - 1)W(\alpha)|_{\alpha=1}$$

$$E_\alpha = \Sigma \times \mathcal{C}_\alpha\,,$$

Σ : two dimensional surface,

$\mathcal{C}_\alpha$: cone with deficit angle $\delta = 2\pi(1-\alpha)$.

$$W(\alpha) = \frac{1}{2}\log\det\Big(\mathcal{H}_{\rm gr}(g_{\mu\nu}, A^c_\mu, \Phi, \Psi)\Big) \propto \underbrace{B_0}_{\Lambda_{\rm cc}}\,k^4 + \underbrace{B_2}_{R}\,k^2 + \underbrace{B_4}_{R^2,\,\mathbf{Ric}^2}\log\left(\frac{k^2}{\mu^2}\right)\Bigg|_{E_\alpha}.$$

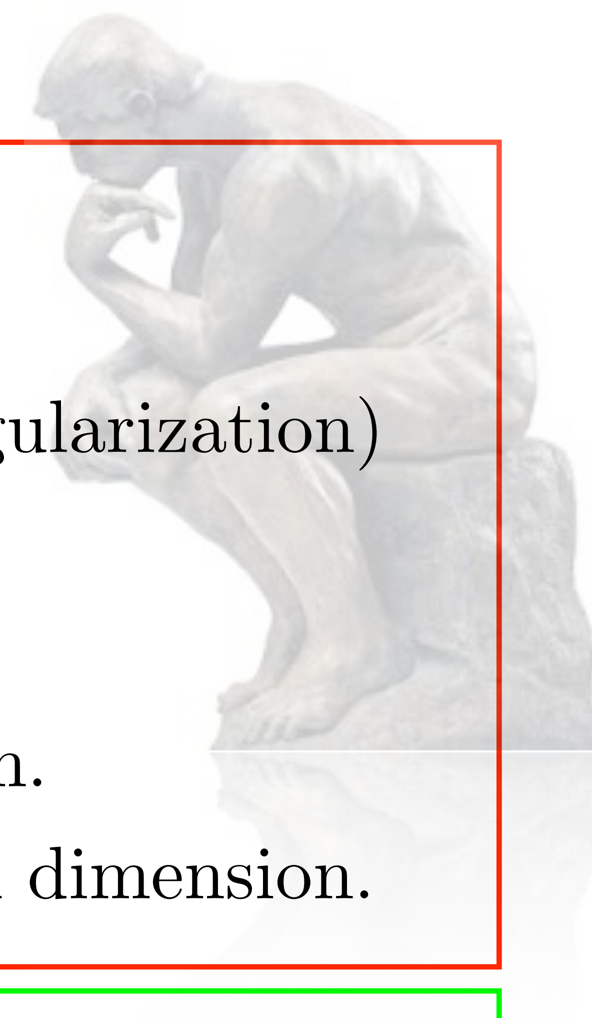
$$S_{\rm ent} = \frac{A}{4G_N} \quad \text{(only from the classical EH operator).}$$

Finite Yang-Mills Gauge theory

L.M. , M. Piva, L. Rachwal.

$$\mathcal{L}_{\mathrm{fin,\,gauge}} = -\frac{\alpha}{4}\mathrm{tr}\Big[\mathbf{F}e^{H(\mathcal{D}^2_\Lambda)}\mathbf{F} + s_g\mathbf{F}^2(\mathcal{D}^2_\Lambda)^{\gamma-2}\mathbf{F}^2\Big]$$

Summary and Conclusions



- Super-renormalizable Gravitational Theories.
  - Unitarity (no ghosts).
  - Renormalizability (Dimensional Regularization)

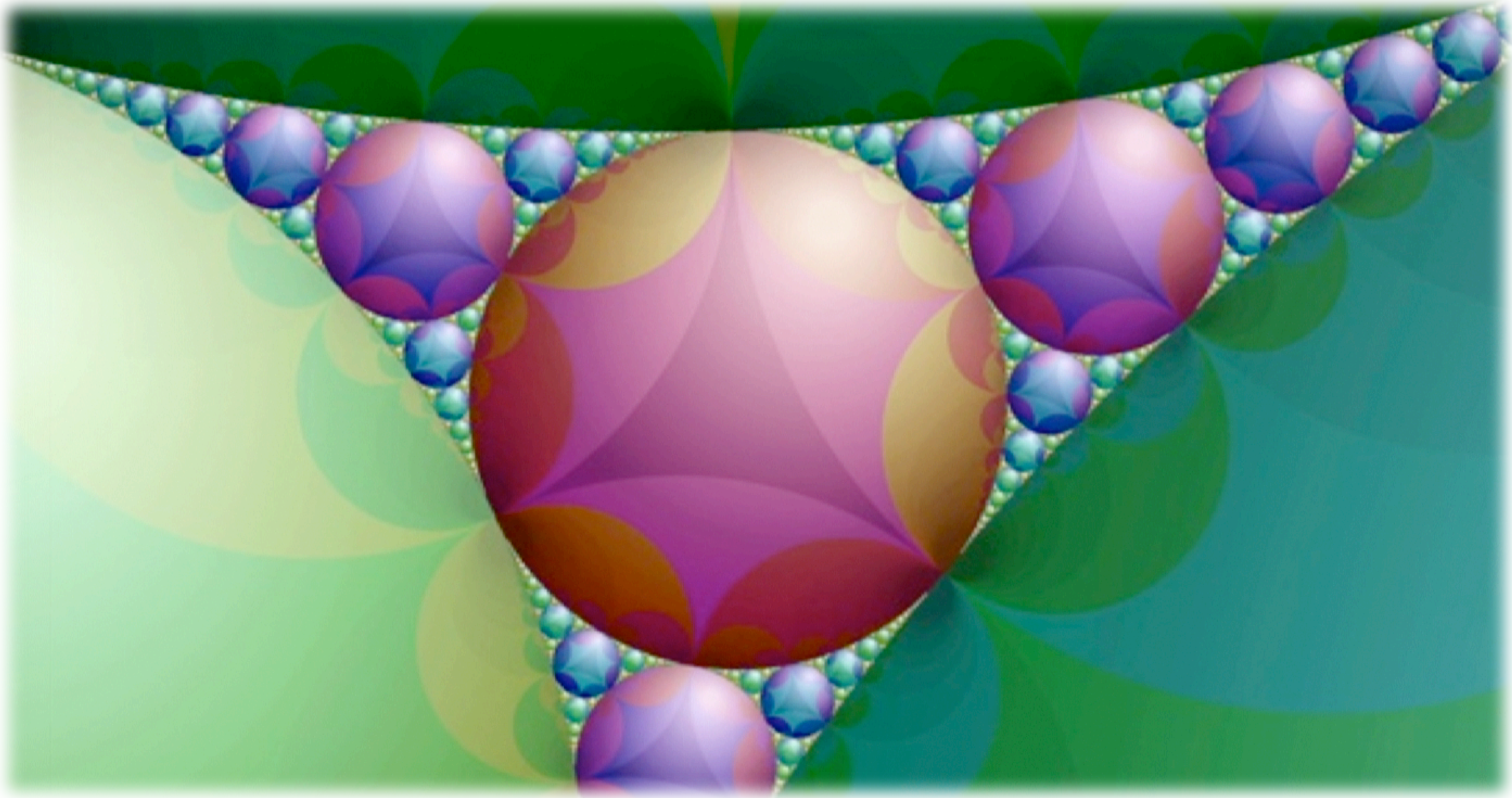
- Finite Gravitational Theories :
  - Quantum Gravity in Odd Dimension.
  - Local Terminating Potential in even dimension.

- Exact solutions and spacetime singularities :
  - Ricci flat, (A)dS, FRW spacetimes.
  - Singularity theorem in nonlocal gravity.

- Scattering amplitudes.

- Conformal invariant quantum gravity.
- Nonsingular spacetimes in Conformal gravity.

We have Finite Quantum Gravity !!!



$$\mathcal{L}_{\rm g} = -2\kappa_D^{-2}\sqrt{-g}\left(R + G_{\mu\nu}\,\frac{e^{H(-\Box_\Lambda)}-1}{\Box}R^{\mu\nu} + s_1 R^2\,\Box^{\gamma-2}R^2 + s_2 R_{\mu\nu}R^{\mu\nu}\,\Box^{\gamma-2}R_{\rho\sigma}R^{\rho\sigma}\right).$$

NONLOCAL GAUGE THEORIES

N. V. Krasnikov

Nonlocal Quantum Gravity

All theories of gravitation based on the use of local Lagrangians are nonr
le. In this section, we construct a superrenormalizable nonlocal Lagrangian
rely gravitational field. The generalization to the supersymmetric case is c

We choose the nonlocal Lagrangian for the gravitational field in the form

$$L=[-{}^{1}/_{2}\gamma R+Rf_1(\Box^{cov})R+R_{\mu\nu}f_2(\Box^{cov})R_{\mu\nu}-{}^{1}/_{3}Rf_2(\Box^{cov})R]\sqrt{-g}.$$

re, $\Box^{cov}$ is the covariant generalization of the operator $\Box=\partial_\mu\partial_\mu$.

Finite nonlocal gravity

Yu. V. Kuz'min

Institute of Nuclear Physics, Moscow State University

(Submitted 23 March 1989)

Yad. Fiz. **50**, 1630–1635 (December 1989)

A version of a nonlocal, superrenormalizable theory
certain values of the Lagrangian parameters the ultra

It is well known that the quantization of general relativity using the background-field formalism leads to a nonrenormalizable theory.[1] The addition to the Lagrangian of terms proportional to the square of the curvature tensor makes the theory renormalizable, but leads to a loss of unitarity at the tree level.[2] Here we abandon the locality of the Lagrangian in order to reconcile the properties of unitarity and renormalizability.

We write the Lagrangian as

$$\mathscr{L}=-\sqrt{-g}\Big[\frac{1}{2\varkappa}R+\alpha Rf_1\Big(\frac{\Box_c}{\mu^2}\Big)R+\beta R^{ik}f_2\Big(\frac{\Box_c}{\mu^2}\Big)R_{ik}+\gamma R^{iklm}f_3\Big(\frac{\Box_c}{\mu^2}\Big)R_{iklm}\Big]. \quad (1)$$

Here $\Box_c$ is the D'Alembertian constructed from covar

UCLA/97/TEP/2
hep-th/9702146

Superrenormalizable Gauge and Gravitational Theories[1]

E. T. Tomboulis[2]
Department of Physics
University of California, Los Angeles
Los Angeles, CA 90095-1547

arXiv:hep-th/9702146v1 20 Feb 1997

Abstract

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Super-renormalizable and finite gravitational theories

Leonardo Modesto *, Lesław Rachwał

Department of Physics & Center for Field Theory and Particle Physics, Fudan University, 200433 Shanghai, China

Received 13 August 2014; received in revised form 7 October 2014; accepted 12 October 2014
Available online 16 October 2014

PHYSICAL REVIEW D **86**, 044005 (2012)

Super-renormalizable quantum gravity

Leonardo Modesto

stitute for Theoretical Physics, 31 Caroline Street, Waterloo, Ontario N2L 2Y5, Canada
12 March 2012; revised manuscript received 27 May 2012; published 3 August 2012)

er we study perturbatively an extension of the Stelle higher derivative gravity involving an
er of derivative terms. We know that the usual quadratic action is renormalizable but suffers
arity problem because of the presence of a ghost (state of negative norm) in the theory. In this
onsider the theory first introduced by Tomboulis in 1997, but we expand and extensively
h the classical and quantum level. This theory is ghost-free, since the introduction of (in
entire functions in the model with the property does not introduce new poles in the
e local high derivative theory is recovered expanding the entire functions to the lowest
ass scale of the theory. Any truncation of the entire functions gives rise to the unitarity
if we keep all the infinite series, we do not fall into these troubles. The theory is
at one loop and finite from two loops on. Since only one-loop Feynman diagrams are
the theory is super-renormalizable. We analyze the fractal properties of the theory at high

Universal Gauge & Gravitational Theories

$$\boxed{-\frac{1}{2g^2}\int d^4x\,\mathrm{tr}F^2 \;\;\text{in}\;\; D=4}\quad \text{like} \quad \boxed{-\frac{2}{\kappa_2^2}\int d^2x\,\sqrt{|g|}R \;\;\text{in}\;\; D=2}$$

$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{4g_{\mathrm{YM}}^2}\left[\mathrm{tr}\,F e^{H(-\mathcal{D}_\Lambda^2)}F + \mathcal{V}_{\mathrm{YM}}\right] \quad (\mathrm{GAUGE-THEORY}).$$

$$\mathcal{L}_{\mathrm{gr}} = -2\kappa_D^{-2}\sqrt{|g|}\left[R + G_{\mu\nu}\,\frac{e^{H(-\Box_\Lambda)}-1}{\Box}R^{\mu\nu} + \mathcal{V}_{\mathrm{gr}}\right] \quad (\mathrm{GRAVITY}).$$

Finite Yang-Mills Gauge theory

L.M. , M. Piva, L. Rachwal.

$$\mathcal{L}_{\text{fin, gauge}} = -\frac{\alpha}{4}\text{tr}\left[\mathbf{F}e^{H(\mathcal{D}^2_\Lambda)}\mathbf{F} + s_g\mathbf{F}^2(\mathcal{D}^2_\Lambda)^{\gamma-2}\mathbf{F}^2\right]$$

Killer Operators

$$1) \quad -\frac{s_g}{4g^2}F^a_{\mu\nu}F^{\mu\nu}_a\Box^{\gamma-2}_\Lambda F^b_{\rho\sigma}F^{\rho\sigma}_b\,,$$

$$2) \quad -\frac{s_g}{4g^2}F^a_{\mu\nu}F^{\mu\nu}_b(\mathcal{D}^2_\Lambda)^{\gamma-2}F^b_{\rho\sigma}F^{\rho\sigma}_a\,.$$

$$\mathcal{L}_{\mathrm{ct}} := -\frac{\alpha}{4}(Z_\alpha - 1)\, F_a^{\mu\nu} F^a_{\mu\nu} = -\mathcal{L}_{\mathrm{div}} = -\frac{1}{\epsilon}\beta_\alpha\, F_a^{\mu\nu} F^a_{\mu\nu}.$$

$$0) \quad \beta_\alpha^{(\gamma)} = -\frac{(5 + 3\gamma + 12\gamma^2)}{192\pi^2} C_2(G)\,, \quad \gamma \geq 2\,,$$

$$1) \quad \beta_\alpha^{(s_g)} = \frac{s_g \Lambda^4}{2\pi^2 \omega},$$

$$2) \quad \beta_\alpha^{(s_g)} = \frac{s_g \Lambda^4}{4\pi^2 \omega}(1 + N_G).$$

$$\beta_\alpha^{(\gamma)} + \beta_\alpha^{(s_g)} = 0 \quad \Longrightarrow \quad s_g^* = -\frac{2\pi^2 \omega \beta_\alpha^{(\gamma)}}{\Lambda^4}\,.$$

$$\mathcal{L}_{\mathrm{fin,\,gauge}} = -\frac{\alpha}{4}\Big[F^a_{\mu\nu} e^{H(\mathcal{D}^2_\Lambda)} F_a^{\mu\nu} + \omega \frac{(5 + 3\gamma + 12\gamma^2)}{96\Lambda^4} C_2(G) F^a_{\mu\nu} F_a^{\mu\nu} (\mathcal{D}^2_\Lambda)^{\gamma-2} F^b_{\rho\sigma} F_b^{\rho\sigma}\Big].$$

Asymptotic Freedom, Finiteness, and Landau Pole

$$s_g \begin{cases} < \omega \frac{(5+3\gamma+12\gamma^2)}{96\Lambda^4} C_2(G)\,, & \text{Landau pole,} \\ = \omega \frac{(5+3\gamma+12\gamma^2)}{96\Lambda^4} C_2(G) \equiv s_g^*\,, & \text{finiteness,} \\ > \omega \frac{(5+3\gamma+12\gamma^2)}{96\Lambda^4} C_2(G)\,, & \text{asymptotic freedom.} \end{cases}$$

Dressed Propagator

$$-i \frac{e^{-H(k^2)}}{k^2 \left(1 + \beta_\alpha\, e^{-H(k^2)} \log(k^2/\mu_0^2)\right)}.$$

Black Supernovae

Classical Solution

Bambi, Malafarina, Marciano, Modesto

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa\, h_{\mu\nu}\,,$$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = dt^2 - a(t)^2\, \delta_{ij} dx^i dx^j\,,$$

$$a^2(t) = 1 - \kappa h(t)\,, \qquad h(t = t_{\rm i}) = 0\,, \qquad g_{\mu\nu}(t = t_{\rm i}) = \eta_{\mu\nu}\,,$$

$$h_{\mu\nu}(t) = h(t)\, \mathrm{diag}(0, \delta_{ij}) =: h(t)\, \mathcal{I}_{\mu\nu}\,;$$

$$\bar{h}_{\mu\nu} := h'_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\, h'^{\lambda}_{\lambda} = h(t)\, \mathrm{diag}(0, -2\delta_{ij}) = -2h(t)\mathcal{I}_{\mu\nu}\,, \qquad \partial^\mu \bar{h}_{\mu\nu} = 0\,.$$

$$\text{Radiation}: \quad a(t)^2 = \left|\frac{t}{t_0}\right|\,,$$

$$\text{Dust}: \quad a(t)^2 = \left|\frac{t}{t_0}\right|^{\frac{4}{3}}\,.$$

$$\tilde{h}(E) = \frac{2\pi\delta(E)}{\kappa} + \frac{2}{\kappa t_0 E^2}, \quad \text{(radiation)}$$

$$\tilde{h}(E) = \frac{2\pi\delta(E)}{\kappa} + \frac{4\Gamma(\frac{4}{3})}{\sqrt{3}\kappa t_0^{4/3}\, |E|^{7/3}}, \quad \text{(dust)}\,.$$

The Calculation

C. Bambi, D. Malafarina, L.M.
G. Calcagni, L. M., P. Nicolini.

$$\Box \bar{h}'_{\mu\nu} = 8 G_N T_{\mu\nu}\,,$$

$$e^{H(\Box)}\Box \bar{h}'^{\,\mathrm{nl}}_{\mu\nu} = 8 G_N T_{\mu\nu}\,.$$

$$e^{H(\Box)}\,\bar{h}'^{\,\mathrm{nl}}_{\mu\nu} = \bar{h}'_{\mu\nu} \quad \Longrightarrow \quad \tilde{\bar{h}}'^{\,\mathrm{nl}}_{\mu\nu}(k) = e^{-H(k^2)}\tilde{\bar{h}}'_{\mu\nu}(k) \quad \Longrightarrow \quad \tilde{h}^{\mathrm{nl}}(E) = e^{-H(E^2)}\tilde{h}(E)\,.$$

Asymptotic Freedom

$$\mathcal{L}_{\mathrm{G}} = \frac{2}{\kappa_D^2(t)}\left(R - G_{\mu\nu}\frac{e^{H(-\Box_\Lambda)}-1}{\Box}R^{\mu\nu}\right) \sim \partial h\, e^{H(-\Box_\Lambda)}\,\partial h + \kappa_D\, h\partial h\,\partial h + O(\kappa_D^2)\,,$$

$$\kappa_D^{-2}(t) = \kappa_{D\,o}^{-2} + \beta_{\kappa_D} t\,.$$

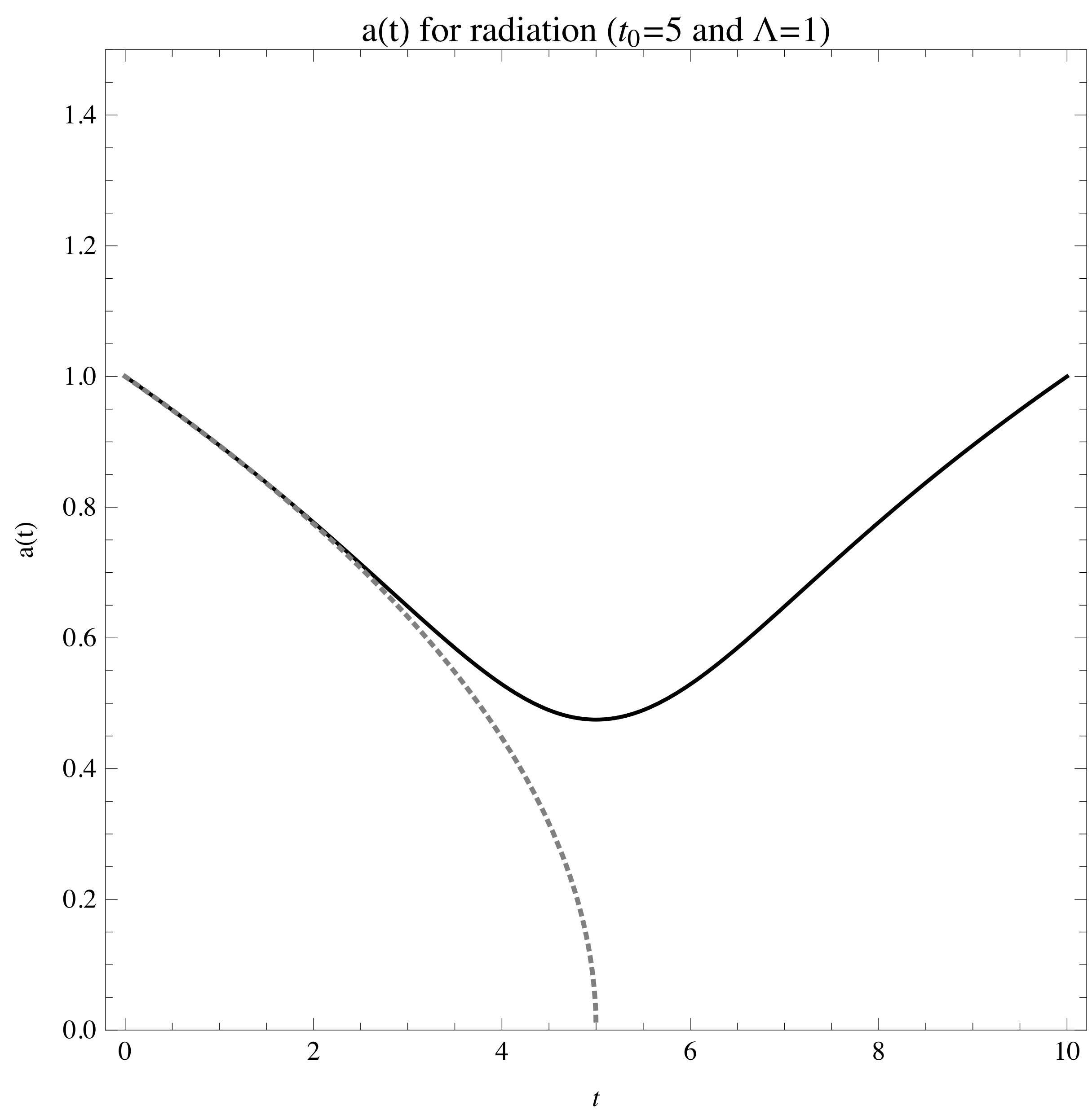
Gravitational Collapse &/or Cosmology

C. Bambi, D. Malafarina, L.M.   G. Calcagni, L. M., P. Nicolini.

From the propagator & asymptotic freedom : $D(k) \sim \frac{V(k)}{k^2}$.

$$n = 1 \implies V(-\Box_\Lambda) = e^{-\Box/\Lambda^2} ,$$

Radiation : $$a^2(t) = \frac{2e^{-\frac{1}{4}\Lambda^2(t-t_0)^2}}{\Lambda\sqrt{\pi}\, t_0} + \frac{(-t+t_0)\,\mathrm{erf}\left(\frac{\Lambda(-t+t_0)}{2}\right)}{t_0}.$$



The Solution

$$a_{\rm cl}(t) = \left|\frac{t}{t_{\rm i}}\right|^{p} \qquad \text{and} \qquad h_{\rm cl}(t) = \frac{1}{\kappa}\left[1 - \left|\frac{t}{t_{\rm i}}\right|^{2p}\right] ,$$

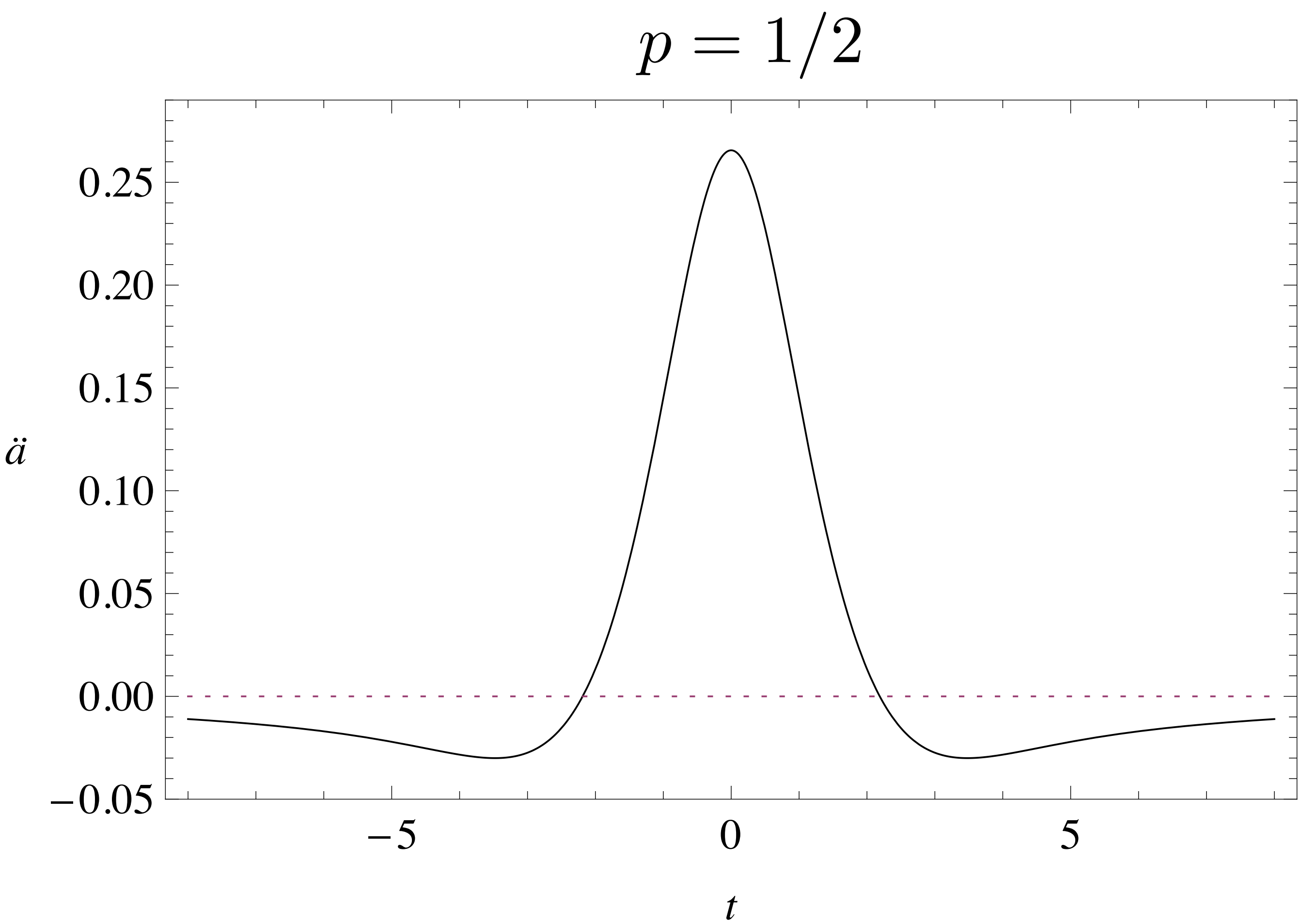
$$\kappa\, h(t) = 1 - \left(\frac{2}{\Lambda t_{\rm i}}\right)^{2p} \frac{\Gamma\left(\frac{1}{2}+p\right)}{\sqrt{\pi}}\, {}_1F_1\left(-p; \frac{1}{2}; -\frac{1}{4}t^2\Lambda^2\right) ,$$

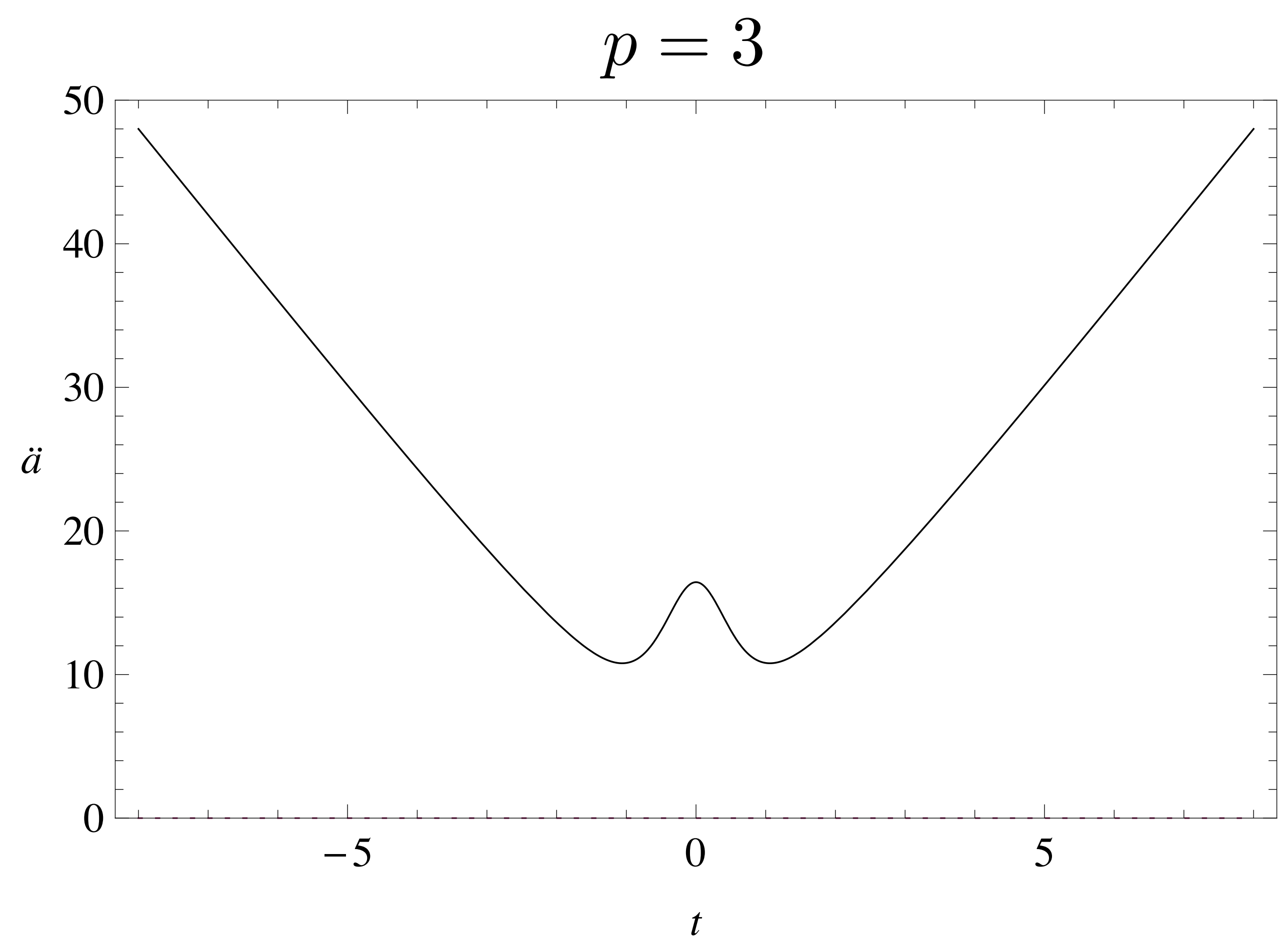
$$a(t) = \left(\frac{2}{\Lambda t_i}\right)^{p} \sqrt{\frac{\Gamma\left(\frac{1}{2}+p\right)}{\sqrt{\pi}}\, {}_1F_1\left(-p; \frac{1}{2}; -\frac{1}{4}t^2\Lambda^2\right)} .$$

Radiation $p = 1/2$:

$$a(t) = \sqrt{\frac{2e^{-\frac{1}{4}\Lambda^2 t^2}}{\sqrt{\pi}\,\Lambda t_{\rm i}} + \frac{t}{t_{\rm i}}\,\mathrm{erf}\left(\frac{\Lambda t}{2}\right)} .$$

Super-acceleration





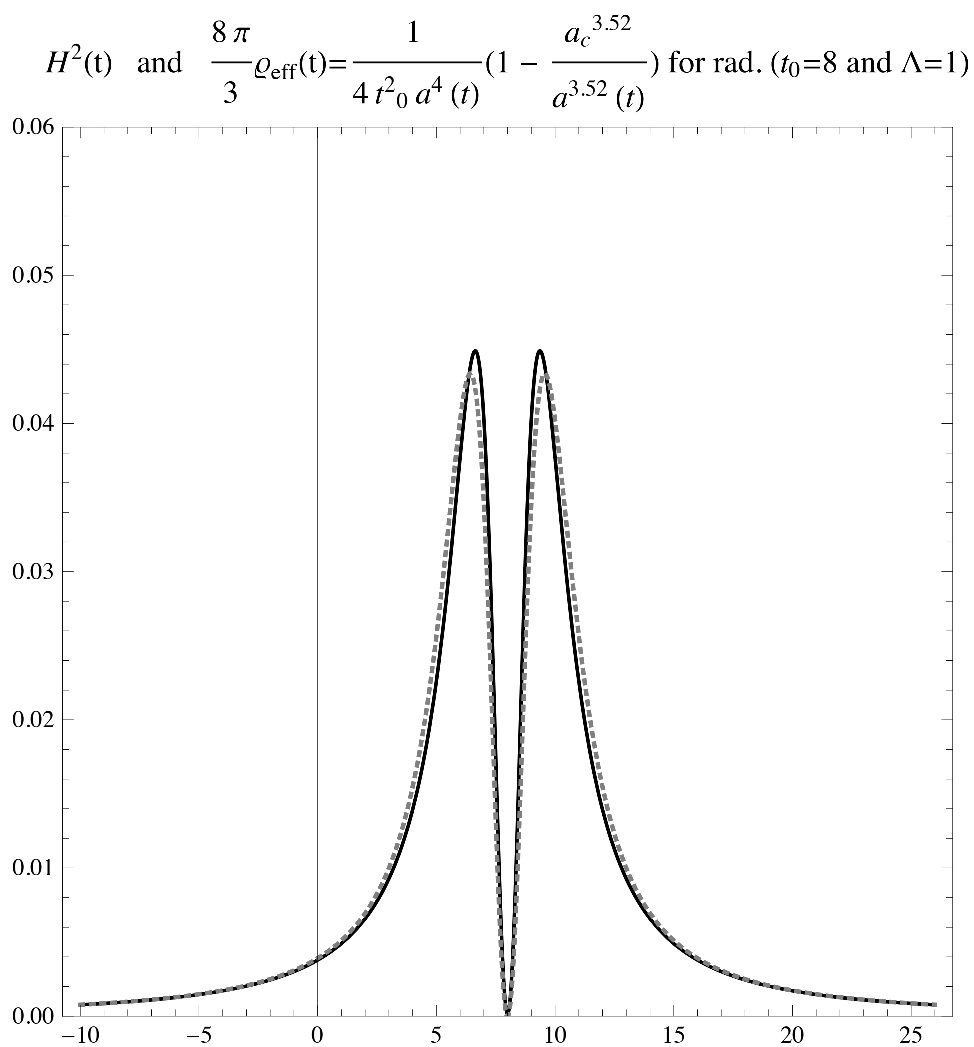
$$H(t) = \frac{\dot{a}}{a} = \frac{p\Lambda^2 t}{2} \frac{{}_1F_1\left(1-p; \frac{3}{2}; -\frac{1}{4}t^2\Lambda^2\right)}{{}_1F_1\left(-p; \frac{1}{2}; -\frac{1}{4}t^2\Lambda^2\right)} \, .$$

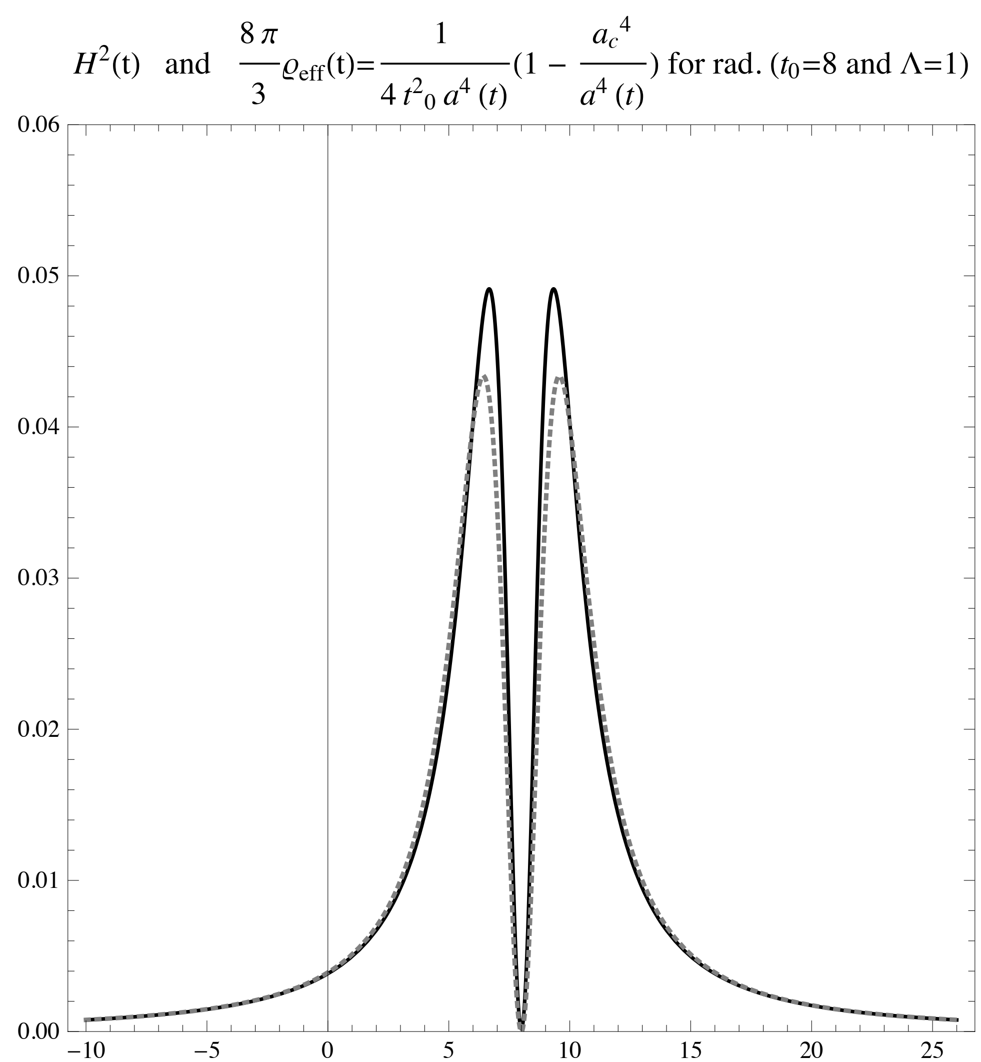
For $t \to 0 \quad H(t) \sim \frac{p\Lambda^2}{2} t$,

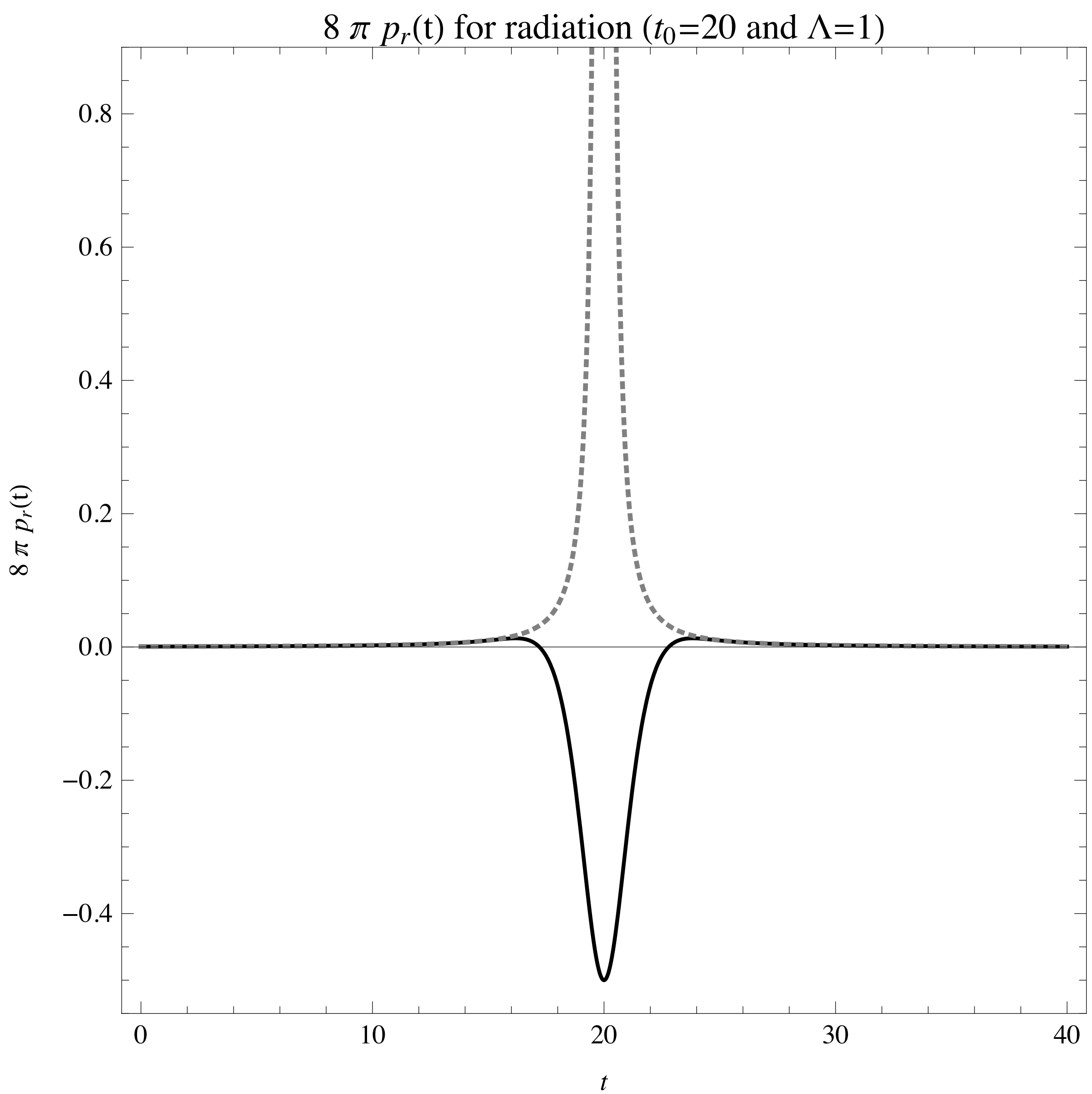
$$a(t) \sim e^{\frac{p}{4}\Lambda^2 t^2} \, .$$

Effective Theory

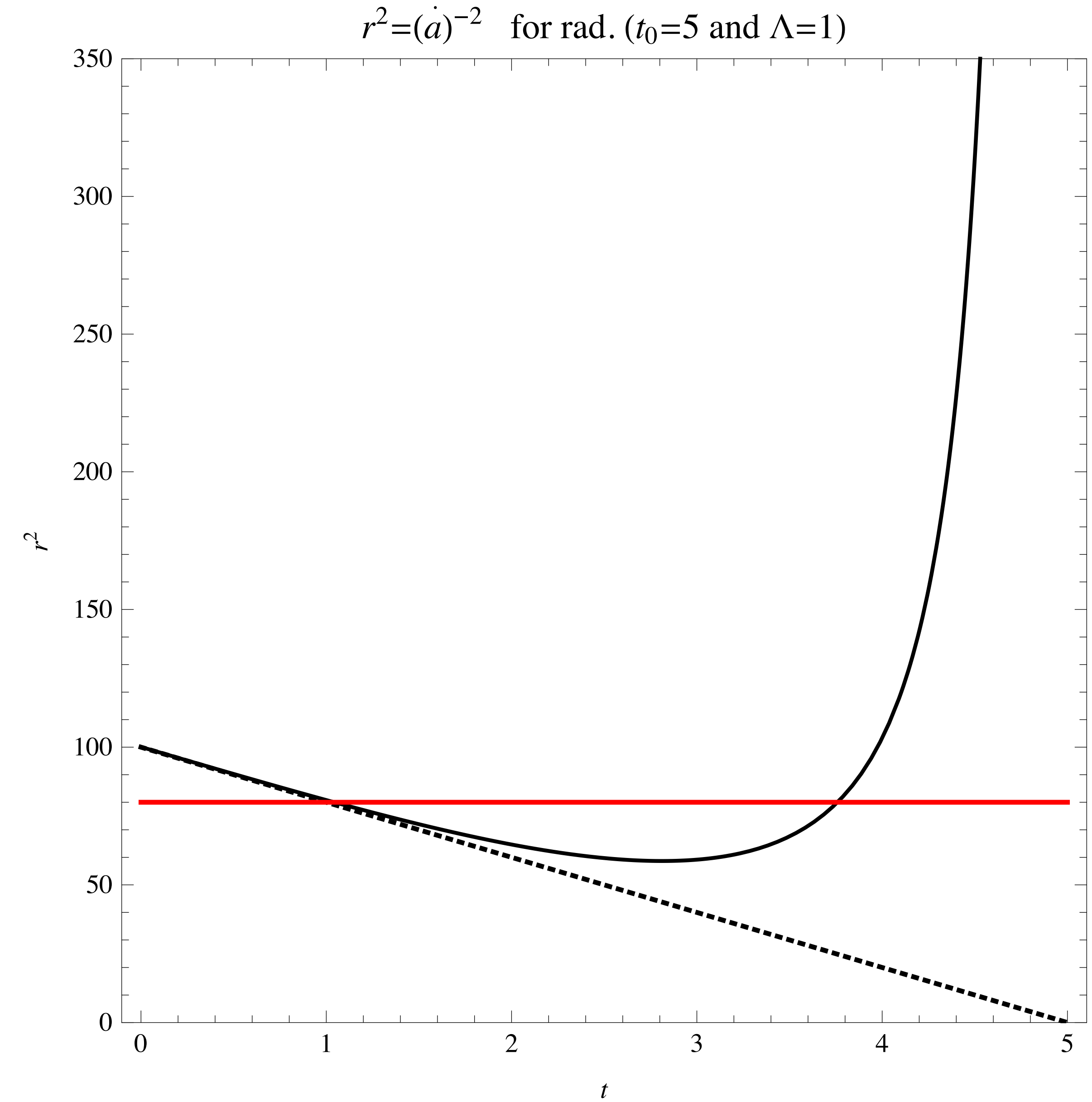
$$H^2 := \frac{8\pi G}{3}\rho_{\rm eff} = \frac{8\pi G}{3}\rho\left(1 - \left(\frac{\rho}{\rho_{\rm cr}}\right)^\alpha\right) \quad \text{where} \quad \rho = \frac{\rho_0}{a^4}\,.$$



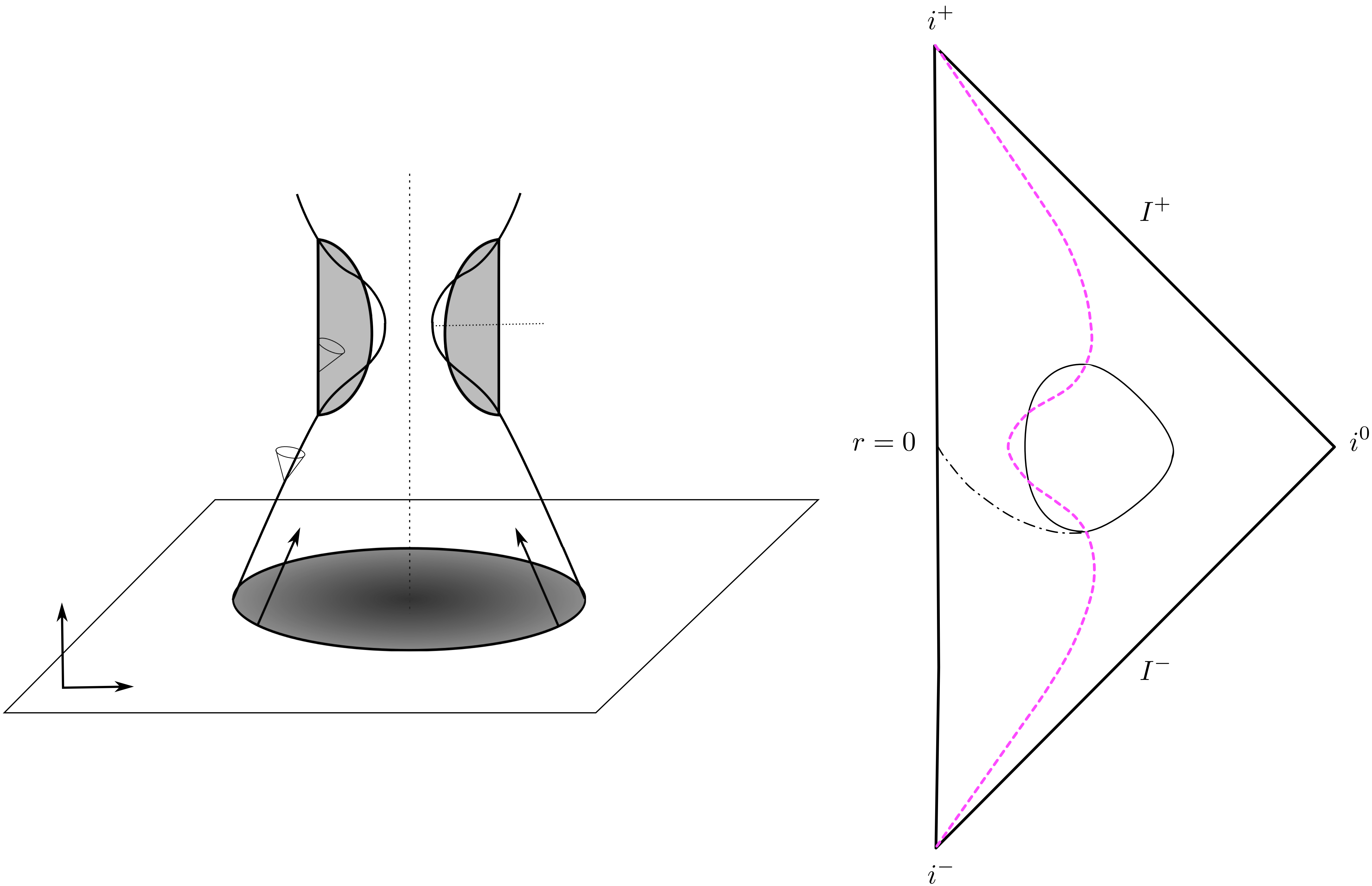




8 π p_r(t) for radiation (t_0=20 and Λ=1)
8 π p_r(t)
t

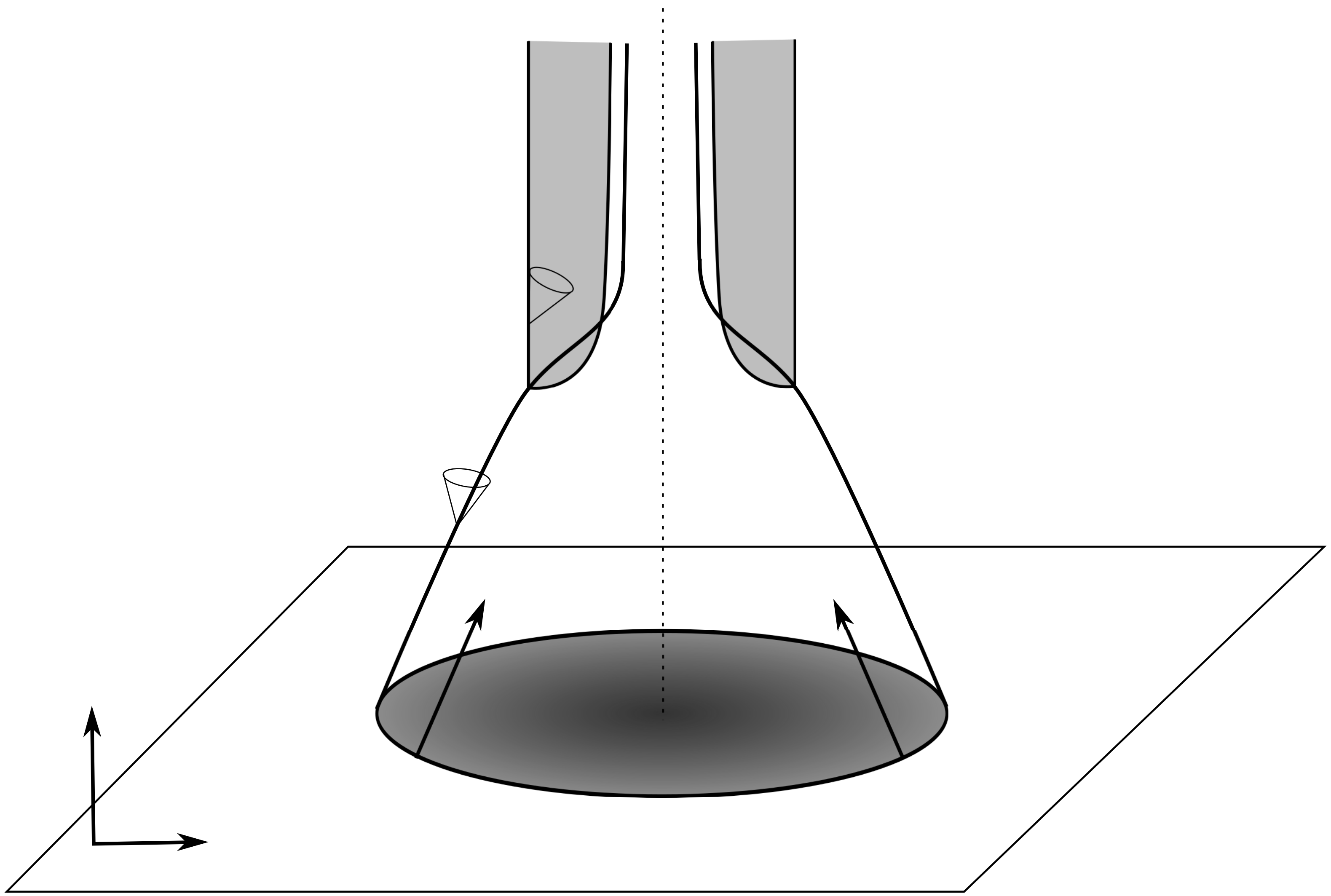


$r^2=(\dot{a})^{-2}$ for rad. (t_0=5 and Λ=1)
r^2
t


i^+
I^+
$r = 0$
i^0
I^-
i^-

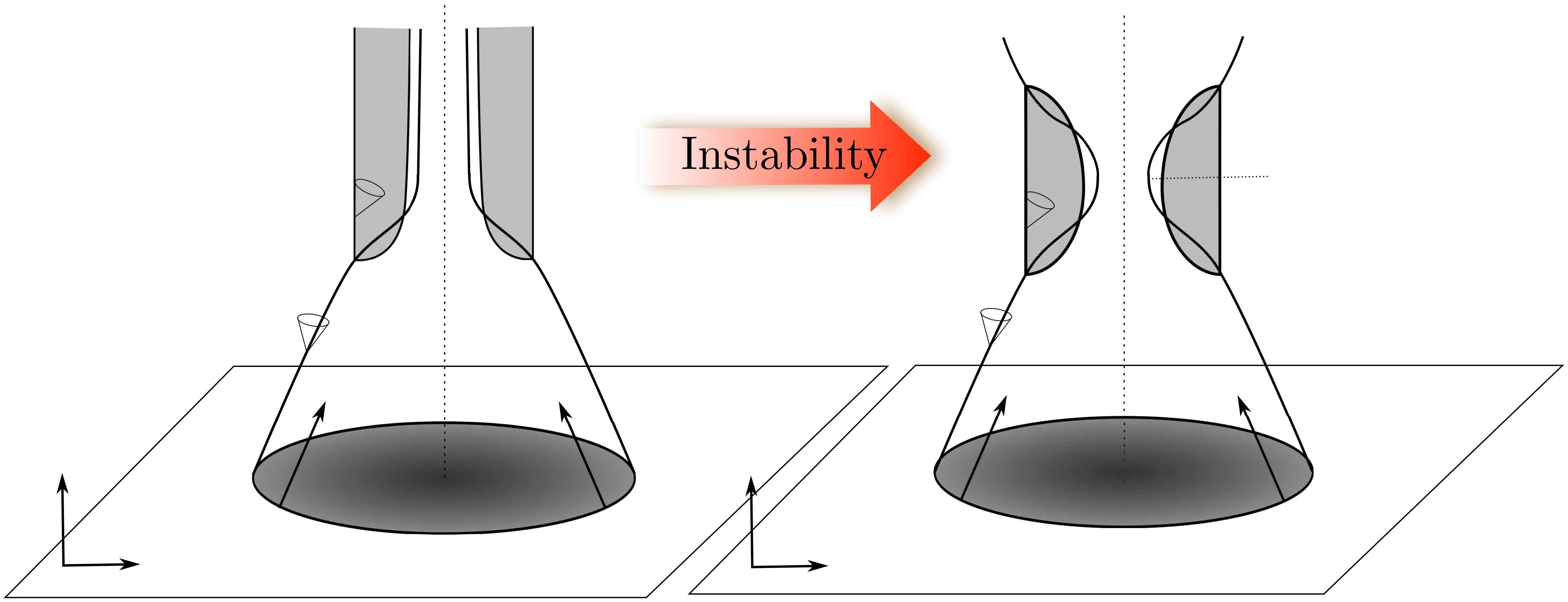
The interior from the exterior

see Torres.



The interior from the exterior

see Torres.



Universal Properties

1. Gravitational potential $\Phi(r) \longrightarrow \text{const.} < 0$,

2. Graviton propagator $G(x) \longrightarrow \text{const.} > 0$,

3. Spherically sym. sol. (black holes) deSitter metric near $r \sim 0$,

$$\forall \quad e^{H(-\Box_\Lambda)} \, .$$

4. The beta functions for the Tomboulis form factor depend only on 1, 2, or 3 parameters.

5. Terminating black holes (no black hole formation). Black Supernova.