Black H●les and Tunneling

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Ian Moss and Ben Withers,1401.0017
Philipp Burda, Ian Moss 1501.04937, 1503.07331, 1601.02152

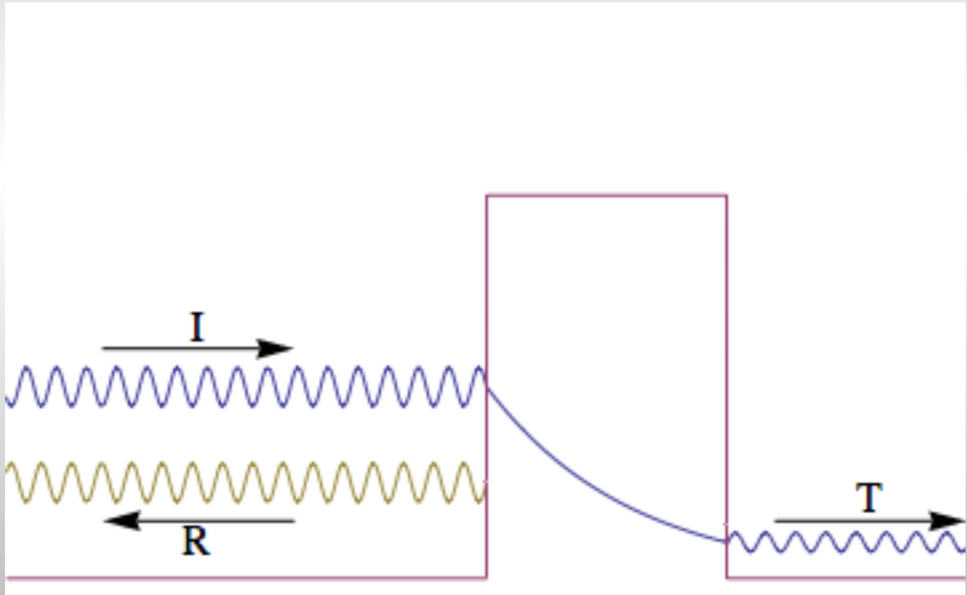
Quantum Tunneling

Tunneling is an example of Quantum Mechanics in action – a classical particle with energy less than barrier height will rebound,

but quantum mechanically the wave function never cuts off under a finite barrier, but decays – meaning that a little emerges through the other side:

Quantum Tunneling

Standard 1+1 Schrodinger tunneling exactly soluble



$$|T|^2 = \frac{1}{1 + \frac{V_0^2 \sinh^2 \Omega d}{4E(V_0 - E)}} \approx e^{-2\Omega d}$$

$$\Omega^2 = \frac{2m}{\hbar^2}(V_0 - E)$$

$$\Omega d = \frac{1}{\hbar}\int_0^d \sqrt{2m(V_0 - E)}dx$$

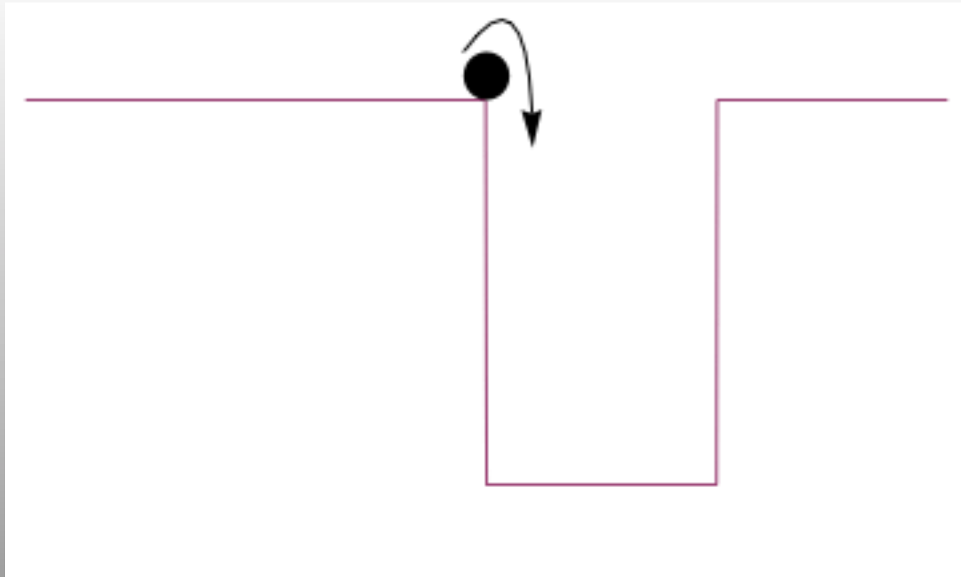
Euclidean Perspective

Now rotate to imaginary time:

$$t \to i\tau$$

A classical particle moving in imaginary time has kinetic energy equal to the potential drop, so the amplitude $|T|^2$ now looks like the action integral for this classical motion.

$$\frac{1}{2}\dot{x}^2 = \Delta V$$



$$\int \sqrt{2\Delta V}\, dx = \int 2\Delta V\, d\tau$$
$$= \int \left(\Delta V + \frac{1}{2}\dot{x}^2\right) d\tau$$

Euclidean Trick

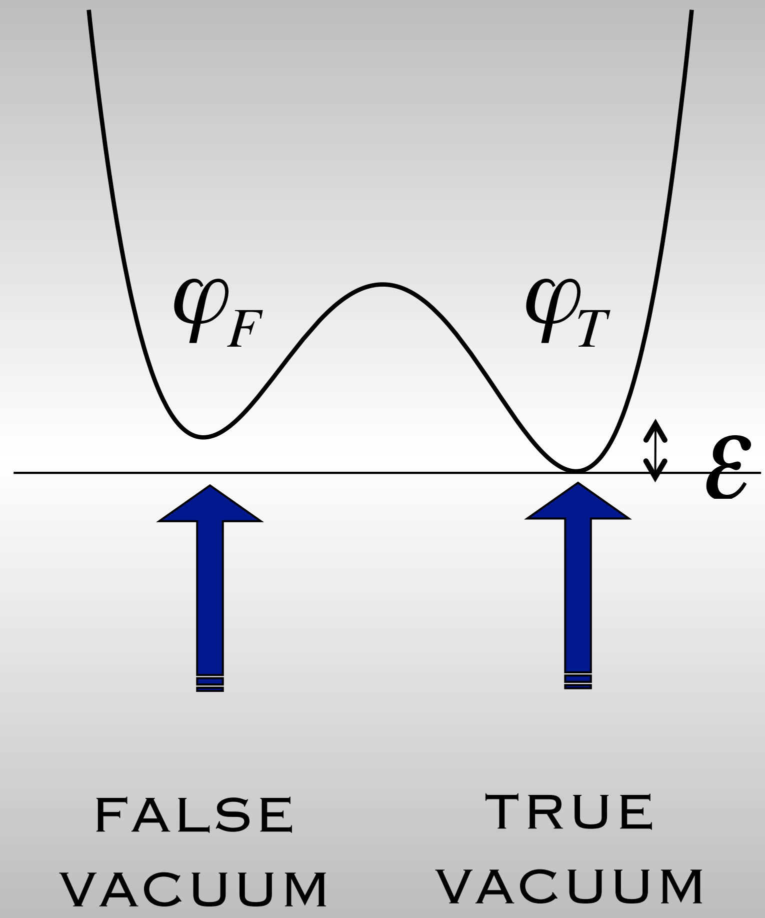
Generally, to compute leading behaviour of a tunneling amplitude take action of a classical particle moving in an inverted potential. The particle rolls from the (now) unstable point to the “exit” and back again – a “bounce”.

The action of this bounce gives the exponent in the amplitude of the wavefunction – a nice way of computing tunneling probability.

The Vacuum

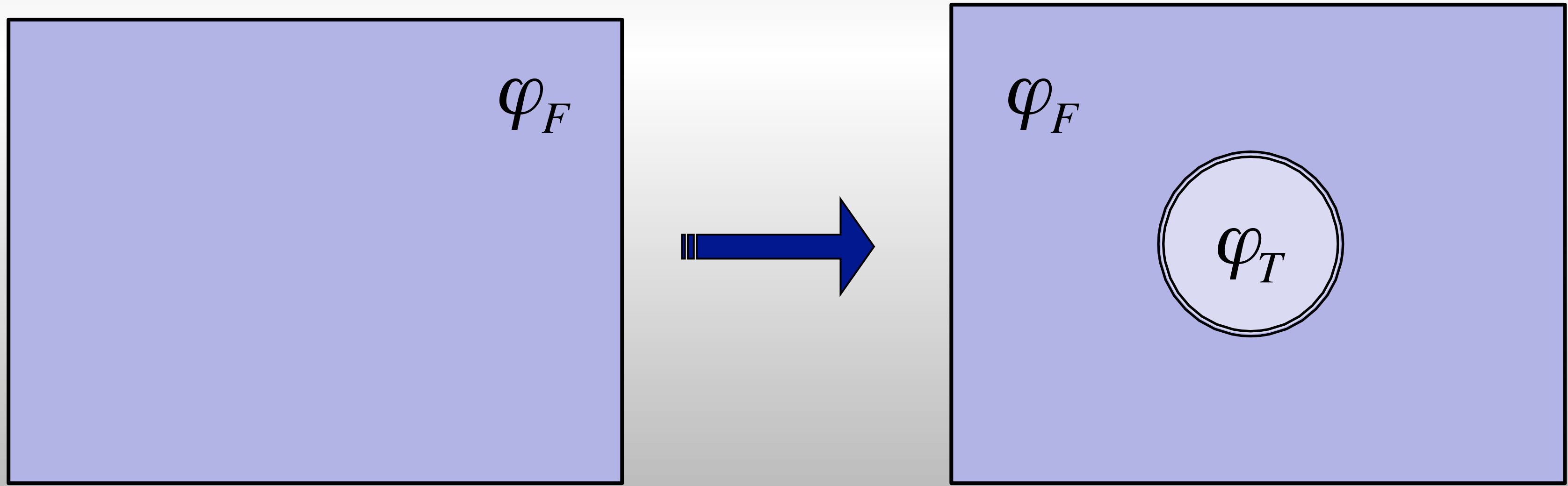
Even maybe not a true ground state at all!

Here, at low energies, if we live in the left "vacuum" we see a 'normal' particle spectrum (vacuum) and do not see it is not a global minimum.



Coleman Bounce

Coleman described this by the Euclidean solution of a bubble of true vacuum inside false vacuum separated by a “thin wall” (cf the Euclidean tunneling)



Like steam – we gain energy from moving to true vacuum, but the bubble wall costs energy

Coleman

Solving the Euclidean field equations should give the saddle point approximation for the tunneling solution.

$$\frac{d^2\phi}{d\tau^2} + \nabla^2\phi = -\frac{\partial V}{\partial \phi} = 2\lambda\phi(\phi^2 - \eta^2) + \mathcal{O}(\epsilon)$$

Original work of Coleman took a field theory with a “false” vacuum: in limit of small energy difference (relative to barrier) transition modeled by a “thin wall” bubble.

$$\phi'' + \frac{3}{\rho}\phi' = 2\lambda\phi(\phi^2 - \eta^2) \qquad [\rho^2 = \tau^2 + \mathbf{x}^2]$$

$$\phi \approx \eta \tanh[\sqrt{\lambda}\eta(\rho - \rho_0)]$$

Euclidean Action

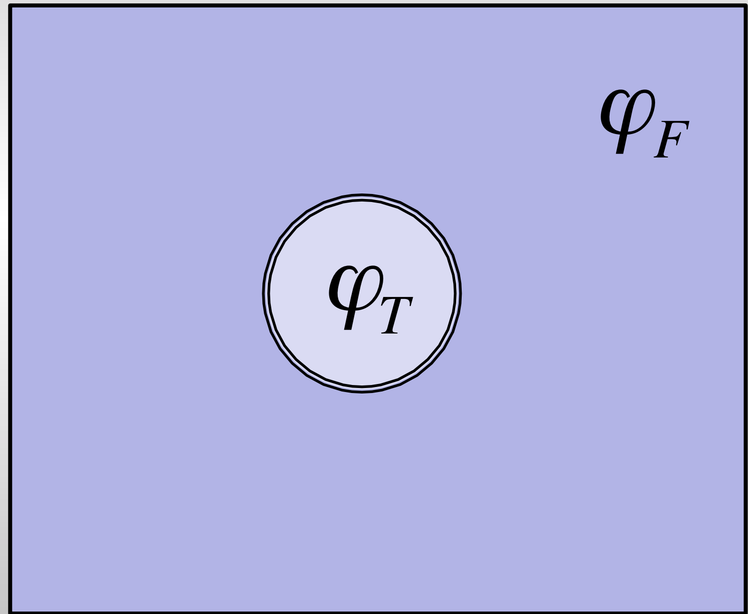
Amplitude determined by action of Euclidean tunneling solution: "The Bounce"

$$\mathcal{B} = \varepsilon \int d^4x\sqrt{g} - \sigma \int d^3x\sqrt{h}$$

$$\sim \frac{\pi^2}{2}\varepsilon R^4 - 2\pi^2\sigma R^3$$

GAIN FROM VACUUM

COST OF WALL



Coleman

Since the bounce is a solution to eqns of motion, it should be stationary under variation of R:

$$R = \frac{3\sigma}{\varepsilon} \quad , \quad \mathcal{B} = \frac{27\pi^2\sigma^4}{2\varepsilon^3}$$

Tunneling amplitude:

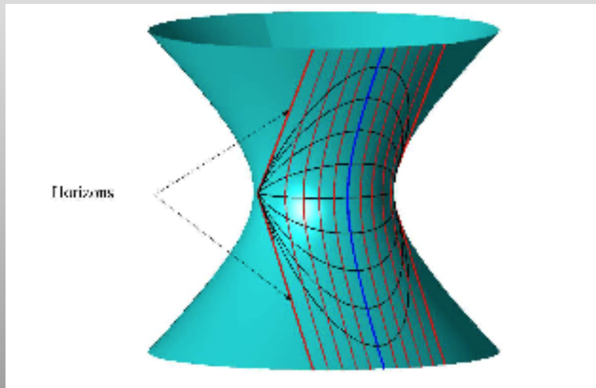
$$\mathcal{P} \sim e^{-\mathcal{B}/\hbar}$$

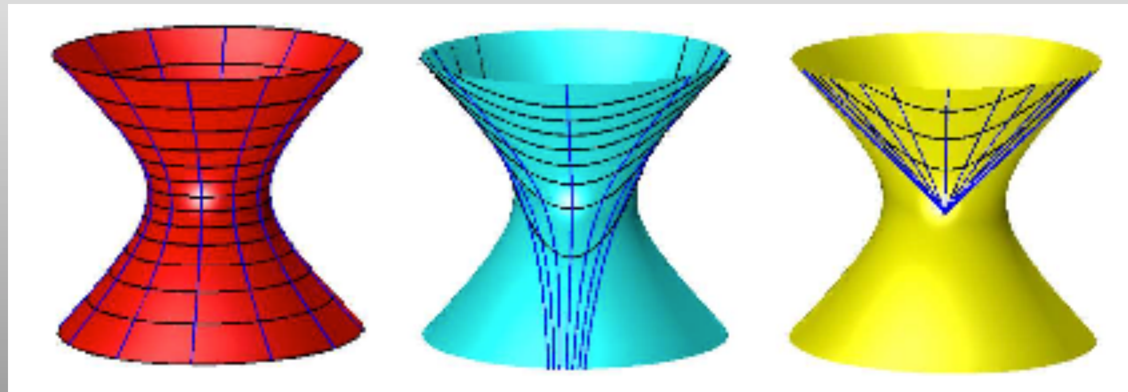
(Notice, R is big, so justifies use of the "thin wall" approximation.)

Gravity and the Vacuum

Vacuum energy gravitates – e.g. our current universe is accelerating – so we must add gravity to our picture.

A cosmological constant gives us de Sitter spacetime.





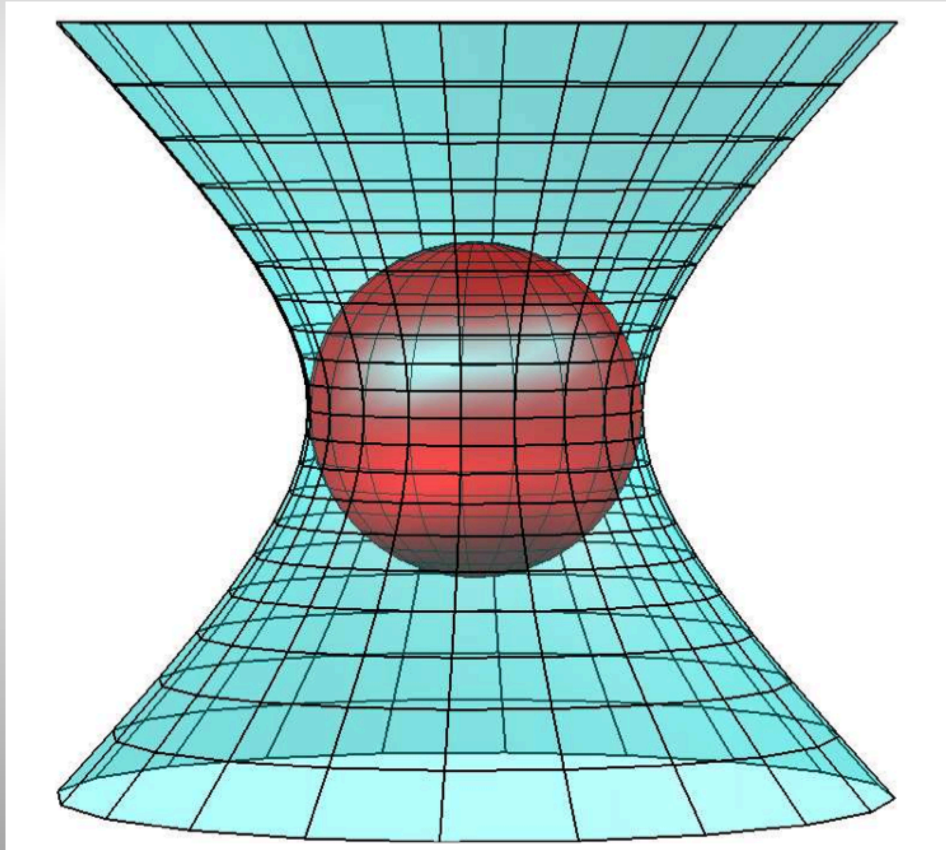
Coleman de Luccia (CDL)

Coleman and de Luccia showed how to do this with a bubble wall.

o The instanton is a solution of the Euclidean Einstein equations with a bubble of flat space separated from dS space by a thin wall.
o The wall radius is ***determined*** by the Israel junction conditions
o The action of the bounce is the difference of the action of this wall configuration and a pure de Sitter geometry.

Coleman and de Luccia, PRD21 3305 (1980)

De Sitter spacetime has a Lorentzian (real time) and Euclidean (imaginary time) spacetime. The real time expanding universe looks like a hyperboloid and the Euclidean a sphere:

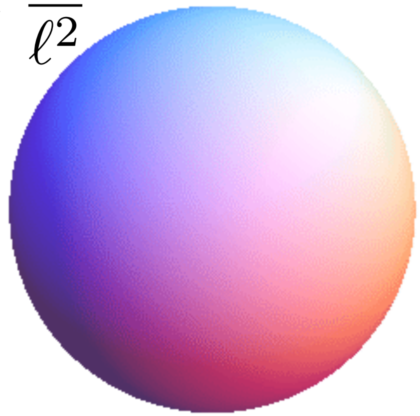


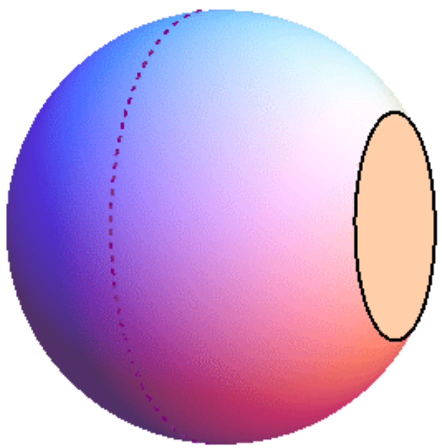
Our instanton must cut the sphere and replace it with flat space (true vacuum).

CDL Instanton

Euclidean de Sitter space is a sphere, of radius ℓ related to the cosmological constant. The true vacuum has zero cosmological constant, so must be flat.

$$\Lambda = \frac{3}{\ell^2}$$



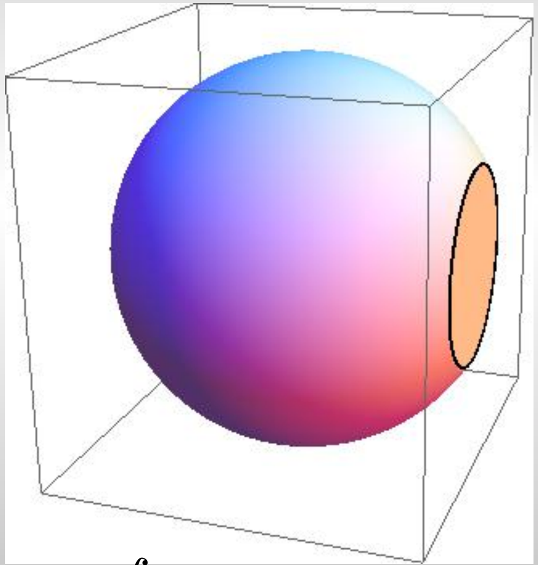


The bounce looks like a truncated sphere.

CDL Action

To compute action, we have to integrate Ricci curvature

$$ds^2_{\rm dS} = d\chi^2 + \ell^2 \sin^2 \frac{\chi}{\ell}\, d\Omega^2_{III} \quad , \quad S_{\rm dS} = -\frac{\ell^2 \pi}{G}$$



Israel conditions give truncation radius:

$$\frac{3}{\ell}\left(\cot \chi_0 - \csc \chi_0\right) = -4\pi G \sigma$$

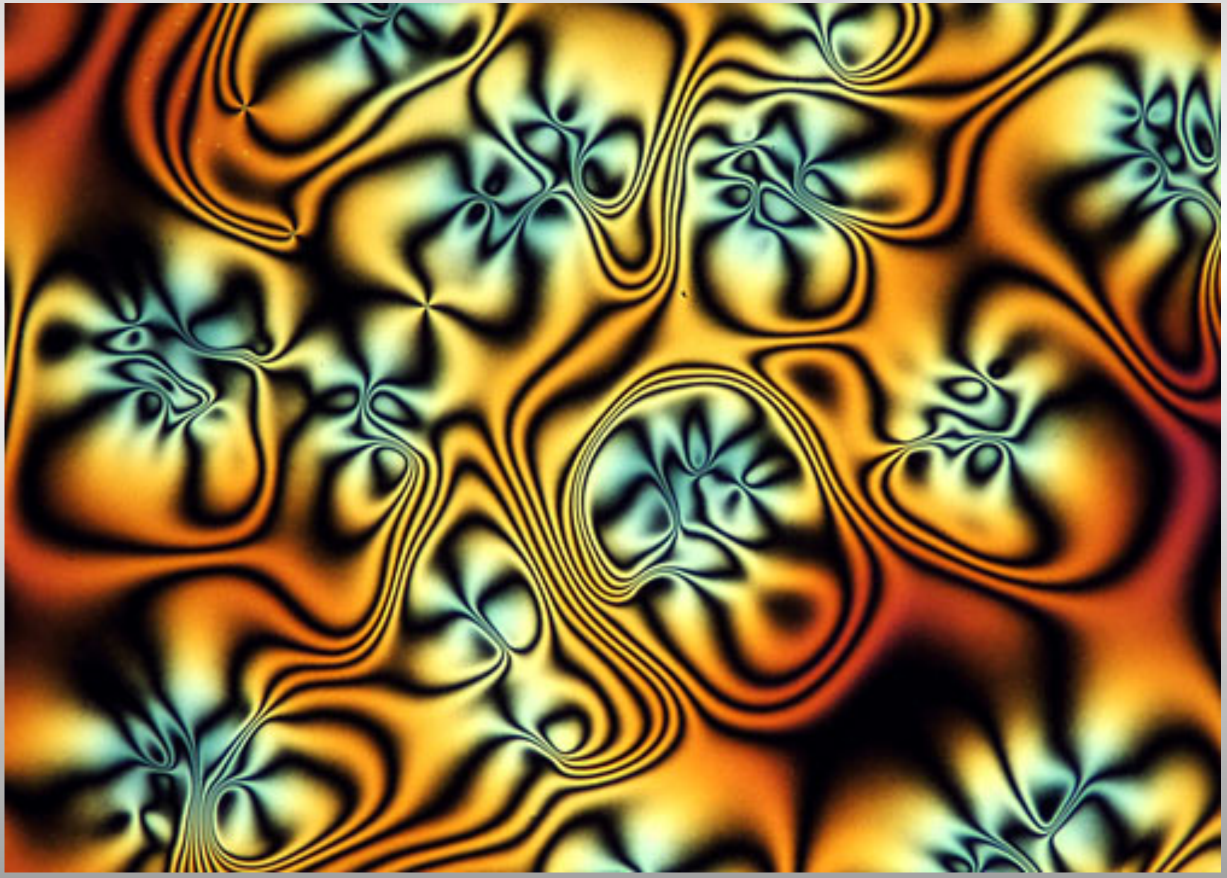
hence bounce action:

$$\mathcal{B} = -\frac{\Lambda}{8\pi G}\int_{\rm int} d^4x\sqrt{g} - \frac{\sigma}{2}\int_{\mathcal{W}} d^3x\sqrt{h}$$

$$= \frac{\pi \ell^2}{4G}(1-\cos\chi_0)^2 = \frac{\pi\ell^2}{G}\frac{16\bar{\sigma}^4\ell^4}{(1+4\bar{\sigma}^2\ell^2)^2}$$

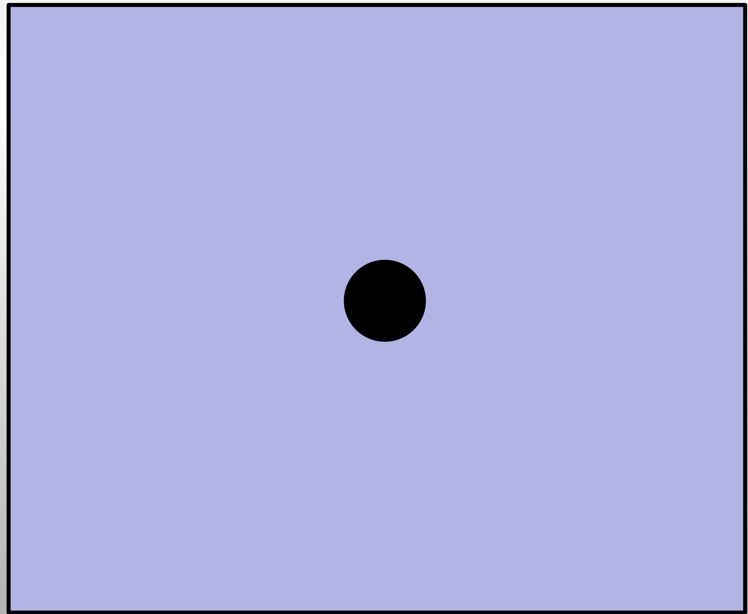
$$\bar{\sigma} = 2\pi G\sigma$$

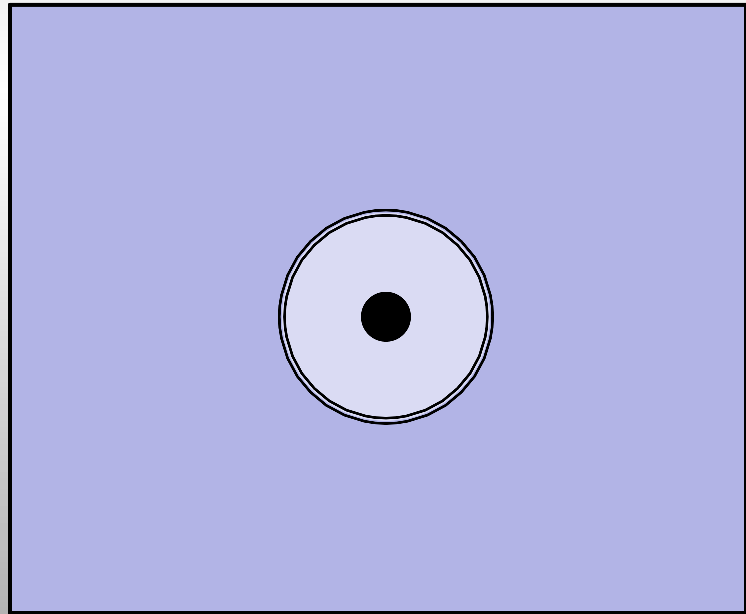
The universe is complex – so how dependent are our results on the assumptions of homogeneity and isotropy? Phase transitions in nature are more “dirty” – how does that affect modelling?



Tweaking CDL

The bubble of true vacuum has a spherical symmetry, so we can add a black hole at "minimal expense"!





RG, Moss & Withers, 1401.0017

A more general bubble

The wall now will separate two different regions of spacetime, each of which solve the Einstein equations:

$$ds^2 = f(r)dt^2 \pm \left[f^{-1}(r)dr^2 + r^2 d\mathbf{x}_\kappa^2\right]$$

$$f(r) = \kappa - \frac{\Lambda}{3}r^2 - \frac{2GM}{r}$$

The regions in general have different cosmological constants, and possibly a black hole mass.

Bowcock, Charmousis, RG: CQG17 4745 (2000)

Wall trajectories

We can compute the wall trajectory and use the Israel junction conditions determine the equation of motion:

Lorentz

Euclid

$$\pm\left(\frac{\dot{R}}{R}\right)^2 = \bar{\sigma}^2 - \frac{\bar{f}}{R^2} + \frac{(\Delta f)^2}{16\bar{\sigma}^2 R^4}$$

- a Friedmann like equation for R. Similar in appearance to CDL, and can match Lorentz and Euclidean solutions at R=0

Bounces

Straightforward to find solutions.

In each case we have to calculate the difference between the background black hole action and the effect of the bubble.

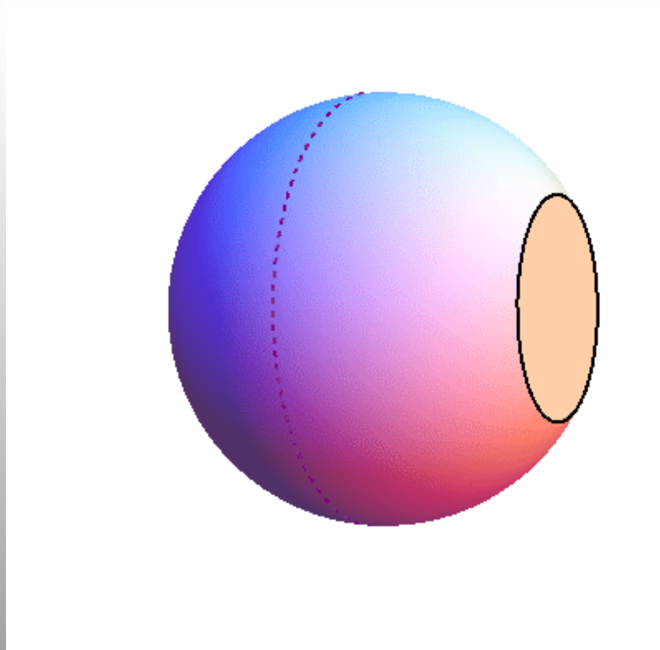
Need to deal with conical singularities (sometimes).

The general action with a black hole on each side is (details vary with Lambda):

$$\mathcal{B} = \underbrace{\frac{\pi(r_+^2 - r_-^2)}{G}}_{\text{Geometry}} \underbrace{- \frac{\bar{\sigma}}{G}\int d\lambda R^2 - \frac{1}{4G}\int d\lambda R^2 \left(f_+'\dot{\tau}_+ - f_-'\dot{\tau}_-\right)}_{\text{Bubble}}$$

Check CDL

Looks rather different to usual CDL – here we are in the "static patch" of de Sitter – or half the sphere. Must be sure it gives the right answer.



Static patch

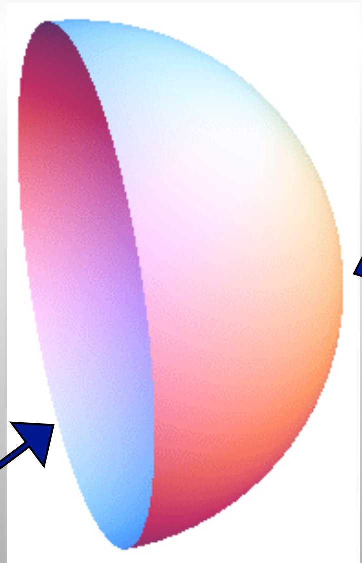
$$f_+ = 1 - \frac{r^2}{\ell^2}$$

$$f_- = 1$$

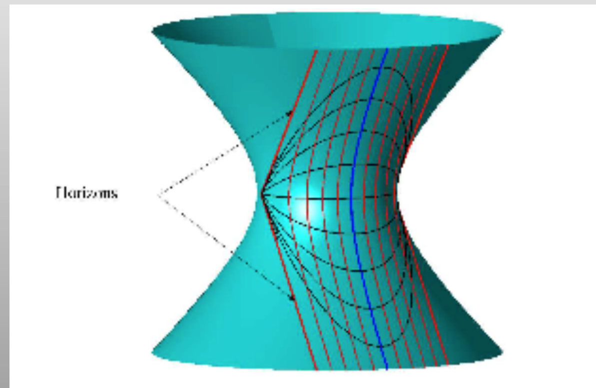
Euclidean de Sitter – Static

For de Sitter in black hole coordinates, we have a "cosmological horizon", and again τ is periodic with a specific value.

r=L



r=0



CDL Wall

$$\pm\left(\frac{\dot{R}}{R}\right)^2+\frac{1}{R^2}=\left(\bar{\sigma}+\frac{1}{4\bar{\sigma}\ell^2}\right)^2$$

Solved by sine/cosine functions:

$$R(\lambda)=\gamma\cos\frac{\lambda}{\gamma}$$

$$t_-(\lambda)=\gamma\sin\frac{\lambda}{\gamma}$$

$$\sqrt{\ell^2-\gamma^2}\,\tan\frac{t_+(\lambda)}{\ell}=\gamma\sin\frac{\lambda}{\gamma}$$

Periodicity *not* the same as static patch

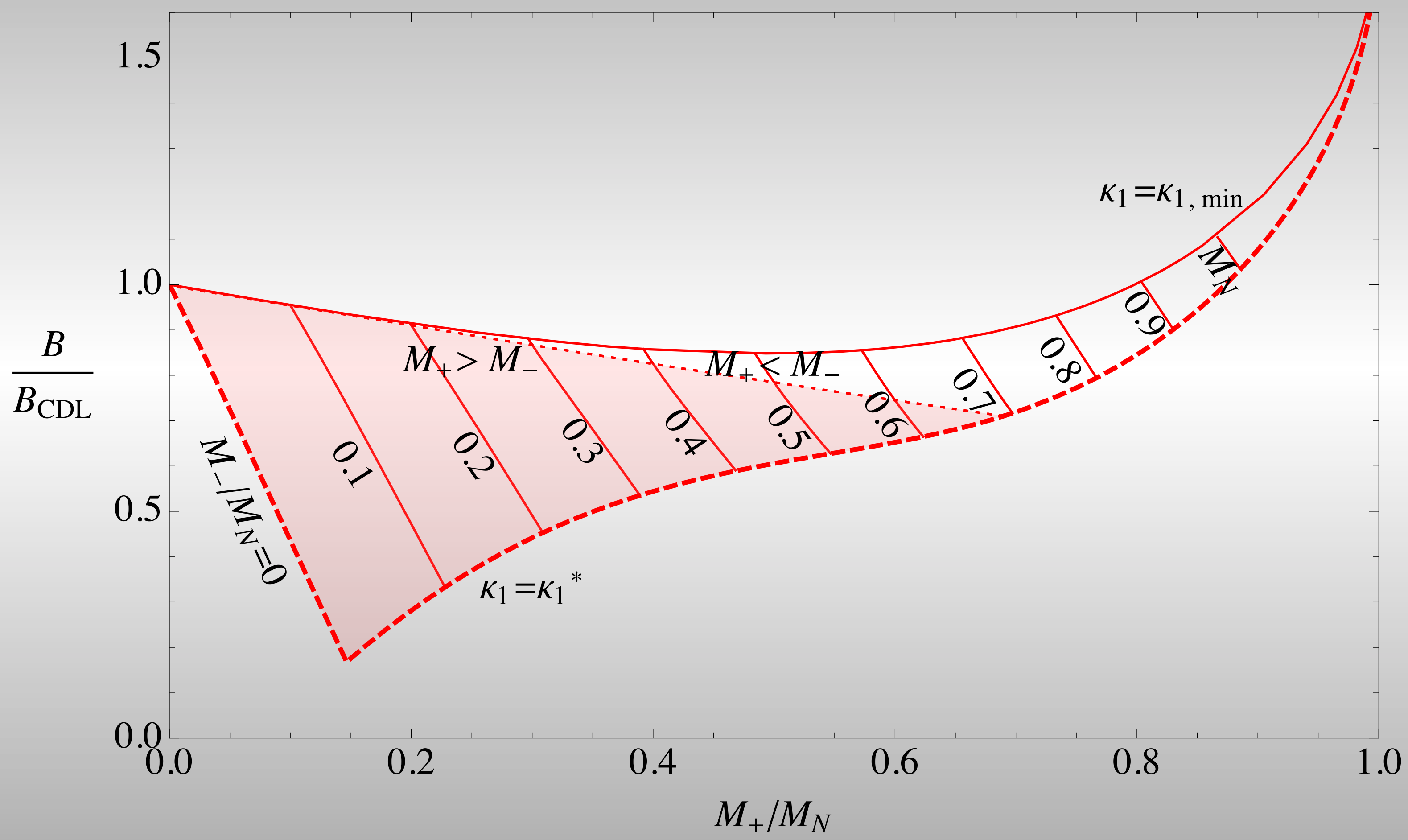
hence conical deficit in bounce

But gives same result.

General Bounce

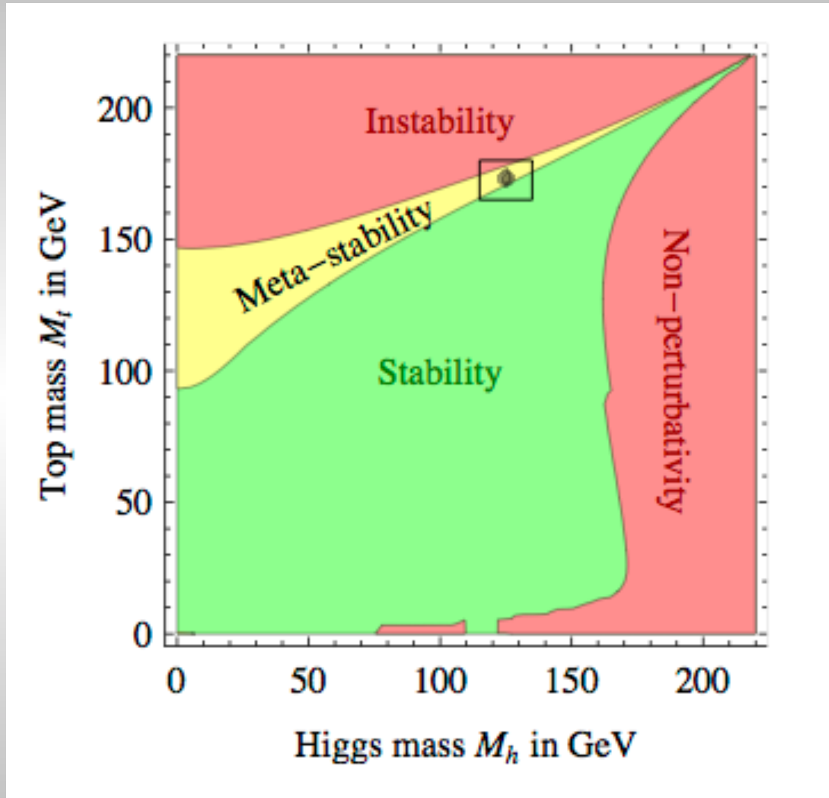
- The general solution has a black hole inside the bubble (remnant) and a mass term outside (seed).
- The solution in general depends on time, but for each seed mass there is a unique bubble with lowest action.
- For small seed masses this is time dependent – a perturbed CDL – with no remnant.
- For larger seed masses this is static and has a remnant.
- For a special Mcrit, there is a static bubble with no remnant.
- Large range of solutions with $B<B_{CDL}$

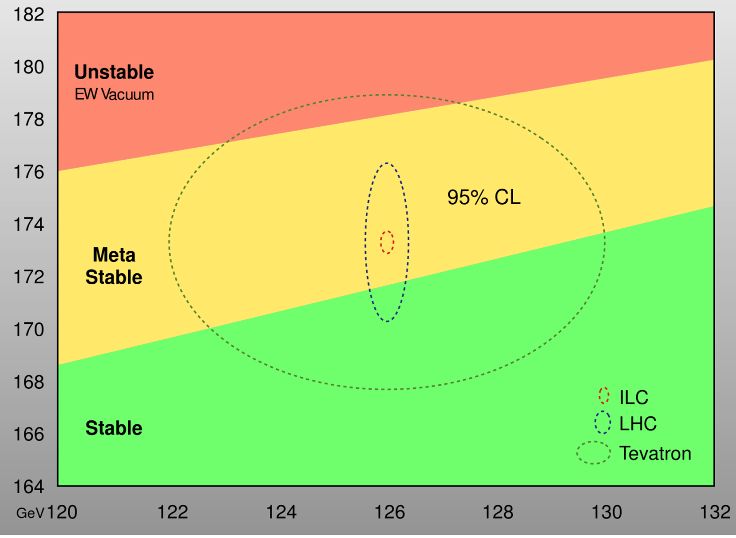
General Bounce



Is Our Vacuum Stable?

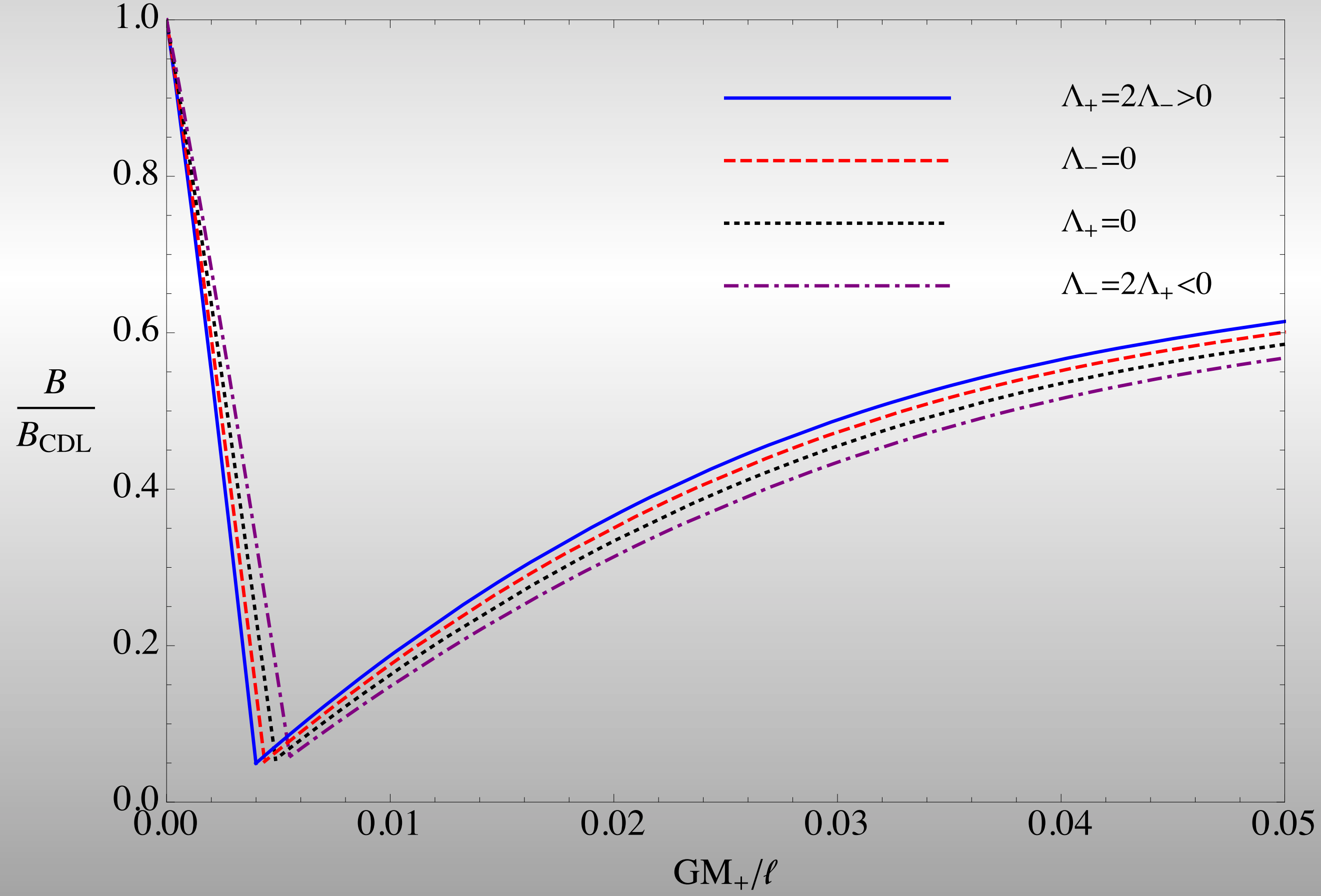
We can calculate the energy of the Higgs vacuum at different scales using masses of other fundamental particles (top quark). The LHC tells us that we seem to be in a sweet spot between stability and instability – metastability.





Generic thin wall Tunneling

Main change is the value of lambda on each side, this changes the action ratio surprisingly little.



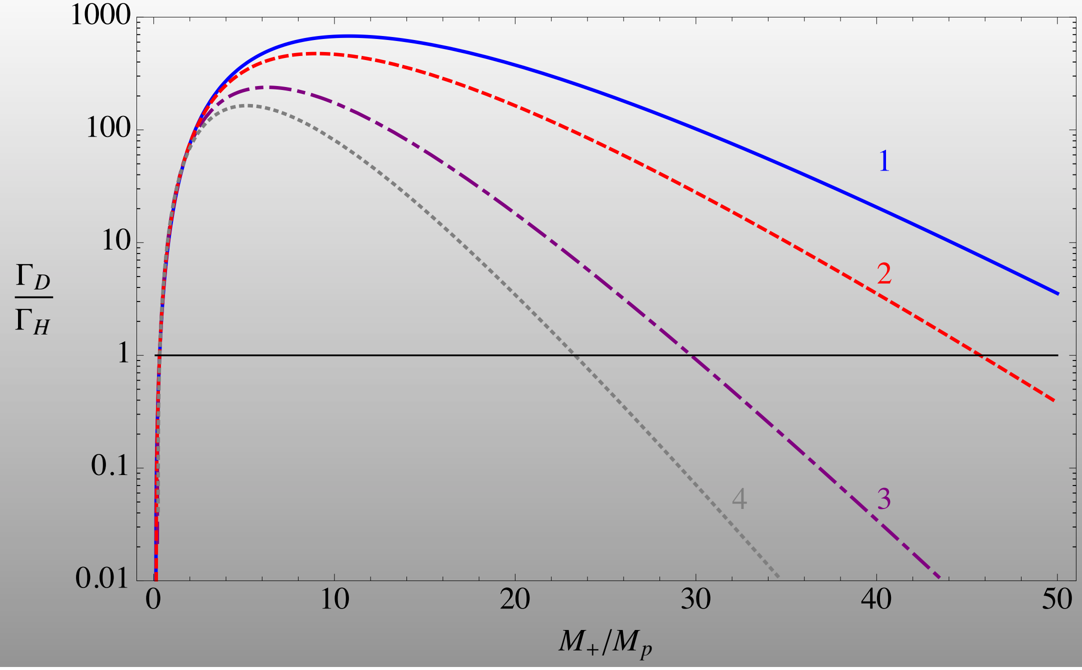
Tunneling v Evaporation:

Black holes can also evaporate – so we must check which process wins. Compare the evaporation rate:

$$\Gamma_H \approx 3.6 \times 10^{-4} (G^2 M_*^3)^{-1}$$

Don PRD-76

to our calculated tunneling rate



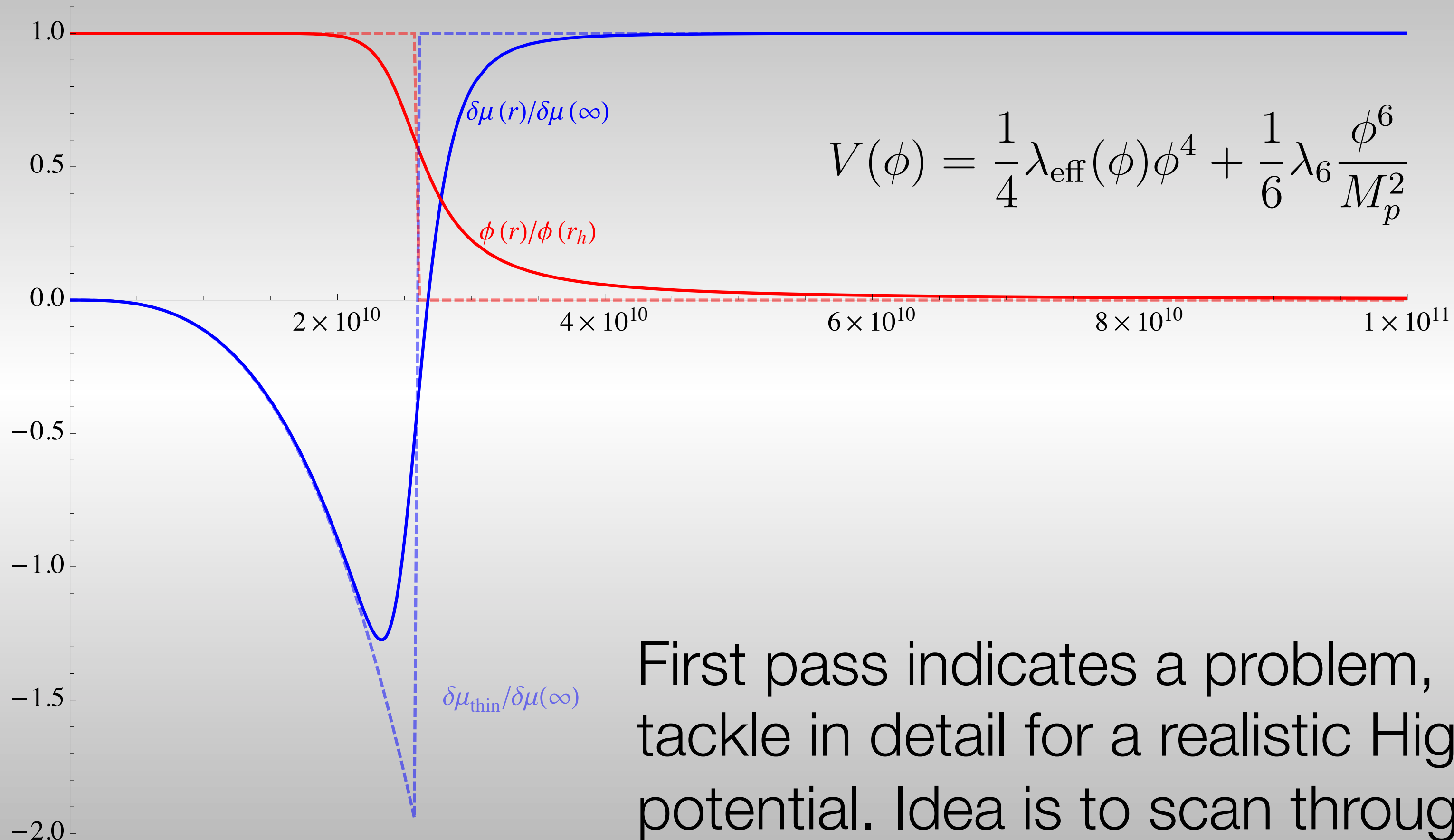
Primordial Black Holes

Plot shows that evaporation (perturbative) is much stronger than decay (nonperturbative) until the black holes are very small.

Decay NOT an issue for astrophysical black holes.

Primordial black holes have a temperature above the CMB, so these do evaporate over time. Eventually, they become light enough that they hit the "danger range" for vacuum decay and WILL catalyse it.

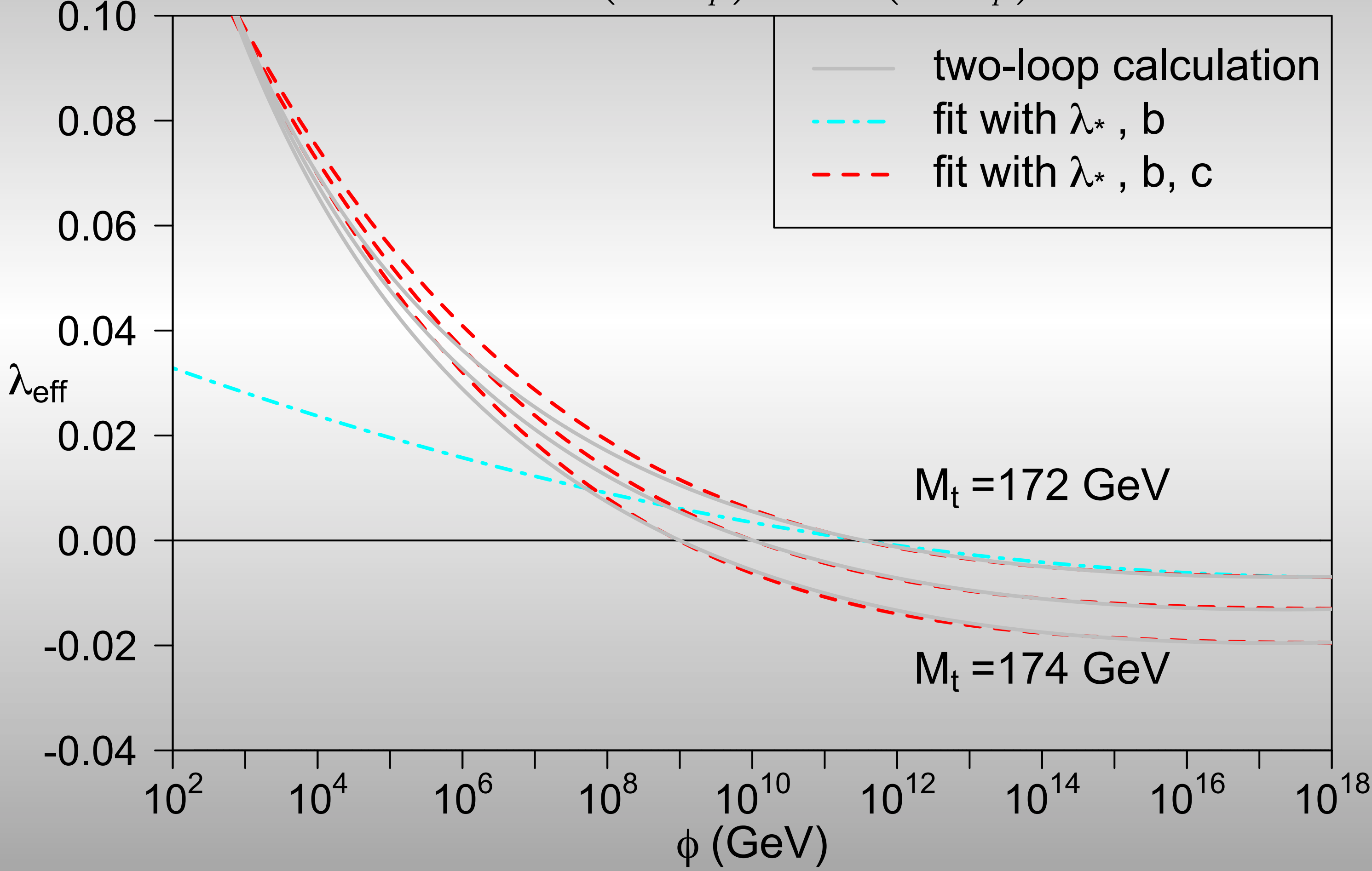
Thin to Thick wall



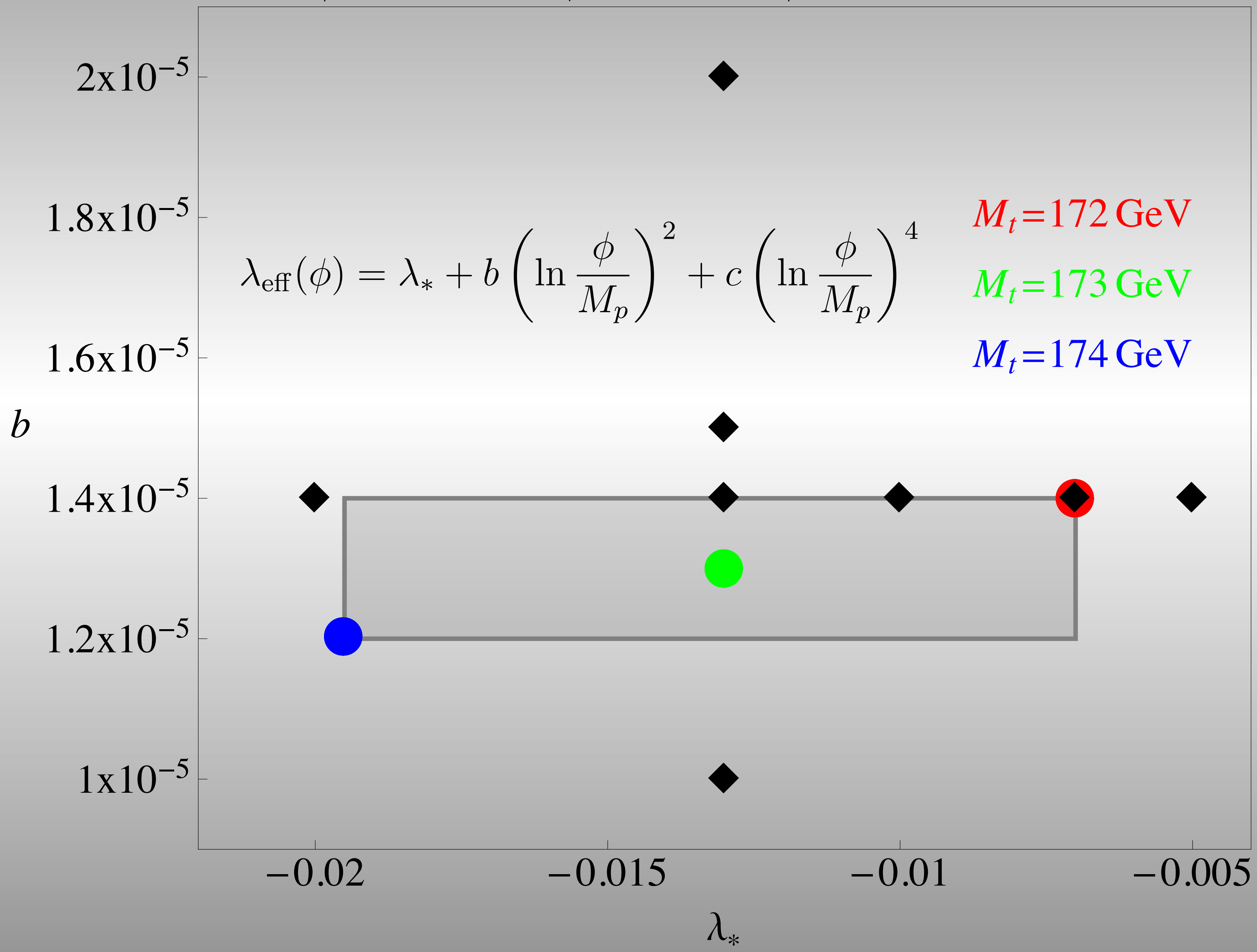
First pass indicates a problem, so tackle in detail for a realistic Higgs potential. Idea is to scan through parameter space (beyond standard model) to see how robust result it.

Fitting the Potential

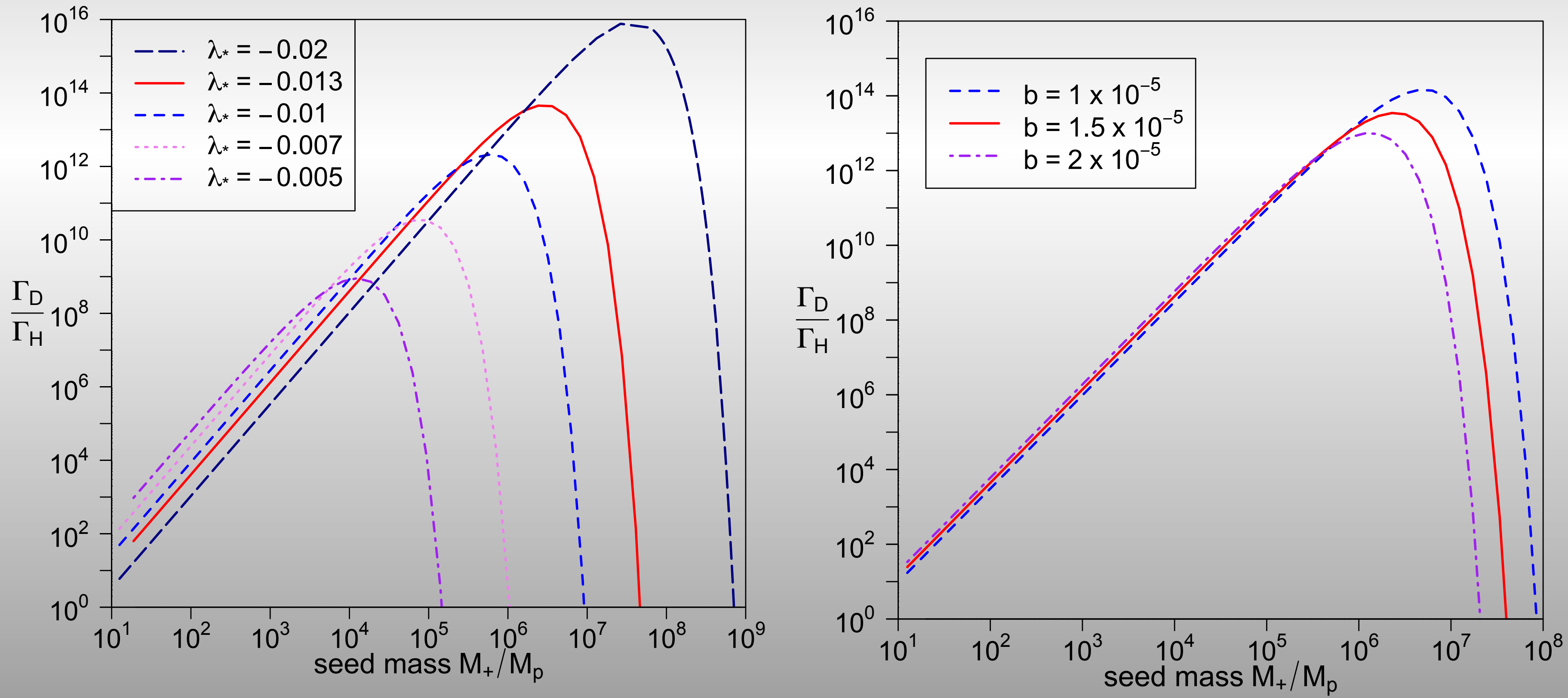
$$\lambda_{\rm eff}(\phi) = \lambda_* + b\left(\ln\frac{\phi}{M_p}\right)^2 + c\left(\ln\frac{\phi}{M_p}\right)^4$$

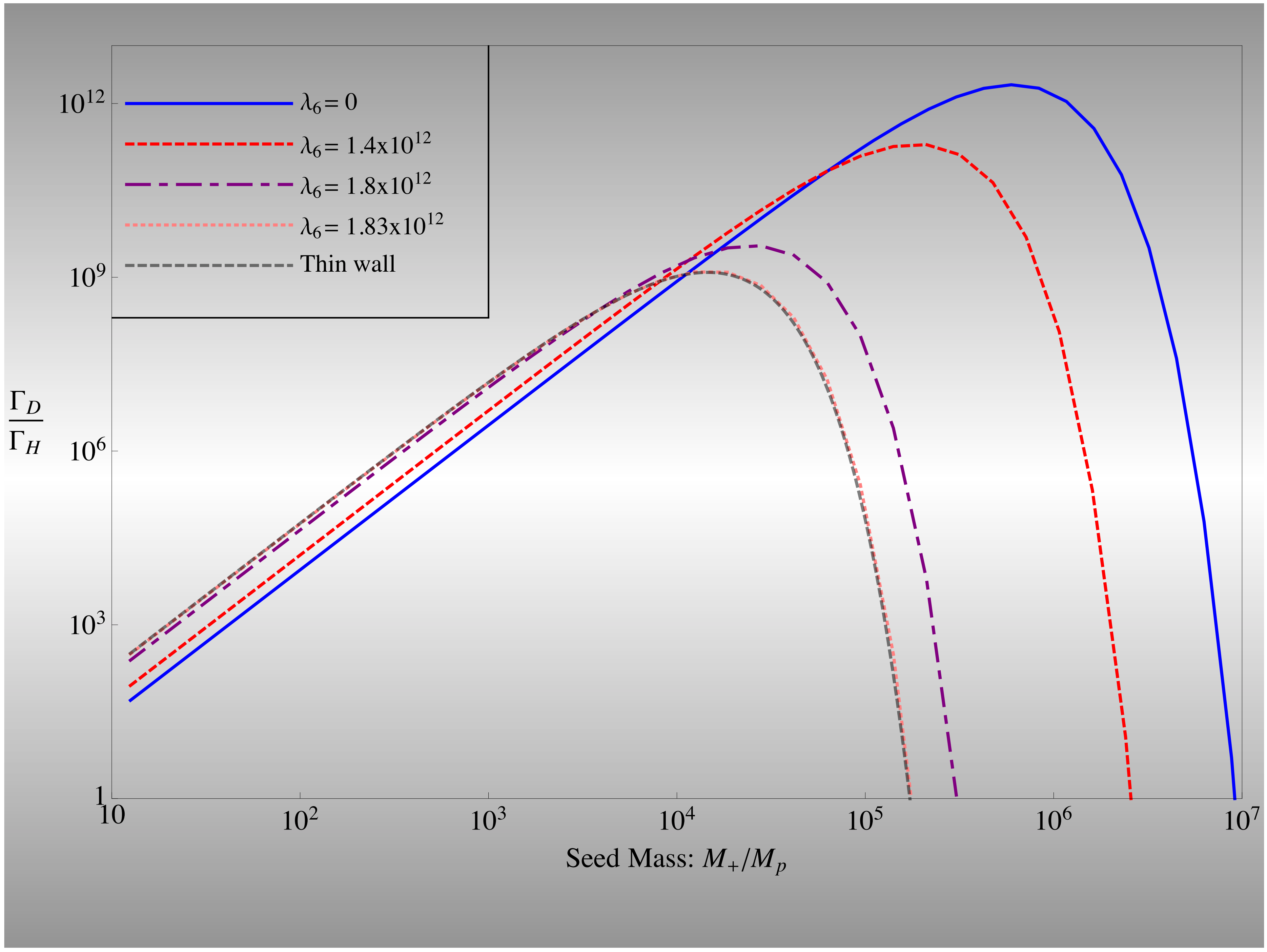


Numerical Integration



Thickening the wall increases the effectiveness of the instanton – the primordial black hole will hit the danger zone much sooner, and the decay will proceed rapidly.





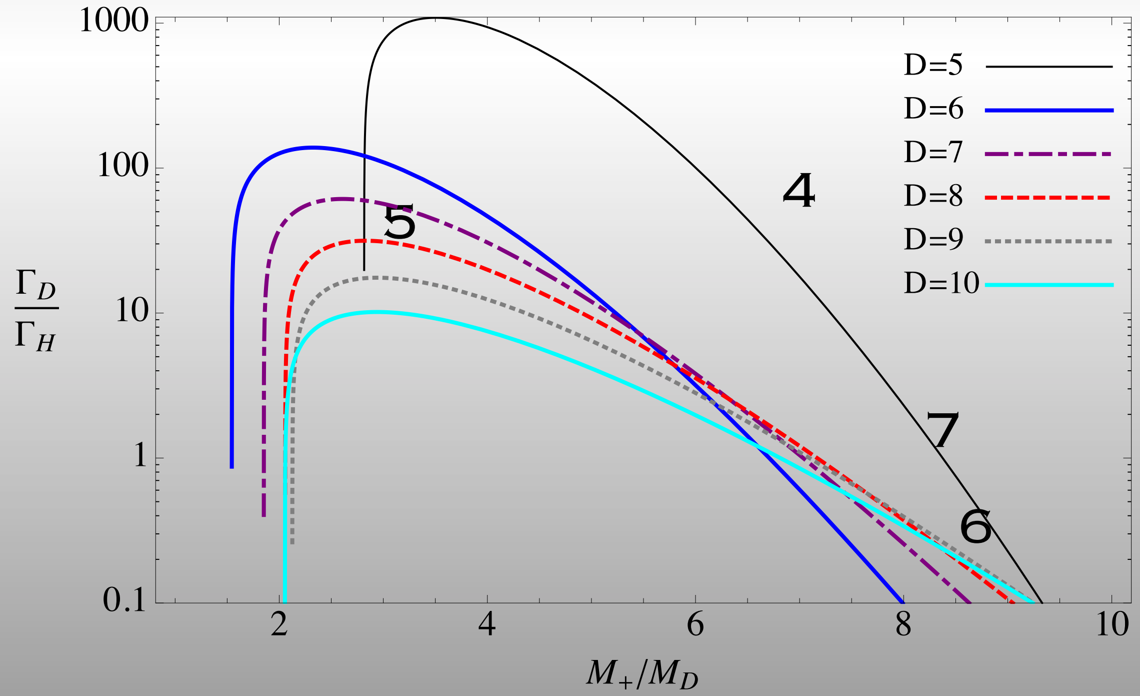
$\lambda_6 = 0$
$\lambda_6 = 1.4\text{x}10^{12}$
$\lambda_6 = 1.8\text{x}10^{12}$
$\lambda_6 = 1.83\text{x}10^{12}$
Thin wall
10^{12}
10^9
10^6
10^3
1
$\frac{\Gamma_D}{\Gamma_H}$
10
10^2
10^3
10^4
10^5
10^6
10^7
Seed Mass: M_+/M_p

Primordial black holes start out with small enough mass to evaporate and will eventually hit these curves.

Can view as a constraint on PBH's or (weak) on corrections to the Higgs potential.

Small black holes also possible in theories with Large Extra Dimensions.

(but the branching ratio seems to drop with D – shown here for thin wall)



Summary

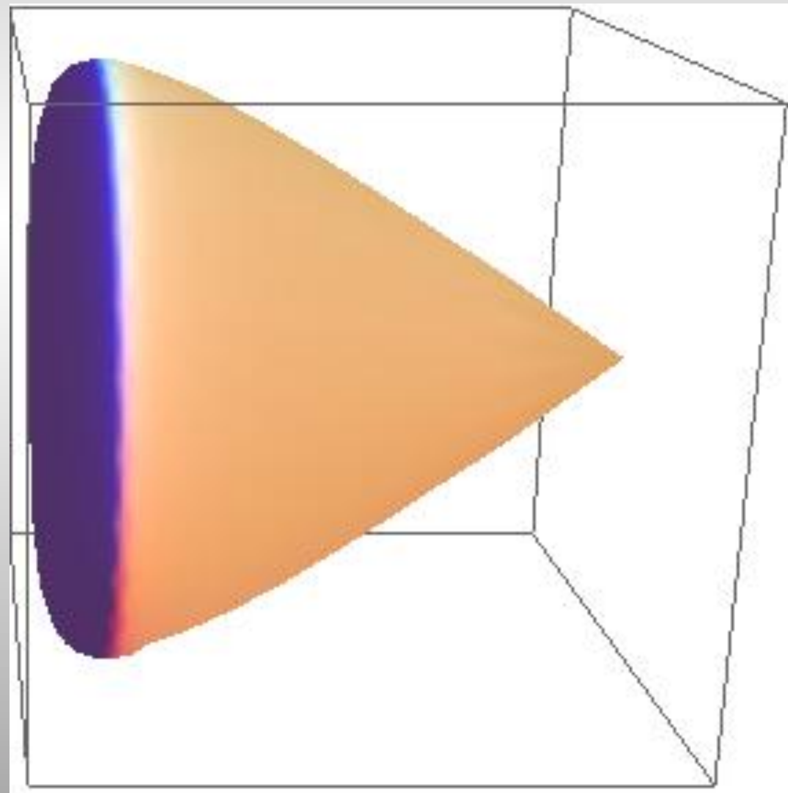
- Depending on higher energy physics, the Higgs vacuum may be unstable.
- We can construct an instanton to describe the decay process – even including gravity.
- Tunneling amplitude significantly enhanced in the presence of a black hole – bubble forms around black hole and can remove it altogether.
- Very efficient for small black holes, so either they don't exist – or the vacuum is stable.

Schwarzschild-De Sitter

Putting a black hole in de Sitter means we can never have a smooth geometry: SdS has a conical deficit/excess on at least one horizon:

$$\Delta\tau = \frac{4\pi}{|f'(r_i)|}$$

$$\frac{\Delta\tau_h}{\Delta\tau_c} = \frac{r_c(1-3\frac{r_h^2}{\ell^2})}{r_h(3\frac{r_c^2}{\ell^2}-1)} \sim \frac{\ell}{2r_h} \quad (r_h \to 0)$$



Conical Actions

The conical deficit has a delta function in the Ricci tensor (caveat – no transverse energy momentum, metric a product space) so can compute the action:

$$ds^2 = d\rho^2 + A^2(\rho)d\chi^2 + C^2(\rho)d\Omega_{II}^2$$

Smooth out A:

$$A'(0) = 1 \quad , \quad A'(\varepsilon) = (1-\delta)$$

$$\mathcal{R} = -\frac{2A''}{A} - \frac{4C''}{C} - \frac{4A'C'}{AC} + \frac{2(1-C'^2)}{C^2} \sim -\frac{2A''}{A} - \frac{8C_2}{C_0} + \frac{2}{C_0^2} + \mathcal{O}(\rho<\varepsilon)$$

$$\int d^4x\sqrt{g}\mathcal{R} \sim (4\pi C_0^2)4\pi[A'(0) - A'(\varepsilon)] + \mathcal{O}(\varepsilon) = (4\pi C_0^2)(4\pi\delta) + \mathcal{O}(\varepsilon)$$

(Geroch-Traschen – not!)

SdS Action

Calculating the action of the SdS black hole now gives an interesting result. For a general periodicity:

$$S_{SdS} = \int d^4x\sqrt{g}(-R+2\Lambda) = -\frac{\pi(r_c^2+r_h^2)}{G} + \frac{\beta}{2G}\left[2\pi\frac{r_c^2}{\beta_c} + 2\pi\frac{r_h^2}{\beta_h} - \frac{r_c^3-r_h^3}{\ell^2}\right]$$

i.e. the result is independent of β

$$r_c^2 + r_h^2 + r_c r_h = \ell^2$$

(as it should be for a physically reasonable solution)

$$\beta_i = \frac{4\pi r_i}{|1-3\frac{r_i^2}{\ell^2}|}$$