Ghost-Free Gravity and Black Holes

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Based on:

"Spherical collapse of small masses in the ghost - free gravity",

V.F, A. Zelnikov and T. Netto, JHEP 1506 (2015) 107;

"Mass - gap for black hole formation in higher derivative and

ghost free gravity", V. F., Phys.Rev.Lett. 115 (2015) 5, 051102;

"Head - on collision of ultrarelativistic particles in the higher -

derivative and ghost - free theories of gravity",

V.F, and A. Zelnikov, Phys.Rev. D93 (2016) 064048;

"Radiation from an emitter in the ghost free scalar theory"

V.F. and A. Zelnikov, e - Print : arXiv : 1603.00826 (2016)

Plan of the talk:

1. Linearized ghost-free gravity;
2. Static field of a point mass;
3. Penrose limit;
4. Spherical collapse of a small mass;
5. BH formation in the collision of UR particles;
6. Time-dependent source
7. Brief summary

Linearized ghost-free gravity

$$S \sim \int d^4x \Big[\frac{1}{2} h_{\mu\nu} a(\Box) h^{\mu\nu} - h_\mu^\sigma a(\Box) \partial_\sigma \partial_\nu h^{\mu\nu} + h c(\Box) \partial_\mu \partial_\nu h^{\mu\nu} - \frac{1}{2} h c(\Box) h + \frac{1}{2} h^{\lambda\sigma} \left(a(\Box) - c(\Box) \right) \Box^{-1} \partial_\sigma \partial_\lambda \partial_\mu \partial_\nu h^{\mu\nu} \Big].$$

IR GR limit : $a(0) = c(0) = 1$

There are no extra poles (ghosts) if a and c are entire functions $\sim \exp(P(\Box))$.

Special cases: Ghost free gravity GF_N

$$a = c = \exp(q(\Box/\mu^2)^N)$$

Static solutions of a point mass

Stress-energy tensor: $\tau_{\mu\nu} = \rho(\vec{r})\delta_\mu^0\delta_\nu^0, \quad \delta^{ij}\tau_{ij} = 0.$

$$ds^2 = -(1+2\varphi)dt^2 + (1-2\psi+2\varphi)d\ell^2 .$$

$$c = a: \quad \psi = \frac{d-1}{d-2}\varphi,$$

$$\kappa_d = 8\pi G^{(D)}, \quad D = d+1$$

Biswas, Gerwick, Koivisto, Mazumdar (2012); V.F. and Zelnikov (2015);

Stelle (1978); Modesto, Netto, Shapiro (2014)

$$a(\Delta)\triangle\psi = \kappa_d \tau_{00} \, .$$

$$ds^2 = -(1+2\varphi)\, dt^2 + (1 - \frac{2}{d-2}\varphi)\, d\ell^2 \, ;$$

$$GR: c = a = 1: \quad \varphi = -\frac{\kappa_d m \Gamma\left(\frac{d}{2}\right)}{2(d-1)\pi^{d/2}} \frac{1}{r^{d-2}} \, ;$$

$$\varphi = -\frac{\kappa_3}{8\pi} \frac{m}{r} \, .$$

Static solutions of linearized GF gravity

$$\psi_d = -\kappa_d m\, D_d(r),$$
$$\exp(-(\Delta/\mu^2)^N)\,\Delta\, D_d(x,x') = -\delta^d(x-x'),$$

Large distance, $r\mu \gg 1$: $\psi_d(r)\,|_{r\to\infty} \sim -\kappa_d m \dfrac{\Gamma\left(\frac{d}{2}-1\right)}{4\pi^{d/2} r^{d-2}}.$

Short distance, $r\mu \ll 1$:

$$\psi_d(r) \sim -\kappa_d m \frac{\mu^{d-2}}{(4\pi)^{d/2}} \frac{1}{N\,\Gamma\left(\frac{d}{2}\right)} \left[\Gamma\left(\frac{d-2}{2N}\right) - \frac{r^2\mu^2}{2d}\Gamma\left(\frac{d}{2N}\right)\right] + O(r^4\mu^4)$$

$$GF_1 : \psi_d(r) = -\kappa_d m \frac{\gamma\left(\frac{d}{2}-1, \frac{r^2\mu^2}{4}\right)}{4\pi^{d/2} r^{d-2}},$$

$\gamma(n, x)$ is the lower incomplete gamma function.

$$\psi_3 = -\kappa_3 m \frac{\operatorname{erf}\left(r\mu/2\right)}{4\pi r};$$

$$\psi_4 = -\kappa_4 m \frac{1-\exp\left(-r^2\mu^2/4\right)}{4\pi^2 r^2}.$$

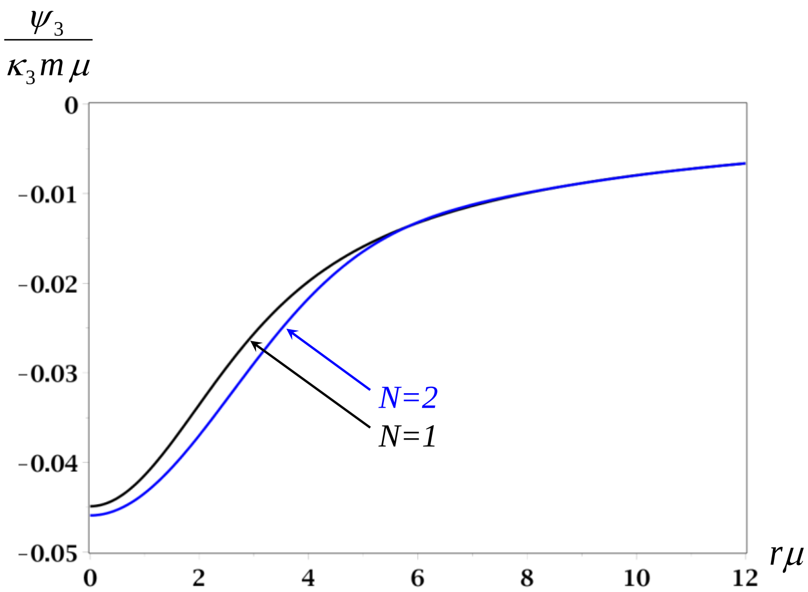
$$GF_2 : a(\Delta) = \exp(\Delta^2 / \mu^4),$$

$$\psi_d(r) = -\frac{\kappa_d m \mu^{d-2}}{d(d-2)2^{\frac{3d}{2}-2}\pi^{\frac{d-1}{2}}}\Big[\frac{d}{\Gamma\left(\frac{d}{4}\right)}\,{}_1F_3\left(\tfrac{d}{4}-\tfrac{1}{2};\tfrac{1}{2},\tfrac{d}{4},\tfrac{d}{4}+\tfrac{1}{2};y^2\right)$$

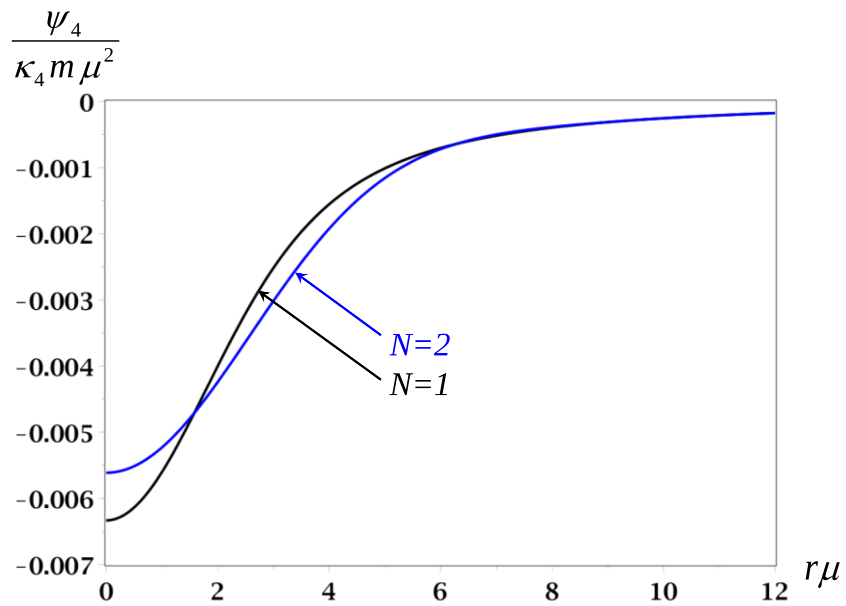
$$-\frac{2(d-2)y}{\Gamma\left(\frac{d}{4}+\frac{1}{2}\right)}\,{}_1F_3\left(\tfrac{d}{4};\tfrac{3}{2},\tfrac{d}{4}+1,\tfrac{d}{4}+\tfrac{1}{2};y^2\right)\Big]$$

$y = \frac{r^2\mu^2}{16}$, ${}_pF_q$ is the generalized hypergeometric function

GF_{2n} : explicit form of ψ_d in terms of generalized hypergeometric functions.



$\frac{\psi_3}{\kappa_3 m \mu}$
0
-0.01
-0.02
-0.03
-0.04
-0.05
0
2
4
6
8
10
12
$r\mu$
N=2
N=1



$\frac{\psi_4}{\kappa_4 m \mu^2}$
0
-0.001
-0.002
-0.003
-0.004
-0.005
-0.006
-0.007
0
2
4
6
8
10
12
$r\mu$
N=2
N=1

Penrose limit

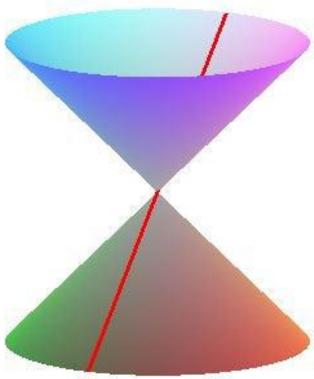
$$E = \gamma m = \text{const}, \ \gamma \to \infty$$

$$ds^2 = -du dv + d\vec{\rho}^{\,2} + F_d(\rho)\,\delta(u) du^2,$$

$$F_d(\rho) = 2\kappa_d E \, D_{d-1}(\rho),$$

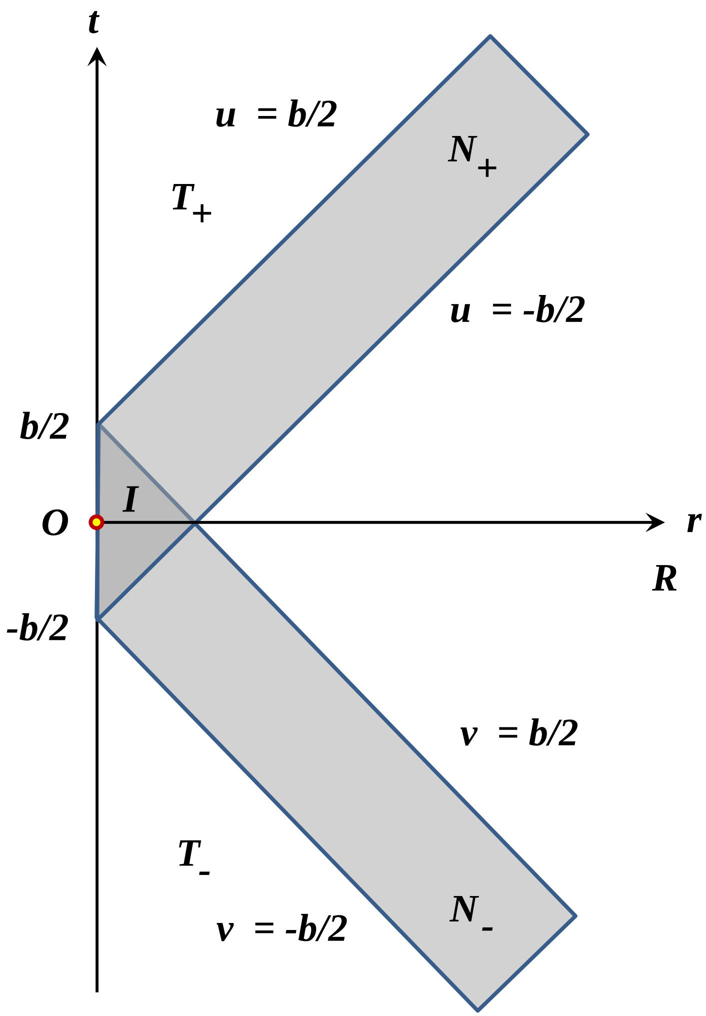
$$\exp(-(\Delta/\mu^2)^N)\,\Delta\, D_d(x,x') = -\delta^d(x-x'),$$

Spherical collapse of small mass



Thin null shell collapse

Thick null shell collapse


t
u = b/2
N+
T+
u = -b/2
b/2
I
O
r
R
-b/2
v = b/2
T-
N-
v = -b/2

1. If $GM\mu \ll 1$ then there is no apparent horizon. This means that there is map gap for mini-black hole creation in the gravitational collapse.

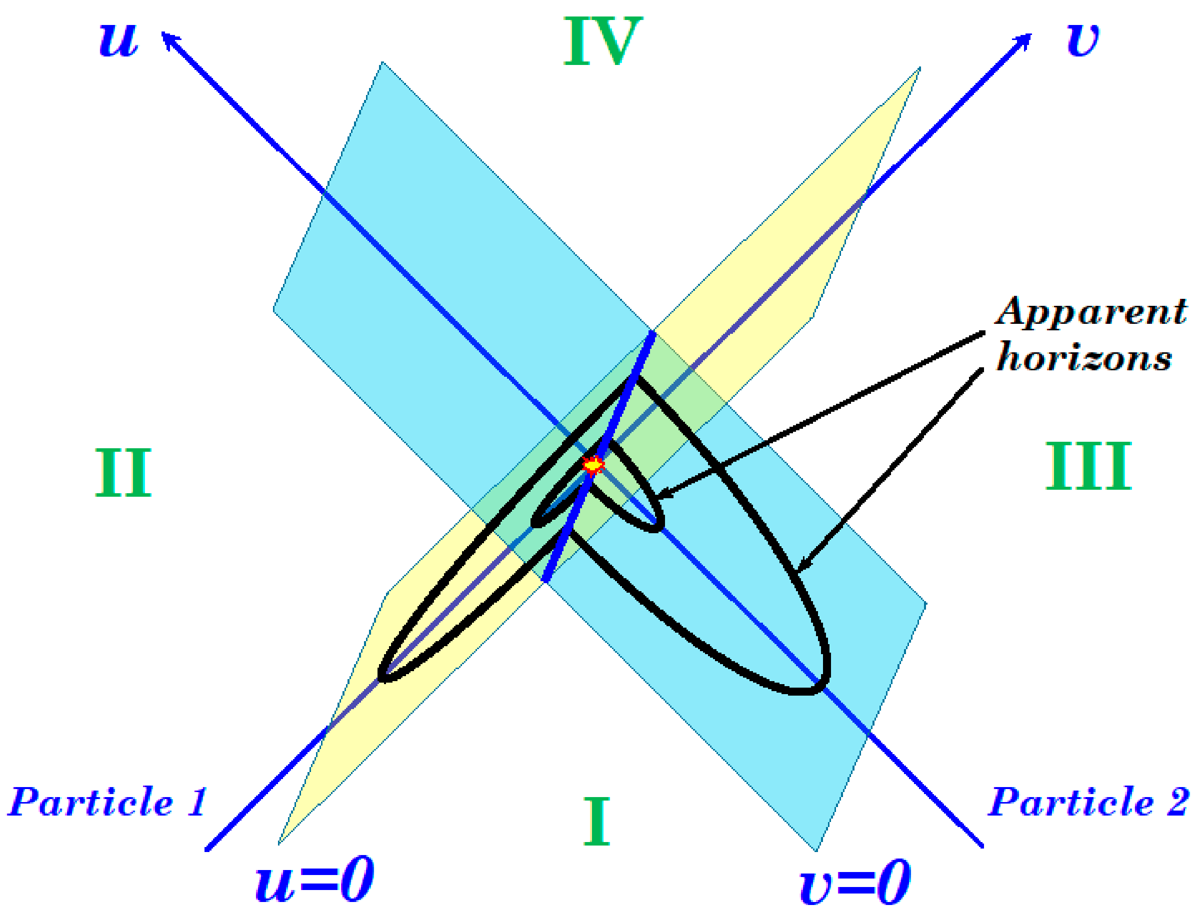
2. Denote $\mu^{-1} = \lambda$. Then $R \sim GM/(T\lambda^2)$.

For $T \geq \lambda \Rightarrow R \leq GM\mu\lambda^{-2}$.

If $GM\mu \ll 1 \Rightarrow R \ll R_{crit} = \lambda^{-2}$. Then one can neglect all higher in curvature corrections (weak field regime!)

BH formation in the collision of UR particles

Head-on collision of
ultra-relativistic particles


u
IV
v
Apparent
horizons
II
III
Particle 1
I
Particle 2
u=0
v=0

Main results:

1. Mass-gap for BH creation in the gravitational collapse and particle collision;
2. Existence of the inner branch of the apparent horizon for particle collision.

Time-dependent source

$$a(\Box)\Box\varphi = qe^{i\Omega t}\delta(\vec{x}), \quad a(\Box) = \exp[(\Box/\mu^2)^N]$$

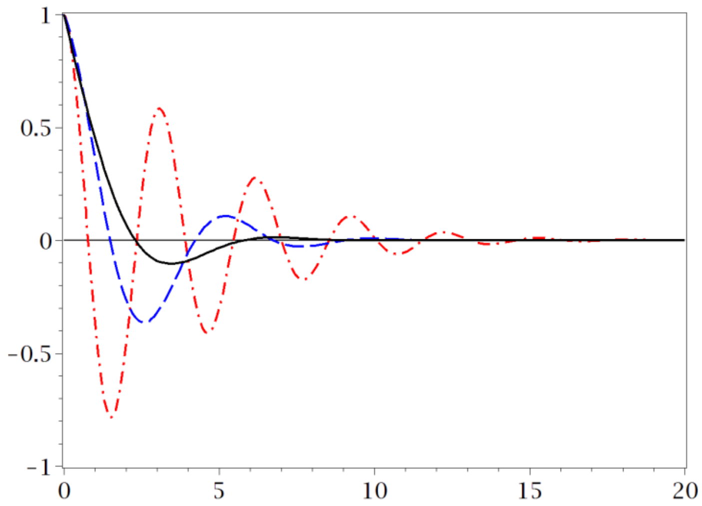
$$\varphi = -\frac{q}{r}e^{i\Omega t}\left[e^{-i\Omega r} + h(\Omega, r)\right],$$

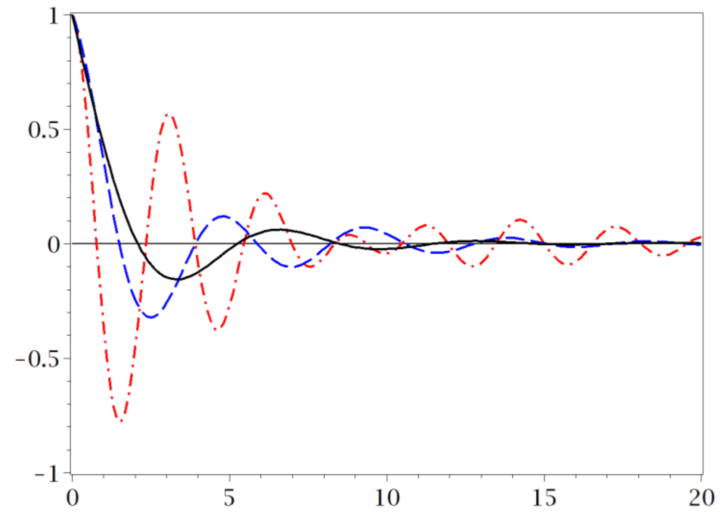
$$h(\Omega, r) = \frac{2}{\pi}\int_0^\infty dp \sin(pr) f(p),$$

$$f(p) = \frac{p}{\Omega^2 - p^2}\left[\exp\left(-((p^2 - \Omega^2)/\mu^2)^N\right) - 1\right]$$

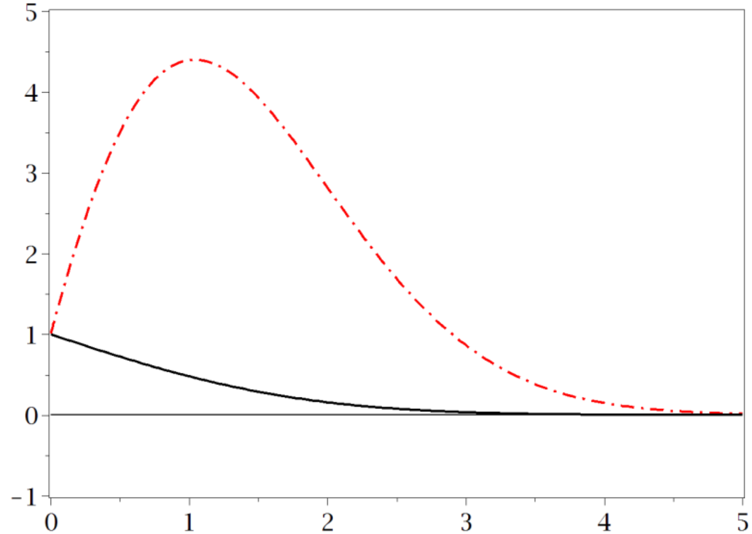
$$\mu \to \infty \Rightarrow h \to 0, \varphi \to -\frac{q}{r}e^{i\Omega(t-r)}.$$

Plots of $h(\Omega, r)$ in GF_2 (left) and GF_4 (right) theory as functions of r for different values of Ω / μ: 0 (solid line), 1 (dashed line), 2 (dotted line).





Plot of $h(\Omega, r)$ in GF_1 theory as functions of r for different values of Ω/μ: 0 (solid line) and 2 (dashed line).



$$h(\Omega, r) \sim \exp(\Omega^2/\mu^2)$$

Brief Summary

1. Gravitational field from a point source in linearized GF gravity is regular for any N.

2. Expression for the metric of an ultra-relativistic source.

3. Spherical collapse for null spherical thick shell is regular.

4. No-horizon for $GM\mu < 1$.

5. Mass gap for mini-black hole production for UR particle collision.

6. Instability of odd-N GF theories for time-dependent source.