Hairy black holes in the XX-th and XXI-st centuries – a status report

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No-hair conjecture

/Ruffini and Wheeler 1969/

- All stationary black holes are completely characterized by their mass M, angular momentum J, and electric charge Q seen from far away in the form of Gaussian fluxes.
- Black holes cannot support hair = any other independent parameters not seen from far away.

All parameters of stationary black holes are associated with the Gauss law.

Logic: gravitational collapse breaks all approximative conservation laws (chemical content, atomic structure, baryon number, etc. – the black hole looses all the memory of them). Only the exact local Lorentz or local $U(1)$ can survive – associated to them mass M, angular momentum J, and electric charge Q – cannot be absorbed by the black hole but remain attached to it as parameters. They give rise to the Gaussian fluxes that can be measured at infinity. Black holes are characterized by only three parameters, and black holes with the same M, J, Q are identically equal.

Evidence in favour of no-hair

- Uniqueness theorems: all electrovacuum black holes are described by Kerr-Newman metrics. This proves the conjecture within the electrovacuum theory.

  /Israel, Carter, Bunting, Mazur ... /

- Black holes in more general theories should either be Kerr-Newman, or they should be characterized by their charges. No-hair theorems

$$
\begin{aligned}
G_{\mu\nu} &= 8\pi G T_{\mu\nu}(\Phi) \\
\Box\Phi &= F(\Phi)
\end{aligned}
$$

If $\Phi \neq 0$, it diverges at the horizon. No black hole solutions with a regular event horizon if Φ is massive scalar, spinor, vector etc. field.

/Chase, Teitelboim, Bekenstein .../

A particular example

- Conformally coupled scalar field

$$\mathcal{L} = \frac{1}{4} R - \frac{1}{2} (\partial\Phi)^2 - \frac{1}{12} R\Phi^2$$

$\Rightarrow$ metric is extreme RN $\in$ Kerr-Newman;

$$ds^2 = -\left(1 - \frac{M}{r}\right)^2 dt^2 + \frac{dr^2}{(1 - M/r)^2} + r^2 d\Omega^2, \quad \Phi = \frac{\sqrt{3}M}{r - M},$$

Φ carries no independent parameter = secondary hair $\Rightarrow$ black holes is completely characterized by the mass
$\Rightarrow$ no-hair holds.

/Bocharova, Bronnikov, Melnikov 1972/
/Bekenstein 1975/

First non-Kerr-Newman black holes /Gibbons 1982/

Dilaton gravity,

$$\mathcal{L} = \frac{1}{4} R - \frac{1}{2} (\partial\Phi)^2 - \frac{1}{4} e^{2\Phi} F_{\mu\nu} F^{\mu\nu}$$

admits exact black hole solutions, not of the Kerr-Newman type,

$$\begin{aligned} ds^2 &= -Ndt^2 + \frac{dr^2}{N} + e^{2\Phi} r^2 d\Omega^2, \qquad N = 1 - \frac{2M}{r}, \\ e^{2\Phi} &= 1 - \frac{P^2}{Mr}, \qquad A_\mu dx^\mu = P \cos\vartheta d\varphi \end{aligned}$$

Independent parameters are the M and the magnetic charge P. The scalar field carries no extra parameters $\Rightarrow$ secondary hair $\Rightarrow$ no-hair holds.

First evidence against no-hair

Skyrme model: scalar triplet $U = \exp\{i\tau^a \Phi_a\}$

$$\mathcal{L} = \frac{1}{2\kappa} R - \frac{\alpha}{2} \operatorname{tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) - \frac{\beta}{4} \operatorname{tr}\left(\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}\right)$$

$\mathcal{F}_{\mu\nu} = \left[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U\right]$. If $\kappa \ll 1 \Rightarrow$ backreaction of Φ^a is small $\Rightarrow \Phi^a$ fulfills equations on a fixed Schwarzschild background. One finds regular solutions for Φ^a which carry an independent parameter not seen from infinity = topological charge $\Rightarrow$ perturbative approximation for hairy black hole solutions in the limit where the backreaction is negligible.

/Luckock and Moss 1986/

XX-th century – black holes with non-Abelian hair

Einstein-Yang-Mills black holes

First recognized example of manifest violation of the no-hair

$$\mathcal{L} = \frac{1}{4}R - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \epsilon_{abc}A^b_\mu A^c_\nu$$

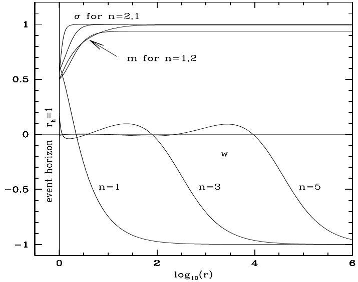
static, spherically symmetric, purely magnetic configuration

$$ds^2 = -\sigma^2(r)N(r)dt^2 + \frac{dr^2}{N(r)} + r^2 d\Omega^2, \quad A^a_i = \epsilon_{aik}\frac{x^k}{r^2}(1 - w(r))$$

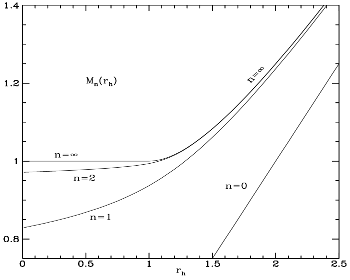
Non-trivial Yang-Mills field outside the horizon but $\boxed{F^a_{ik} \sim 1/r^3}$ $\Rightarrow$ no Yang-Mills charge. Solutions are labeled by their mass M and the number n of oscillations of $w(r)$. For a given M there are black holes with different n's $\Rightarrow$ the Yang-Mills hair is *primary*, it carries an independent parameter not visible from far away $\Rightarrow$ uniqueness is violated. /M.S.V., Gal'tsov '89/ /Bizon '90 /Kunzle, Masood-ul-Alam '90/

Axisymmetric and stationary generalizations /Kleihaus, Kunz '97/

Solutions



σ for n=2,1
m for n=1,2
r_h=1
event horizon
w
n=1
n=3
n=5
log10(r)
1
0.5
0
−0.5
−1
0
2
4
6



M_n(r_h)
n=∞
n=∞
n=2
n=1
n=0
r_h
1.4
1.2
1
0.8
0
0.5
1
1.5
2
2.5

Generalizations

- Einstein-Yang-Mills-Higgs (dublet or triplet)

$$\mathcal{L} = \frac{1}{4} R - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} - \frac{1}{2} D_\mu \Phi D^\mu \Phi - \frac{\lambda}{4} (\Phi^2 - \Phi_0^2)^2$$

  gravitating monopoles or sphalerons containing a black hole (horizon inside a classical lump).

- Einstein-Yang-Mills-dilaton

$$\mathcal{L} = \frac{1}{4} R - \frac{1}{2} \nabla_\mu \Phi \nabla^\mu \Phi - \frac{1}{4} e^{2\Phi} F^a_{\mu\nu} F^{a\mu\nu}$$

- Einstein-Skyrme
- Einstein-dilaton-Gauss-Bonnet

$$\mathcal{L} = \frac{1}{4} R - \frac{1}{2} \nabla_\mu \Phi \nabla^\mu \Phi - \alpha e^{2\Phi} \mathcal{G}_{\mathrm{GB}}$$

- Einstein + YM + ... + cosmological term

XX-th century hairy black holes

- Hairy black holes (static and stationary) generically arise for gravity-coupled non-Abelian gauge fields.
- Can support in addition a scalar Higgs field or stringy-inspired features – dilaton and curvature correction.
- Can be static, but not necessarily spherically symmetric, or stationary and axially symmetric (not necessarily spinning).
- Can be unstable, but not necessarily.
- Shrinking horizon $\Rightarrow$ regular gravitating "lumps" $\Rightarrow$ "horizon inside a classical lump".
- For stable solutions (magnetic monopole, sphaleron, Skyrmion) horizon size is bounded from above.

A review – /M.S.V., Gal'tsov, Phys.Rep.'98/

XXI-st century – black holes in modified gravity theories

Cosmic acceleration

- Our universe is actually accelerating, which indicates the presence of a dark energy.
- One can explain this in GR by introducing a small cosmological term or by modifying the GR equations.
- Most popular DE models consider a cosmic scalar field. It couples to gravity, but not necessarily minimally, and its energy may be not positive.
- Such field may violate the condition of back hole no-hair theorems.
- There are other models, for example massive gravity.

A. Models with a scalar field

Violating strong energy condition

$$\mathcal{L} = \frac{1}{4} R - \frac{1}{2} \nabla_\mu \Phi \nabla^\mu \Phi - V(\Phi)$$

- No spherically symmetric black holes if $V > 0$ (strong energy condition) /Heussler '96/
- Hairy black holes if V is not positive definite; for example

$$\begin{aligned} V(\Phi) &= 3\sinh(2\Phi) - 2\Phi\,[\cosh(2\Phi) + 2\,] \\ ds^2 &= -Ndt^2 + \frac{dr^2}{N} + R^2\, d\Omega^2\,, \qquad R^2 = r(r+2Q), \\ N &= 1 - 4\,[Q(Q+r) - R^2\,\Phi], \qquad e^{2\Phi} = 1 + \frac{2Q}{r}\,. \end{aligned}$$

Many other examples (with more or less the same $V(\Phi)$ (!)). /Bronnikov, Nucamendi, Salgado, Zloschastiev, Mann, Anabalon, Kolyvaris, Gonzalez, ... /

Violating weak energy condition

Phantom black holes

$$\mathcal{L} = \frac{1}{4}\,R + \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi - V(\Phi)$$

Conditions on $V(\Phi)$ are still needed – it cannot be positive definite.
/Bronnikov, Fabris '05/, /Dzhunushaliev et al '08/, ...

Abandoning staticity – spinning black holes with scalar hair

$$\mathcal{L} = \frac{1}{4} R - \nabla_\mu \Phi^* \nabla^\mu \Phi - \mu^2 \Phi^* \Phi$$

No static black holes with $\Phi \neq 0$ /Pena, Sudarsky '97/
There are stationary bound states on the Kerr background for

$$\Phi = F(r, \vartheta) \exp\{i\omega t + im\varphi\}$$

with $\omega = m\Omega_H$ (scalar clouds) /Hod/. These can be promoted to fully backreacting solutions with

$$ds^2 = -N\, dt^2 + \frac{1}{\Delta} (d\varphi + W\, dt)^2 + R\, (dr^2 + r^2 d\vartheta^2)$$

N, Δ, W, R depend on r, ϑ. Properties:

- Characterized by M, J and the global Noether charge Q.
- Do not have static limit
- Reduce to scalar clouds if $Q \to 0$.
- Can have $J > M^2$
- Shrinking horizon $\Rightarrow$ spinning boson star with $J = mQ$

/Herdeiro and Radu '14/

Non-minimal couplings – Horndeski theory

Most general theory of a gravity-coupled scalar field with second order e.o.m. $\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5$ with

$$\begin{aligned}
\mathcal{L}_2 &= G_2(X,\Phi), \qquad \mathcal{L}_3 = G_3(X,\Phi)\,\Box\Phi\,, \\
\mathcal{L}_4 &= G_4(X,\Phi)\,R + \partial_X G_4(X,\Phi)\,\delta^{\mu\nu}_{\alpha\beta}\,\nabla^{\alpha}{}_{\mu}\Phi\nabla^{\beta}{}_{\nu}\Phi\,, \\
\mathcal{L}_5 &= G_5(X,\Phi)\,G_{\mu\nu}\nabla^{\mu\nu}\Phi - \frac{1}{6}\,\partial_X G_5(X,\Phi)\,\delta^{\mu\nu\rho}_{\alpha\beta\gamma}\,\nabla^{\alpha}{}_{\mu}\Phi\nabla^{\beta}{}_{\nu}\Phi\nabla^{\gamma}{}_{\rho}\Phi\,,
\end{aligned}$$

where $X = -\frac{1}{2}(\partial\Phi)^2$ and $G_k(X,\Phi)$ are arbitrary.
One might expect interesting results due to the non-minimal coupling.

If $G_k = G_k(X) \Rightarrow$ Galileon shift symmetry $\Phi \to \Phi + \Phi_0 \Rightarrow$ scalar equation is

$$\nabla_\mu J^\mu = 0 \quad \text{with} \quad J^\mu = \frac{\partial\mathcal{L}}{\partial\partial_\mu\Phi}$$

A no-go result for Galileons

If $\Phi = \Phi(r)$ and

$$ds^2 = -Ndt^2 + \frac{dr^2}{N} + R^2 d\Omega^2$$

then

$$J^\mu = \delta^\mu_r J^r \quad \Rightarrow \quad (R^2 J^r)' = 0 \quad \Rightarrow \quad J^r = \frac{C}{R^2}$$

One should have $C = 0$ since otherwise $J_\mu J^\mu$ diverges at the horizon $\Rightarrow$ $J^r = 0$ everywhere. Therefore, assuming that Φ' enters minimum quadratically,

$$J^r = R^2 \frac{\partial \mathcal{L}}{\partial \Phi'} = \Phi' F(\Phi', N, N', R, R', R'') = 0.$$

For asymptotically flat solutions $F \to const.$ at infinity $\Rightarrow$ black holes must have $\Phi' = 0$ everywhere. /Hui, Nicolis '12/
Loopholes in this proof allow one to have hairy black holes.

Galileon black holes

One can choose parameters of the Horndeski theory such that

$$\mathcal{L} = \frac{1}{4} R - \frac{1}{2} \nabla_\mu \Phi \nabla^\mu \Phi - \alpha \Phi \, \mathcal{G}_{\rm GB}$$

$$\mathcal{G}_{\rm GB} = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 = \nabla_\mu \mathcal{G}^\mu$$

$\Rightarrow$ Lagrangian contains linear in Φ' term $\Rightarrow$

$$J^r = A + B\Phi' + C\Phi'^2 + \dots$$

$\Rightarrow$ $J^r = 0$ does not imply $\Phi' = 0$ $\Rightarrow$ there are hairy black holes /Sotiriou, Zhou '12/ very similar to

- Einstein-dilaton-Gauss-Bonnet black holes

$$\mathcal{L} = \frac{1}{4} R - \frac{1}{2} \nabla_\mu \Phi \nabla^\mu \Phi - \alpha e^{2\Phi} \mathcal{G}_{\rm GB}$$

/Mignemi '93/, /Torii, ... Alexeev, ... Kanti, ... '96-97/

Time-dependent scalar

A particular Horndeski model

$$\mathcal{L} = \mu\, R - (\sigma\, G_{\mu\nu} + \varepsilon\, g_{\mu\nu})\nabla^{\mu}\Phi\nabla^{\nu}\Phi - 2\Lambda\,.$$

The no-go does not apply if the scalar is time-dependent, $\Phi = Q\,t + \phi(r)$. Setting $\mu = \epsilon = \Lambda = 0$ gives an exact solution,

$$\begin{aligned} ds^2 &= -Ndt^2 + \frac{dr^2}{N} + r^2 d\Omega^2, \qquad N = 1 - \frac{2M}{r}\,, \\ \Phi &= Q\,t \pm Q \int \frac{\sqrt{1-N}}{N}\, dr \end{aligned}$$

geometry is Schwarzschild. /Babichev, Charmousis/
Various generalizations (complex scalars, vectors, etc.), solutions are generically unstable.

Non-asymptotically flat Galileon black holes

$$\mathcal{L} = \mu\, R - (\sigma\, G_{\mu\nu} + \varepsilon\, g_{\mu\nu})\nabla^{\mu}\Phi\nabla^{\nu}\Phi - 2\Lambda\,.$$

The no-go does not apply if $\Phi = \Phi(r)$ but solutions are not asymptotically flat

$$\begin{aligned} ds^2 &= -Ndt^2 + \frac{dr^2}{H} + r^2 d\Omega^2, \quad H = \frac{(\eta\, r^2 + 1)N}{(rN)'}, \\ N &= \eta\,(\mu-\lambda)^2 r^2 + 3(\mu-\lambda)(3\mu+\lambda) \\ &+ 3(\lambda+\mu)^2 \frac{\arctan(\sqrt{\eta}\, r)}{\sqrt{\eta}\, r} - \frac{2M}{r}\,, \quad \Phi'^2 = \frac{\eta\,(\lambda+\mu) r^2}{\sigma\,(\eta\, r^2+1)H}\,, \end{aligned}$$

where $\eta = -\varepsilon/\sigma$ and $\lambda = \Lambda/\eta$. One can adjust the parameters such this corresponds to a black hole, but Φ becomes complex-valued either inside or outside. Other solutions – solitons. /Anabalon, ... '14/, /Kolyvaris, ... '12/, /Rinaldi '12/

B. Other modified gravity models

Higher order equations; broken diff.

- Higher order in curvature

$$\mathcal{L} = \frac{1}{4}R - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi - V(\Phi) + f_1(\Phi)R^2 + f_2(\Phi)R_{\mu\nu}R^{\mu\nu} + f_3(\Phi)R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + f_4(\Phi)R_{\mu\nu\rho\sigma}R^{*\mu\nu\rho\sigma}$$

  Partial cases – Gauss-Bonnet Gravity; Chern-Simons gravity /Alexander, Younes '09/
  Black holes in conformal gravity

$$\mathcal{L} = \gamma R + \alpha R^2 + \beta C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}$$

  /Lu, Perkins, Pope, Stelle '15/

- Horava-Lifshiz black holes /many authors/

$$R^{(4)} = K_{ik}K^{ik} - K^2 + R^{(3)} \to$$

$$\to K_{ik}K^{ik} - \lambda K^2 + \alpha_1 R^{(3)} + \alpha_2 (R^{(3)})^2 + \ldots$$

Ghost-free massive bigravity

Two dynamical metrics $g_{\mu\nu}$ and $f_{\mu\nu}$

$$
\begin{aligned}
S &= \frac{1}{2\kappa_1}\int R(g)\sqrt{-g}\,d^4x + \frac{1}{2\kappa_2}\int R(f)\sqrt{-f}\,d^4x \\
&- m^2\int \mathcal{U}\sqrt{-g}\,d^4x
\end{aligned}
$$

ghost-free interaction /Hassan and Rosen 2012/

$$
\mathcal{U} = b_0 + b_1\sum_a \lambda_a + b_2\sum_{a<b}\lambda_a\lambda_b + b_3\sum_{a<b<c}\lambda_a\lambda_b\lambda_c + b_4\lambda_0\lambda_1\lambda_2\lambda_3
$$

where λ_a are eigenvalues of the matrix $\gamma^\mu{}_\nu = \sqrt{g^{\mu\alpha}f_{\alpha\nu}}$.

$$
G_{\mu\nu}(g) = m^2\kappa_1 T_{\mu\nu}(g,f)
$$

$$
G_{\mu\nu}(f) = m^2\kappa_2\mathcal{T}_{\mu\nu}(g,f)
$$

massive + massless graviton with $7 = 2 + 5$ DoF. If $g_{\mu\nu} = f_{\mu\nu} \Rightarrow$ $G_{\mu\nu}(g) = G_{\mu\nu}(f) = 0 \Rightarrow$ usual vacuum black holes – mildly unstable.

Hairy black holes

$$ds_g^2 = -Q^2 dt^2 + \frac{R'^2}{N^2} dr^2 + R^2 d\Omega^2$$
$$ds_f^2 = -q^2 dt^2 + \frac{U'^2}{Y^2} dr^2 + U^2 d\Omega^2$$

Event horizon at $r = r_h$

$$N^2 = \sum_{n\geq 1} a_n (r-r_h)^n, \quad Y^2 = \sum_{n\geq 1} b_n (r-r_h)^n, \quad U = u_h + \sum_{n\geq 1} c_n (r-r_h)^n$$

- Horizon is common for both metrics, surface gravities and temperatures are the same for both metrics.
- Black hole support massive graviton hair outside the horizon, asymptotically approach the AdS. /M.S.V. 2012 /. For discrete values of r_h and u_h they can be asymptotically flat /Brito, Cardoso, Pani '13/.

Conclusion

There exist plenty of hairy black holes in various systems.

Is there still any sense in the ho-hair conjecture ?

Answer

Yes

- No-hair applies for astrophysical black holes, because Einstein-Maxwell is valid at the macroscopic scale, but in this theory the conjecture is proven.
- At the microscopic scale, where other theories apply, there could be stable hairy black holes, but they are always microscopically small and loose their hair when grow beyond a certain size.
- If there exists a cosmic scalar field, this might perhaps modify the macroscopic black hole structure, but no stable hairy black holes are known at present.
- In massive gravity black holes are almost Kerr-Newman, apart from a narrow region in the horizon vicinity where a slow accretion of massive graviton modes takes place.

It seems the known astrophysically relevant black holes obey the no-hair conjecture.