Gravitational action with null boundaries

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Variational principle for general relativity

The standard action functional for general relativity,

$$S = \int_{\mathscr{V}} (R - 2\Lambda)\, dV + 2 \oint_{\partial\mathscr{V}} K\, d\Sigma$$

applies when the boundary $\partial\mathscr{V}$ is smooth and nowhere null.

When $\partial\mathscr{V}$ is made up of two segments joined at a codimension-2 surface $\mathcal{B}$ [Hayward (1993)],

$$\oint_{\partial\mathscr{V}} K\, d\Sigma \to \int_{\partial\mathscr{V}_1} K\, d\Sigma + \int_{\partial\mathscr{V}_2} K\, d\Sigma + \oint_{\mathcal{B}} \eta\, dS$$

where η is the boost parameter relating the unit normals.

Variational principle with null boundaries

The standard formulation does not apply when $\partial\mathscr{V}$ has a null segment: K is no longer defined.

This situation was examined by Parattu, Chakraborty, Majhi, and Padmanabhan (2015) in the case of a single null hypersurface.

We completed their work by including the joints between a null segment of the boundary and other (spacelike, timelike, or null) segments.

Description of a null hypersurface $\mathscr{N}$

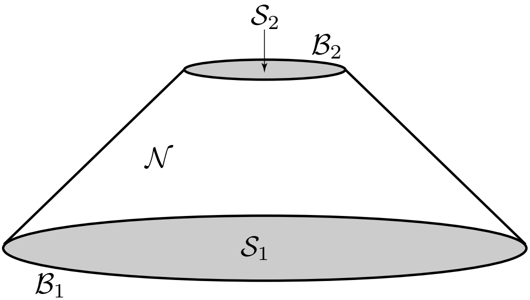
The vector k^α is tangent to $\mathscr{N}$'s null generators.

The generators are arbitrarily parametrized by λ.

Failure of λ to be an affine parameter is measured by $\kappa(\lambda)$:

$$k^\beta \nabla_\beta k^\alpha = \kappa k^\alpha$$

Boundary with a null segment



Boundary action

$$S_{\partial \mathscr{V}} = -2\int_{\mathcal{S}_2} K\, d\Sigma - 2\int_{\mathcal{N}} \kappa\, dS d\lambda + 2\int_{\mathcal{S}_1} K\, d\Sigma + 2\oint_{\mathcal{B}_2} \ln(-n_2\cdot k)\, dS - 2\oint_{\mathcal{B}_1} \ln(-n_1\cdot k)\, dS$$

Ambiguity of the action

The value of the gravitational action for a given region of a given spacetime (the on-shell action) is **ill-defined**: it depends on the parametrization of the null generators.

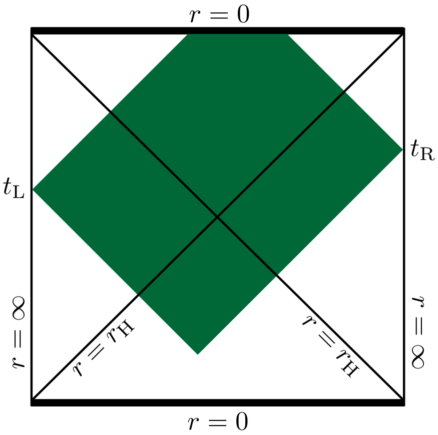
The contribution to the action from $\mathcal{N}$ can be eliminated by choosing λ to be an affine parameter.

The joint contributions remain.

The variation of the action is **well-defined**: the parametrization is fixed during the variation.

"Complexity equals action" conjecture

The complexity C of a state $|\psi(t_\mathrm{L}, t_\mathrm{R})\rangle$ of a conformal field theory on the boundary of an asymptotically anti de Sitter spacetime is the minimum number of quantum gates required to produce the state from a reference state.



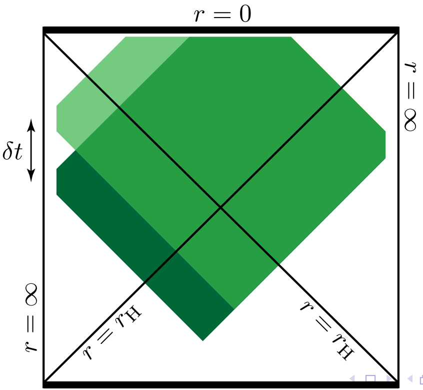
This is related to the gravitational action S evaluated for a Wheeler-deWitt patch of the spacetime: $C = S/(\pi\hbar)$.

[Brown, Roberts, Susskind, Swingle, Zhao (2015)]

Action grows linearly with time

The conjecture is supported by the expectation that C increases linearly with time, together with the calculation

$$\frac{dS}{dt} = 2M_{\mathrm{ADM}}$$



Suspicions

We were suspicious of the calculations reported by Brown *et al*.

- Contributions from null surfaces were computed as if they were timelike surfaces, with no consideration of the fact that K is not defined in the null case.
- Contributions from joints were ignored.

This motivated us to provide a proper formulation of the gravitational action in the presence of null boundaries.

We identified the correct methods of calculation, and eliminated the parametrization ambiguity by choosing an affine parametrization ($\kappa = 0$) for the generators of all null surfaces.

We recalculated dS/dt for Schwarzschild-anti de Sitter.

Our final result

$$\frac{dS}{dt} = 2M_{\text{ADM}}$$

What do you know!

Conclusion

We gave a proper formulation of the gravitational action for a region of spacetime bounded in part by a segment of null surface.

We build on the work of Parattu *et al*, but incorporate the contribution from joints between a null segment of the boundary and other (spacelike, timelike, null) segments.

The on-shell action is ill-defined: it depends on the parametrization of the null generators.

Choosing an affine parametrization, we recalculated dS/dt for a Wheeler-deWitt patch of Schwarzschild-anti de Sitter.

Unexpectedly, our result agrees with Brown *et al*.

Agreement may not result for other spacetimes.