Electromagnetic energy in regular black hole spacetimes

V. S. Manko
(Work in collaboration with E. Ruiz)

Black Holes' New Horizons
Oaxaca, Mexico
19 May, 2016

Plan of talk:

- Introduction
- Electromagnetic energy in Ayón-Beato-García solutions
- Electromagnetic energy in Bardeen spacetime
- Concluding remarks

1. Introduction

Static regular black holes

* Bardeen (1968) [Borde (1994)]
* Ayón-Beato & García (1998, 1999)

Rotating regular black holes

* Toshmatov et al (2014)

2. The EFE in ABG solutions

The structure of solutions:

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega^2$$

The first ABG solution:

$$f(r) = 1 - \frac{2Mr^2}{(r^2+Q^2)^{3/2}} + \frac{Q^2 r^2}{(r^2+Q^2)^2},$$

$$E = Qr^4 \left(\frac{r^2 - 5Q^2}{(r^2+Q^2)^4} + \frac{15}{2} \frac{M}{(r^2+Q^2)^{7/2}} \right).$$

The ED of EF:

$$T_{\alpha\beta} u^\alpha u^\beta = (-g_{tt})^{-1} T_{tt} = -T_t^t,$$
$$u^\alpha = (-g_{tt})^{-1/2} \xi^\alpha.$$

The simplest way of calculating $T_{\alpha\beta}$:

$$T_{\alpha\beta} = \frac{1}{8\pi} G_{\alpha\beta}$$

The total EM energy (t =const):

$$\mathcal{E}_{e/m} = \int_{R^3} \left(-T_t^t\right) \sqrt{-g} dr d\vartheta d\varphi,$$
$$\sqrt{-g} = r^2 \sin\vartheta.$$

For the 1st ABG solution

$$\begin{aligned} -T_t^t &= -\frac{1}{8\pi} G_{tt} g^{tt} \\ &= \frac{Q^2(r^2 - 3Q^2 + 6M\sqrt{r^2+Q^2})}{8\pi(r^2+Q^2)^3}, \end{aligned}$$

(positive definite if $2M > |Q|$), and

$$\begin{aligned} \mathcal{E}_e(r) &= \int_0^r \int_0^\pi \int_0^{2\pi} \left(-T_t^t\right) r^2 \sin\vartheta dr d\vartheta d\varphi \\ &= -\frac{Q^2 r^3}{2(r^2+Q^2)^2} + \frac{M r^3}{(r^2+Q^2)^{3/2}}. \end{aligned}$$

Therefore

$$\mathcal{E}_e(\infty) = M$$

Alternative way:

$$4\pi T^{\alpha}_{\beta} = \mathcal{H}_P P_{\beta\mu} P^{\alpha\mu} - \delta^{\alpha}_{\beta}(2P\mathcal{H}_P - \mathcal{H}),$$

where

$$\begin{aligned} P_{\alpha\beta} &= 2\delta^t_{[\alpha}\delta^r_{\beta]}\frac{Q}{r^2}, \qquad P^{\alpha\beta} = -2\delta^{[\alpha}_t\delta^{\beta]}_r\frac{Q}{r^2}, \\ P &= -\frac{Q^2}{2r^4}, \\ \mathcal{H}_P &= \frac{r^6(2r^2 - 10Q^2 + 15M\sqrt{r^2+Q^2})}{2(r^2+Q^2)^4}, \\ \mathcal{H} &= -\frac{Q^2(r^2 - 3Q^2 + 6M\sqrt{r^2+Q^2})}{2(r^2+Q^2)^3}. \end{aligned}$$

The Komar mass function

$$M_K(r) = \frac{1}{4\pi}\int_{S_r} \boldsymbol{\omega},$$

with

$$\boldsymbol{\omega} = -\tfrac{1}{2}\eta_{\alpha\beta\nu\mu}\nabla^{\nu}\xi^{\mu}dx^{\alpha}\wedge dx^{\beta}.$$

Specifically,

$$\boldsymbol{\omega} = \frac{1}{2}\omega_{\vartheta\varphi} d\vartheta \wedge d\varphi,$$

where

$$\begin{aligned} \omega_{\vartheta\varphi} &= -\frac{2r^3 \sin\vartheta}{(r^2+Q^2)^3}[Q^2(r^2-Q^2) \\ &\quad -M(r^2-2Q^2)\sqrt{r^2+Q^2}]. \end{aligned}$$

Then

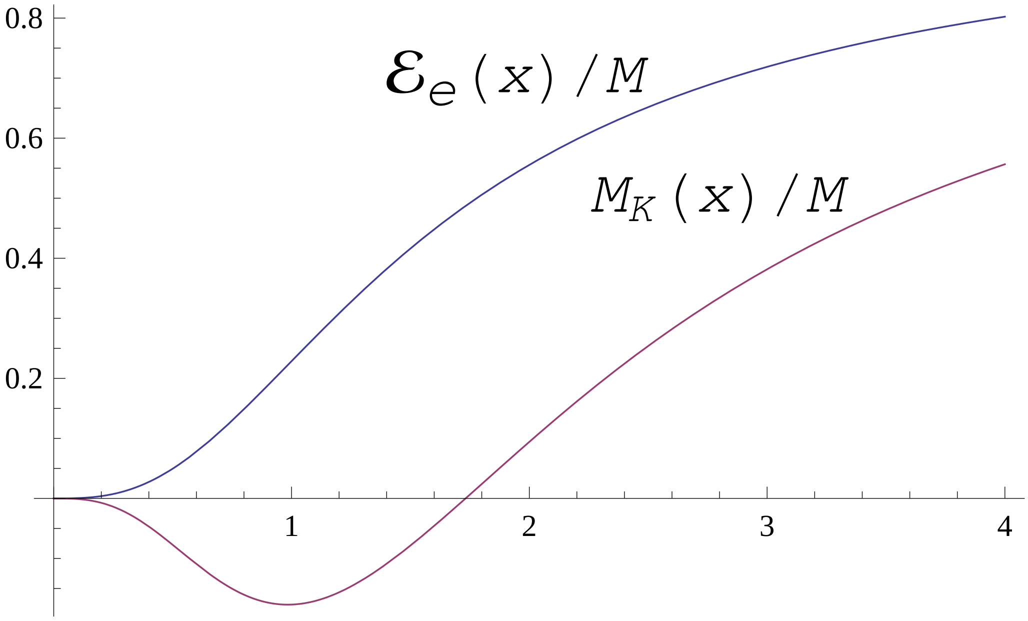
$$M_K(r) = \frac{1}{4}\int_0^\pi \omega_{\vartheta\varphi} d\vartheta$$

$$= -\frac{r^3[Q^2(r^2-Q^2) - M(r^2-2Q^2)\sqrt{r^2+Q^2}]}{(r^2+Q^2)^3},$$

and

$$M_K(\infty) = M$$

The case $|Q|/M = 1$ $(x = r/|Q|)$



The second ABG solution

$$f(r) = 1 - \frac{2M}{r}\left(1 - \tanh\frac{Q^2}{2Mr}\right),$$

and

$$\begin{aligned} E &= \frac{Q}{4Mr^3}\left(1 - \tanh^2\frac{Q^2}{2Mr}\right) \\ &\quad \times \left(4Mr - Q^2 \tanh\frac{Q^2}{2Mr}\right). \end{aligned}$$

The ED of EF:

$$-T^t_t = \frac{Q^2}{8\pi r^4}\,\mathrm{sech}^2\frac{Q^2}{2Mr},$$

so that

$$\mathcal{E}_e(r) = M - M\tanh\frac{Q^2}{2Mr}.$$

For large r

$$\mathcal{E}_e(r) \approx M - \frac{Q^2}{2r} + O\left(\frac{1}{r^2}\right),$$

hence

$$\mathcal{E}_e(\infty) = M.$$

The Komar mass function

$$\begin{aligned}\omega_{\vartheta\varphi} \quad = \quad & \sin\vartheta \left[2M\left(1 - \tanh\frac{Q^2}{2Mr}\right)\right. \\ & \left. -\frac{Q^2}{r}\,\mathrm{sech}^2\frac{Q^2}{2Mr}\right],\end{aligned}$$

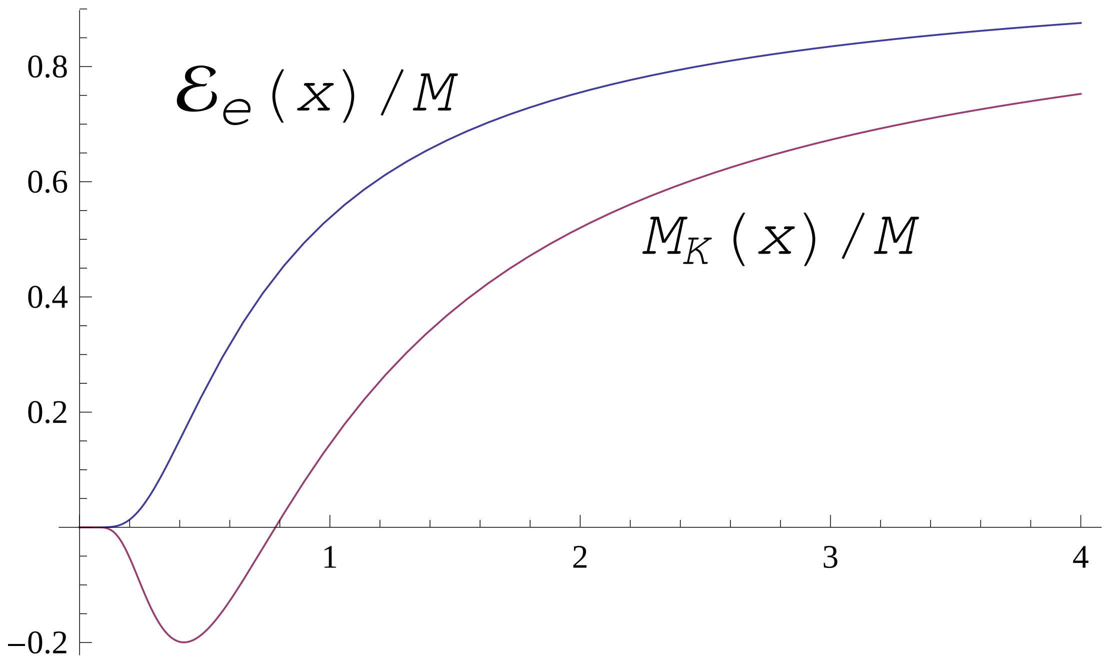
so that

$$M_K(r) = M - \frac{Q^2}{r\left(1 + \cosh\dfrac{Q^2}{Mr}\right)} - M\tanh\frac{Q^2}{2Mr},$$

and

$$M_K(\infty) = M.$$

Behavior ($x = Mr/Q^2$):



3. The MFE in Bardeen spacetime

$$f(r) = 1 - \frac{2Mr^2}{(r^2 + Q^2)^{3/2}},$$

and

$$F_{\alpha\beta} = 2\delta^{\vartheta}_{[\alpha}\delta^{\varphi}_{\beta]} Q \sin\vartheta.$$

The ED of MF:

$$-T^t_t = \frac{3MQ^2}{4\pi(r^2 + Q^2)^{5/2}},$$

so that

$$\mathcal{E}_m(r) = \frac{Mr^3}{(r^2 + Q^2)^{3/2}}.$$

Therefore,

$$\mathcal{E}_m(\infty) = M.$$

The Komar mass function

$$\omega_{\vartheta\varphi} = \frac{2Mr^3(r^2 - 2Q^2)\sin\vartheta}{(r^2 + Q^2)^{5/2}}.$$

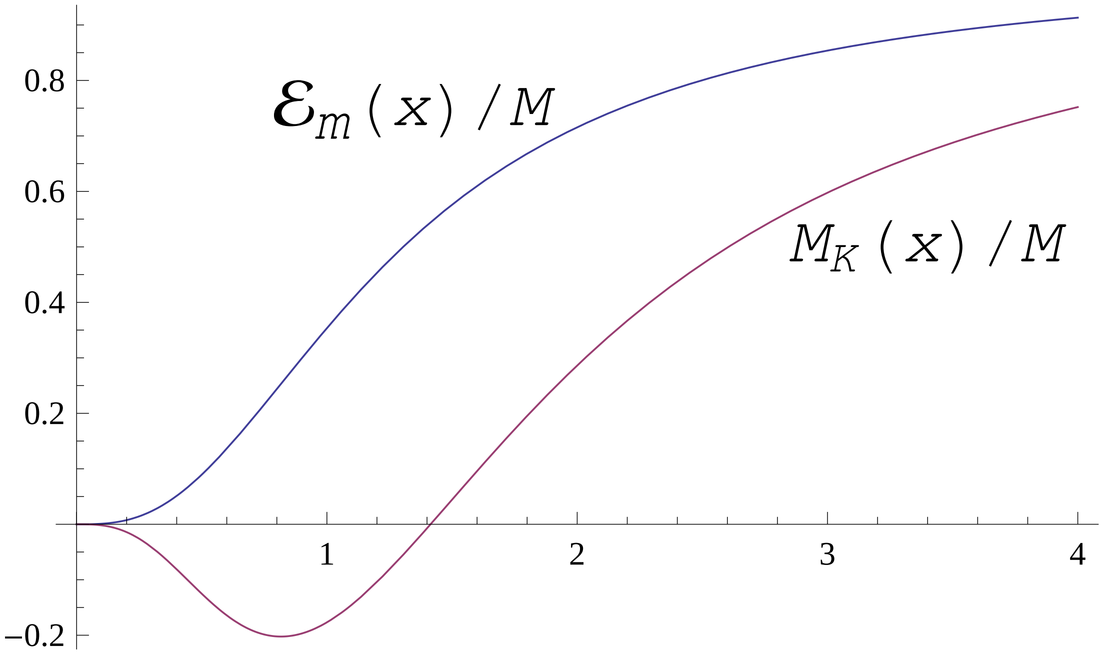
Thus

$$M_K(r) = \frac{Mr^3(r^2 - 2Q^2)}{(r^2 + Q^2)^{5/2}},$$

and

$$M_K(\infty) = M.$$

Behavior ($x = r/|Q|$):



In the Schwarzschild case

$$\omega_{\vartheta\varphi} = 2M \sin\vartheta \;\Rightarrow\; M_K(r) = M \text{ for } \forall\, r > 0.$$

4. Conclusions

• The Bardeen and ABG models do not describe the field of a point charge, but rather of some distributions of charges.

• The total EME in these models is equal exactly to the ADM mass M, thus being independent of the charge parameter Q.

• The entire "mass" in these spacetimes comes from the electromagnetic field.

• These solutions seem to reproduce the old idea of Born and Infeld (Nature, 1933) to use nonlinear electrodynamics for proving the electromagnetic origin of inertia.

THANK YOU !