Stochastic Analysis and its Applications

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1 An Overview of the Field

The field of stochastic analysis started with KIYOSHI ITÔ'S fundamental work on stochastic integration and stochastic differential equations in the mid-twentieth century. It arose from the need to study random processes whose typical behavior is so irregular that the tools of classical analysis simply do not apply. Having both pure mathematical, as well as applied, facets to it, stochastic analysis has retained its importance and appeal ever-since.

One of the most important areas of application of stochastic analysis is the mathematics of finance. In addition, mathematical finance is one of the most active and exciting areas of contemporary applied mathematics. It provides the mathematical community with a rich source of challenges, which, in turn, enrich the society's understanding of the financial system and help both regulators and the financial industry make better and more informed choices. Its proximity to practice, in addition to its purely mathematical appeal, makes this field especially attractive to young mathematicians. Moreover, it provides for wider employment opportunities both within the academic world and outside of it.

In addition to finance and economics, stochastic analysis has found an application in many other areas, such as biology (molecular biology, integrative biology, neuroscience), chemistry (polymers, chemical reactions), physics (diffusion, turbulence), and many others.

Intrinsic and pure research in stochastic analysis remains strong, with interesting new problems appearing faster than we can solve them. Some of the recent breakthroughs in this world include the work on stochastic representations for PDEs, stochastic partial differential equations, backward stochastic differential equations, and analysis on manifolds.

2 Scientific Progress Made

Rough volatility models. Volatility modeling has been one of the driving forces behind the development of mathematical finance. These models are of paramount interest to practitioners, but also require a deep understanding of stochastic-analytic tools. One of the most important developments in this area is the reintroduction of nonsemimartingales (most importantly, the small-Hurst-parameter fractional Brownian motion) into the framework in a seminal paper of GATHERAL, JAISSON and ROSENTHAL. Such models seem to exhibit an extraordinarily good fit to the observed data, and provide a parsimonious explanation for a number of empirically observed features.

The talks of CHRISTA CUCCHIERO on stochastic Volterra processes, BLANKA HORVATH on a functional central limit theorem, ANTOINE JACKIER on pricing and hedging in rough volatility models and STEPHANO DE MARCO on volatility derivatives in those models made it clear how much the theory has advanced in the span of only two years.

Multi-agent models. Formerly a staple of economic theory and deemed too difficult to analyze in the continuous-time setting, models with multiple agents have become one of the most active areas of mathematical finance. Such models, based both on game-theoretic foundations, as well as on the general equilibrium theory featured quite prominently in our meeting, too.

On the game-theoretic side, several researchers presented results in the so-called mean-field framework. Introduced by LASRY and LIONS to the mathematics community, this framework allows for a large number of identical competing agents and provides probabilistic and analytic tools for their study. This model has recently been recognized as especially relevant for applications and it is developing rapidly. The talks of MARCEL NUTZ on convergence issues and MICHAILO SHKOLNIKOV on particle systems that interact through their hitting times provide excellent evidence. Moreover, more theoretical talks of GONCALO DOS REIS on the large-deviations aspects of McKean-Vlasov equations or TOMOYUKI ICHIBA on an infinite-dimensional stochastic McKean-Vlasov equation explored some theoretical questions related to the mean-field framework. GIORGIA CALLEGARO considered nonzero-sum stochastic differential games with impulse controls.

On the other hand, several presenters incorporated elements of classical competitive economic models - as well as related principal-agent problems - into their talks. Connections with the theory of backward stochastic differential equations - one of the important sub-areas of stochastic analysis - have been made. THIBAUT MASTROLIA considered a principal-agent problem under equilibrium conditions, while SCOTT ROBERTSON presented his results on an equilibrium model with heterogeneous information. KIM WESTON talked about an equilibrium model for an annuity price in an incomplete market.

Optimal investment and related topics. The roots of the optimal-investment (utility maximization) theory in continuous-time market models can be traced to the seminal work of ROBERT MERTON. The theory has been adopted by the financial-mathematics community through the work of KARATZAS, KRAMKOV, LEHOCZKY, SCHACHERMAYER, SHREVE, XU and many others, during the 1990s. In the early 2000s it joined with the axiomatic theory of risk measures, introduced by ARTZNER, DELBAEN, EBER and HEATH, and further developed by these authors, as well as FÖLLMER with collaborators. This area has been represented by the talk of BRUNO BOUCHARD on simple bounds for transaction cost problems, and the talk of MIHAI SIRBU on stability in a general optimal-investment model.

Another important area at the intersection of optimal investment and arbitrage theory is stochastic portfolio theory. Introduced by ROBERT FERNHOLZ and later developed by FERNHOLZ, KARATZAS and others, this theory studies the properties of financial markets and a class of simply-generated trading strategies in them from a descriptive—as opposed to normative—point of view. It was represented in the meeting by the talks of SOUMIK PAL on the multiplicative Schrödinger problem, and MARTIN LARSSON on short- and long-term relative arbitrage.

Pricing and price-impact models. Option pricing is a fundamental problem in mathematical finance, and is the focus of almost all of its early activity. The fact that the area is still relevant to researchers is best gathered from several intriguing talks on the subject. The talk of ERIC TREVIÑO AGUILAR tackles the pricing problem for American options.

Due to their intrinsic mathematical interest, as well as the demand from the financial industry, price-impact models are an especially popular area of concentration. Talks of MICHAIL ANTHROPELOS on optimal investment under price impact, DANIEL HERNANDEZ- HERNANDEZ on periodic strategies in a multiplicative price-impact model and of SERGIO PULIDO on an endogenous price impact model, illustrate the scope of the theory and the range of mathematical tools it uses.

Optimal transportation. The classical problem of optimal transportation, posed by GASPARD MONGE and developed by LEONID KANTOROVICH has seen a recent mathematical renaissance in the recent past. The stochastic version of the problem - the so-called martingale optimal transport problem - with its deep connections both to the classical theory, as well as numerous applications, is studied by some of the top stochastic analysis. Our meeting featured several talks in the area: JULIO BACKHOFF gave probabilistic versions of some of the central theorems of the classical optimal transport, TONGSEOK LIM talked about multidimensional formulations in the same context, JAN OBLOJ presented new numerical methods for the martingale optimal transport problem, and TEEMU PENNANEN discussed the role of convex duality in nonlinear optimal transport theory.

Other areas. Several mathematically interesting issues that do not fit neatly into the categories singled out above were discussed in the meeting. UMUT CETIN talked about a new class of path transformations for one-dimensional diffusions, SIGRID KÄLLBLAD introduced a class of measure-valued processes related to the Skorokhod Embedding Problem and DANIEL LACKER presented a useful result on weak approximations by adapted processes. MARK PODOLSKIJ listed a number of interesting high-dimensional problems of statistical nature, JOHANNES RUF gave a compelling counterexample in the theory of filtration expansions, while JOSEF TEICHMANN discussed the reservoir paradigm and regularity structures in machine learning applied to stochastic analysis and finance.

3 Presentation Highlights

MICHAIL ANTHROPELOS

Optimal investment and derivative demand and pricing under price impact

Abstract. This paper studies the effects of price impact upon optimal investment, as well as the pricing of, and demand for, derivative contracts. Assuming market makers have exponential preferences, we show for general utility functions that the large investor's optimal investment problem with price impact can be re-expressed as a constrained optimization problem in fictitious market without price impact. While typically the (random) constraint set is neither closed nor convex, in several important cases of interest, the constraint is non-binding. In these instances, we explicitly identify optimal demands for derivative contracts, and state three notions of an arbitrage free price. Due to price impact, even if a price is not arbitrage free, the resulting arbitrage opportunity only exists for limited position sizes, and might be ignored due to hedging considerations. Lastly, in a segmented economy where large investors interact with local market makers, we show optimal positions in derivative contracts are inversely proportional to the market makers' representative risk aversion. Thus, large positions endogenously arise either as market makers approach risk neutrality, or as the number of market makers becomes large. This is a joint work with S. Roberson and K. Spiliopoulos (BU).

JULIO BACKHOFF

Martingale Benamou-Brenier: a probabilistic perspective

Abstract. In classical optimal transport, the contributions by Benamou, Brenier and McCann (among others) regarding the time-dependent version of the problem, have had a lasting impact in the field and led to many applications. It is remarkable that this is achieved even if in continuous time classical optimal transport mass/particles only travel in straight lines. Of course this fails to happen when we consider (continuous-time) martingale optimal transport. In this talk we discuss the existence of a martingale analogue to McCann's interpolation and the Benamou-Brenier formula, from a probabilistic point of view. This remarkable martingale is characterized by very natural optimality and geometric properties, leading us to say, that it provides a canonical martingale way to connect two measures in convex order. This is joint work with M. Beiglbck, M. Huesmann and S. Kallblad.

BRUNO BOUCHARD

Simple Transaction cost bounds

Abstract. Using elementary arguments, we derive $L_p$error bounds for the approximation of frictionless wealth process in markets with proportional transaction costs. For utilities with bounded risk aversion, these estimates yield lower bounds for the frictional value function, which pave the way for its asymptotic analysis using stability results for viscosity solutions. Using tools from Malliavin calculus, we also derive simple sufficient conditions for the regularity of frictionless optimal trading strategies, the second main ingredient for the asymptotic analysis of small transaction costs.

GIORGIA CALLEGARO

Nonzero-sum stochastic differential games with impulse controls: a verification theorem with applications

Abstract. We consider a general nonzero-sum impulse game with two players. The main mathematical contribution of the paper is a verification theorem which provides, under some regularity conditions, a suitable system of quasi-variational inequalities for the value functions and the optimal strategies of the two players. As an application, we study an impulse game with a one-dimensional state variable, following a real-valued scaled Brownian motion, and two players with linear and symmetric running payoffs. We fully characterize a Nash equilibrium and provide explicit expressions for the optimal strategies and the value functions. We also prove some asymptotic results with respect to the intervention costs. Finally, we consider two further non-symmetric examples where a Nash equilibrium is found numerically.

UMUT CETIN

Diffusion transformations, Black-Scholes equation and optimal stopping

Abstract. I will introduce a new class of path transformations for one-dimensional diffusions that are tailored to alter their long-run behaviour from transient to recurrent or vice versa. It turns out that these transformations are very useful in improving the convergence rate of Euler schemes for killed diffusions, simplifying the solutions of optimal stopping problems with discounting, and characterising the stochastic solutions of Cauchy problems defined by the generators of strict local martingales, which are well-known for not having unique solutions. I will give a description of these transformations and discuss their connections with h-transforms and Schroedinger semigroups, and how one can use them to solve the above problems.

CHRISTA CUCHIERO

Markovian representations of stochastic Volterra processes

Abstract. We consider stochastic (partial) differential equations appearing as Markovian lifts of Volterra processes with jumps. In particular we provide existence and uniqueness results for Markovian lifts of affine rough volatility models of general jump diffusion type. We also discuss extensions to polynomial Volterra processes and provide a moment formula.

STEFANO DE MARCO

Volatility derivatives in rough forward variance models

Abstract. Forward variance models are models for the joint dynamics of an asset price and the implied volatility of its variance swaps. The works in this field of research and its applications can be traced back to the early 90's (see B. Dupire [93]); important contributions have been given by (among others) H. Buehler [2006] and L. Bergomi [2004+]. Since 2008, these models have been successfully applied to the derivatives market on the VIX index. More recently, the new stream of research on rough volatility modeling has pushed forward new instances within this family notably, the rough Bergomi model of [Bayer, Friz, Gatheral 2016]. In this talk, we will first consider a class of log normal forward variance models that embeds the models above, and present some of its major properties, with a focus on the model-generated term structure of volatilities of volatilities. Then, we will present a non-log normal extension that is able to accommodate VIX derivatives in this (rough) forward variance setting (which was so far a limitation of the log normal class above), and present its appealing properties in the calibration to the VIX market.

GONCALO DOS REIS

Large Deviations for McKean Vlasov Equations and Importance Sampling

Abstract. We discuss two Freidlin-Wentzell large deviation principles for McKean-Vlasov equations (MV-SDEs) in certain path space topologies. The equations have a drift of polynomial growth and an existence/uniqueness result is provided. We apply the Monte-Carlo methods for evaluating expectations of functionals of solutions to MV-SDE with drifts of super-linear growth. We assume that the MV-SDE is approximated in the standard manner by means of aninteracting particle system and propose two importance sampling (IS) techniques to reduce the variance of the resulting Monte Carlo estimator. In the "complete measure change" approach, the IS measure change is applied simultaneously in the coefficients and in the expectation to be evaluated. In the "decoupling" approach we first estimate the law of the solution in a first set of simulations without measure change and then perform a second set of simulations under the importance sampling measure using the approximate solution law computed in the first step.

DANIEL HERNANDEZ-HERNANDEZ

Periodic strategies in optimal execution with multiplicative impact.

Abstract. In this talk we study the optimal execution problem with multiplicative price impact in algorithm trading, when an agent holds an initial position of shares of a financial asset. The inter-selling-decision times are modelled by the arrival times of a Poisson process. The criterion to be optimised consists in maximising the expected net present value of gains of the agent, and it is proved that an optimal strategy has a barrier form, depending only on the number of shares left and the level of asset price. Joint work with H. Moreno-Franco and J.L. Prez.

BLANKA HORVATH

Blanka Horvath: Functional central limit theorems for rough volatility

Abstract. We extend Donsker's approximation of Brownian motion to fractional Brownian motion with Hurst exponent $H \in (0,1)$ and to Volterra-like processes. Some of the most relevant consequences of our 'rough Donsker (rDonsker) Theorem' are convergence results for discrete approximations of a large class of rough models. This justifies the validity of simple and easy-to-implement Monte-Carlo methods, for which we provide detailed numerical recipes. We test these against the current benchmark Hybrid scheme BLP15 and find remarkable agreement (for a large range of values of H). This rDonsker Theorem further provides a weak convergence proof for the Hybrid scheme itself, and allows to construct binomial trees for rough volatility models, the first available scheme (in the rough volatility context) for early exercise options such as American or Bermudan.

TOMOYUKI ICHIBA

An Infinite-dimensional McKean-Vlasov Stochastic Equation

Abstract. We consider large linear systems of interacting diffusions and their convergence, as the number of diffusions goes to infinity. Our limiting results contain two complementary scenarios, (i) a mean-field interaction where propagation of chaos takes place, and (ii) a local chain interaction where neighboring components are highly dependent. We describe them by an infinite-dimensional, nonlinear stochastic differential equation of McKean-Vlasov type. Furthermore, we determine a dichotomy of presence or absence of mean-field interaction, and we discuss the problem of detecting its presence from the observation of a single component process. This is joint work with J.-P. Fouque and N.-C. Detering.

ANTOINE JACQUIER

Pricing and Hedging in rough volatility models

Abstract. We discuss the pricing and hedging of volatility options in some rough volatility models. First, we develop efficient Monte Carlo methods and asymptotic approximations for computing option prices and hedge ratios in models where log-volatility follows a Gaussian Volterra process. While providing a good fit for European options, these models are unable to reproduce the VIX option smile observed in the market, and are thus not suitable for VIX products. To accommodate these, we introduce the class of modulated Volterra processes, and show that they successfully capture the VIX smile. This is based on joint works with Blanka Horvath, Aitor Muguruza and Peter Tankov.

SIGRID KÄLLBLAD

Measure-valued martingales and optimality of solutions to the Skorokhod embedding problem

Abstract. We consider (probability) measure valued processes, which we call MVMs, which have a natural martingale structure. Following previous work such processes are known to have a close connection to solutions to the Skorokhod Embedding Problem. Here, we consider two key properties of these processes, and in particular, we are able to show that the MVMs connected to the Bass and Root embeddings possess natural optimality properties. Based on joint work with M. Beiglbck, A. Cox and M. Huesmann.

DANIEL LACKER

Weak approximation by adapted process

Abstract. This talk highlights a useful lemma on how and when one can approximate a pair of stochastic processes (X, Y) in distribution by (X, Y_n), where Y_n is adapted to X. This appears naturally in the study of weak formulations of stochastic control problems (particularly in mean field contexts), causal optimal transport, and progressive enlargement of filtrations.

MARTIN LARSSON

Short- and long-term relative arbitrage in stochastic portfolio theory

Abstract. A basic result in Stochastic Portfolio Theory states that a mild nondegeneracy condition suffices to guarantee long-term relative arbitrage, that is, the possibility to outperform the market over sufficiently long time horizons. A longstanding open question has been whether short-term relative arbitrage is also implied. Fernholz, Karatzas & Ruf recently showed that it is not, without giving tight bounds on the critical time horizon. We connect existence of relative arbitrage to a certain geometric PDE describing mean curvature flow, and use properties of such flows to compute the critical time horizon.

TONGSEOK LIM

Various formulations of martingale optimal transport problem in multi dimension

Abstract. We would like to introduce a few different setups of martingale optimal transport problem in dimension greater than one. Optimal correlations of martingales become an interesting issue, and each problem has its own unique feature in this regard.

THIBAUT MASTROLIA

Principal-Agent problem with common agency without communication

Abstract. In this paper, we consider a problem of contract theory in which several Principals hire a common Agent and we study the model in the continuous time setting. We show that optimal contracts should satisfy some equilibrium conditions and we reduce the optimisation problem of the Principals to a system of coupled Hamilton-Jacobi-Bellman (HJB) equations. We provide conditions ensuring that for risk-neutral Principals, the system of coupled HJB equations admits a solution. Further, we apply our study in a more specific linear-quadratic model where two interacting Principals hire one common Agent.

MARCEL NUTZ

Convergence to the Mean Field Game Limit: A Case Study

Abstract. We study the convergence and multiplicity of equilibria in a tractable game of optimal stopping. If the mean field game has a unique equilibrium, any sequence of n-player equilibria converges to it as $n \to \infty$. Whereas in the case of non-uniqueness, it is shown that an additional stability condition is needed to ensure that a mean field equilibrium is the limit of n-player equilibria. (Joint work with Xiaowei Tan)

JAN OBLOJ

Computational Methods for Martingale Optimal Transport problems

Abstract. We develop numerical methods for solving the martingale optimal transport (MOT) problem. We prove that the MOT problem can be approximated through a sequence of linear programming (LP) problems which result from a discretisation of the marginal distributions combined with a suitable relaxation of the martingale constraint. Specialising to the one-step model in dimension one, we provide an estimation of the convergence rate. We adopt two computational algorithms to solve the LP problem that are related to a tailored discretisation of the marginals preserving the increasing convex order, based respectively on the iterative Bregman projection and stochastic averaged gradient method. Joint work with Gaoyue Guo.

SOUMIK PAL

Multiplicative Schrdinger problem in stochastic portfolio theory

Abstract. We are interested in constructing relative arbitrage portfolios in high-dimensional equity markets that outperform the market portfolio in the long run without statistical assumptions. There is a natural Monge-Kantorovich optimal transport problem via which such portfolios can be constructed. We show that the dynamics of the transport is a multiplicative version of the Schrdinger problem for the classical Wasserstein transport (due to Schrdinger, Fllmer, and, recently, Lonard) where the role of Brownian motion is played by the gamma subordinator. The corresponding Lagrangian is related to the Wasserstein diffusion put forward by Sturm and coauthors. The financial interpretation helps us derive a Talagrand type relative entropy bound on the transport cost. Based on joint work with T.-K. L. Wong.

TEEMU PENNANEN

Convex duality in nonlinear optimal transport

Abstract. We study problems of optimal transport, by embedding them in a general functional analytic framework of convex optimization. This provides a unified treatment of a large class of related problems in probability theory and allows for generalizations of the classical problem formulations. General results on convex duality yield dual problems and optimality conditions for these problems. When the objective takes the form of a convex integral functional, we obtain more explicit optimality conditions and establish the existence of solutions for a relaxed formulation of the problem. This covers, in particular, the mass transportation problem and its nonlinear generalizations.

MARK PODOLSKIJ

High dimensional problems for continuous time models

Abstract. In modern research many statistical and probabilistic problems are high dimensional in nature. Quite often, the issue of high dimensionality is discussed in the setting of discrete stochastic processes. In this talk we will highlight some natural high dimensional questions in the framework of diffusion models, which are particularly interesting for applications in mathematical finance. We will review some existing results, but we will mostly concentrate on future research directions.

SERGIO PULIDO

Optimal investment in an endogenous price impact model

Abstract. We study the optimal investment problem in a price impact model where an influential investor trades illiquid assets with a representative market maker with exponential preferences. The impact of the demand of the investor on the prices of the illiquid assets is derived endogenously through an equilibrium mechanism. Employing recent findings on the density of the set of probability measures satisfying a backward formulation of the martingale representation property, we prove approximate (endogenous) completeness results for this price impact mechanism. As a consequence, we obtain explicit formulas for the value function of the influential investor's optimal investment problem. This is joint work with Dmitry Kramkov.

SCOTT ROBERTSON

Equilibrium with Heterogeneous Information.

Abstract. We consider the problem of establishing equilibrium in the presence of information asymmetry. There are three investor types: informed, who initially sees a noisy version of the terminal asset value; uninformed who see no signal; and noise, who think they are seeing a signal, but are seeing pure noise. All three types seek to maximize expected utility form terminal wealth. Equilibrium consists of a market filtration, asset dynamics, and optimal strategies satisfying the clearing condition. Under minimal conditions, (multiple) equilibria are established using filtration enlargement techniques. Explicit identification of the market signal, the asset dynamics, and optimal trading strategies is given, as well as a comparison of the fully and partially revealing equilibria. Joint work with Marcel Rindisbacher and Jerome Detemple of Boston University.

JOHANNES RUF

Filtration shrinkage, the structure of deflators, and the failure of market completeness

Abstract. We analyse the structure of stochastic discount factors (SDFs) projected on smaller filtrations. Via use of a Bayesian filtering approach, we demonstrate the exact mechanism of how updates on the possible class of models under less information result in the strict supermartingale property of projections of SDFs. In a general continuous-path setting, we show that the local martingale part in the multiplicative Doob-Meyer decomposition of projected SDFs are themselves SDFs in the smaller information market. Finally, we demonstrate that these projections are unable to span all possible SDFs in the smaller information market, by means of an interesting example where market completeness is not retained under filtration shrinkage.

This is joint work with Kostas Kardaras.

MYKHAYLO SHKOLNIKOV

Particles interacting through their hitting times: neuron firing, supercooling and systemic risk

Abstract. I will discuss a class of particle systems that serve as models for supercooling in physics, neuron firing in neuroscience and systemic risk in finance. The interaction between the particles falls into the mean-field framework pioneered by McKean and Vlasov in the later 1960s, but many new phenomena arise due to the singularity of the interaction. The most striking of them is the loss of regularity of the particle density caused by the the self-excitation of the system. In particular, while initially the evolution of the system can be captured by a suitable Stefan problem, the following irregular behavior necessitates a more robust probabilistic approach. Based on joint work with Sergey Nadtochiy.

MIHAI SIRBU

Optimal investment and consumption with labor income or liability streams in incomplete markets

Abstract. We consider the problem of optimal investment and consumption for an investor endowed with a stream of income/liabilities, where the constraint that wealth be positive is imposed not only at terminal times, but at all times. We consider a general semi-martingale market which is incomplete (even without the constraints). While in a Markovian framework the problem can be attacked as an HJB with state constraints (Duffie, Flemming, Soner, Zariphopoulou 97), in the non-Markovian case a duality theory is needed to solve the problem. In a Brownian model, such analysis is developed in He,Pages 93 and El-Karoui-Jeanblanc 98. The dual problem is a singular control-problem related to the martingale measure for the unconstrained problem. In order to deal with the general incomplete case we parametrize the labor income stream by a time-dependent multiplier q, in the spirit of Hugonnier-Kramkov. We can show that the primal and dual value functions are conjugate and optimizers exist. Under appropriate conditions we show the the primal value function is differentiable (with respect to q). Based on joint work in progress with Oleksii Mostovyi.

JOSEF TEICHMANN

Reservoir Computing, Regularity Structures and learning of Dynamics in Finance

Abstract. The reservoir computing paradigm reduces the complex task of learning highly non-linear maps by sophisticated splitting procedures: the Ito-Lyons map on rough paths, or solution maps in the theory of regularity structures are abstract examples of this phenomenon. We follow this approach in several examples from mathematical Finance.

ERICK TREVINO

On the partial hedging of American options

Abstract. We will review recent advances on the minimax problem of partial hedging of American options. This is a difficult problem due to its minimax characteristic and we will show that it is possible to handle this problem by convex duality.

KIM WESTON

Incomplete Equilibrium with Stochastic Interest Rates

Abstract. An interest rate conveys the value of current versus future consumption. In a Radner equilibrium, individual agent running consumption problems require an interest rate to be determined endogenously. I will discuss how the presence of a traded annuity leads to a desirable form for the individual agent problems, which provides a pathway to study equilibrium. Under smallness conditions, I will discuss the existence of an equilibrium with endogenously determined stochastic interest rates. (This is joint work with Gordan Zitkovic.)

Other invited participants were BEATRICE ACCIAIO, DYLAN POSSAMAÏ, MATHIAS BEIGLBOCK,