Formal Analysis of Binarized Deep Neural Networks

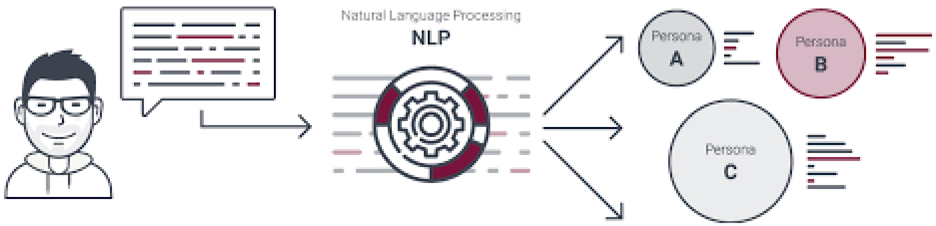
Nina Narodytska

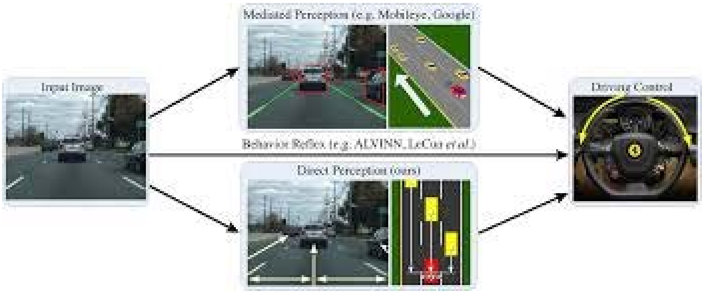
Outline

1. Motivation
2. Adversarial attacks on Neural Networks
3. Verification of Neural Networks
4. Few observations on properties/networks

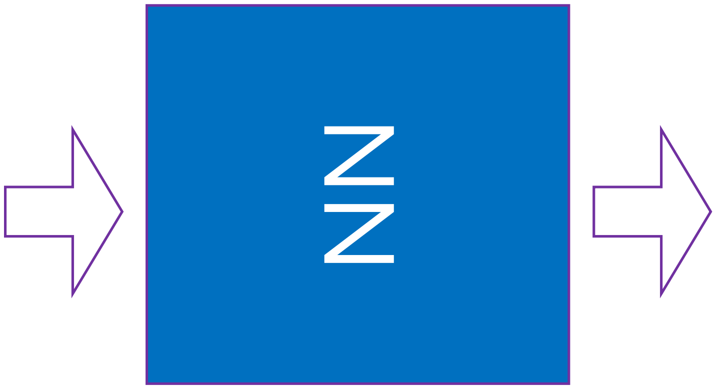
Motivation

Machine Learning





Vulnerability of NN



Function

Vulnerability of NN

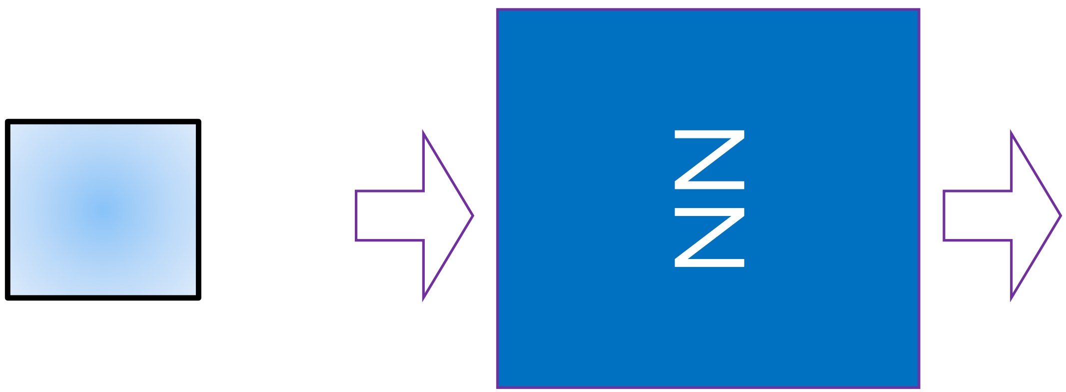
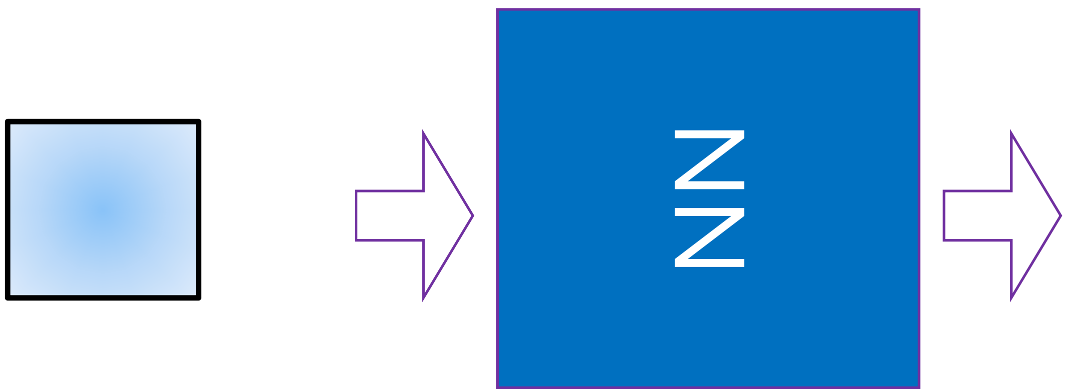


Image	Function

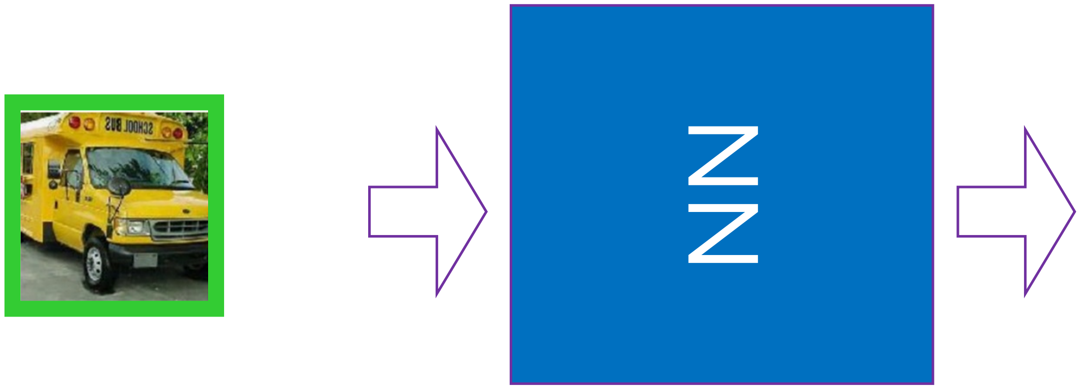
Vulnerability of NN



[
Class C (0.6),
Class B (0.2),
Class A (0.2)
]

Image	Function	Output

Vulnerability of NN



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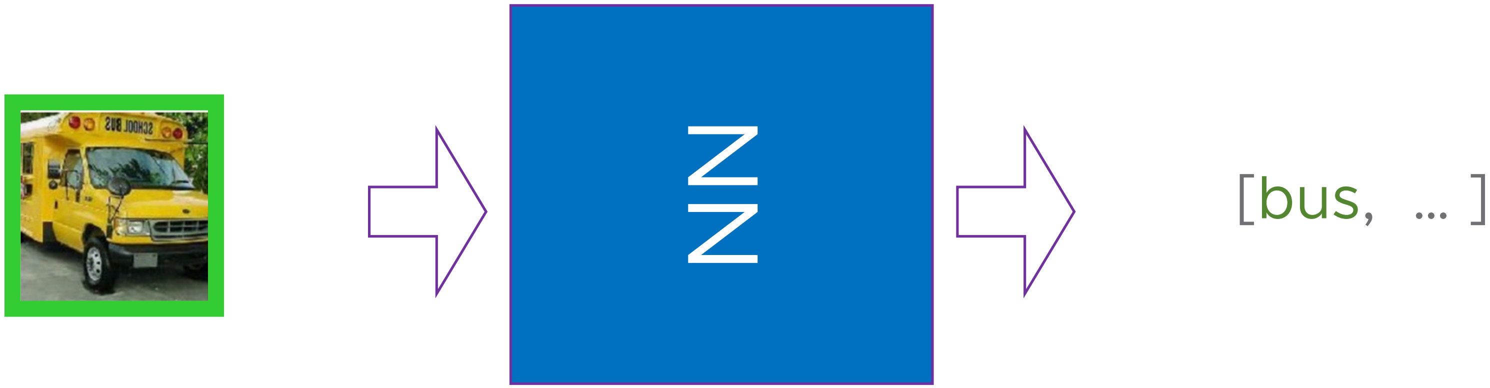

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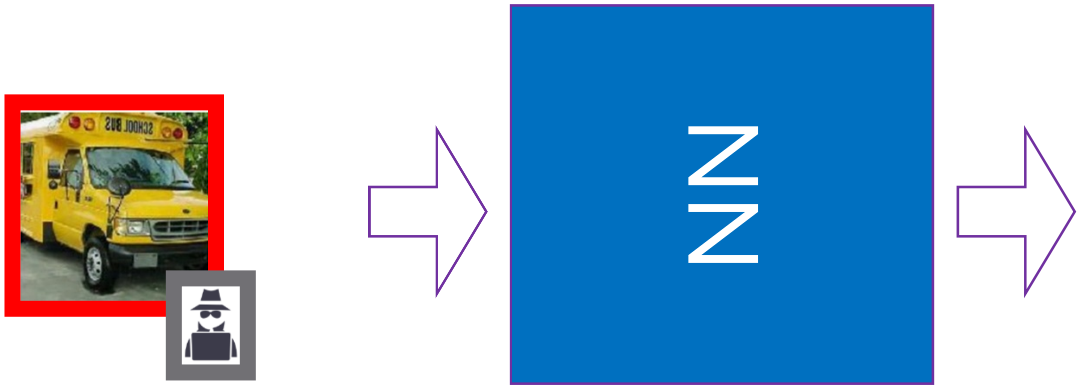

Vulnerability of NN

Adversarial attacks

[Szegedy *et al.*] *Intriguing properties of neural networks*

Untargeted adversarial examples

Given an input (X, C), an input $X' = X + P$ is an untargeted adversarial example iff NN misclassifies X' and P is small according to some metric.

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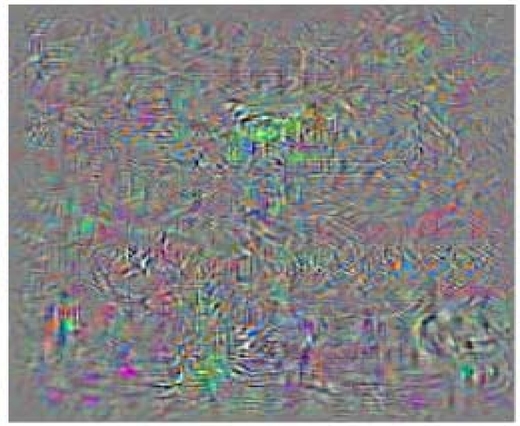
Untargeted adversarial examples

Original image

1.Bus
2. …

Untargeted adversarial examples

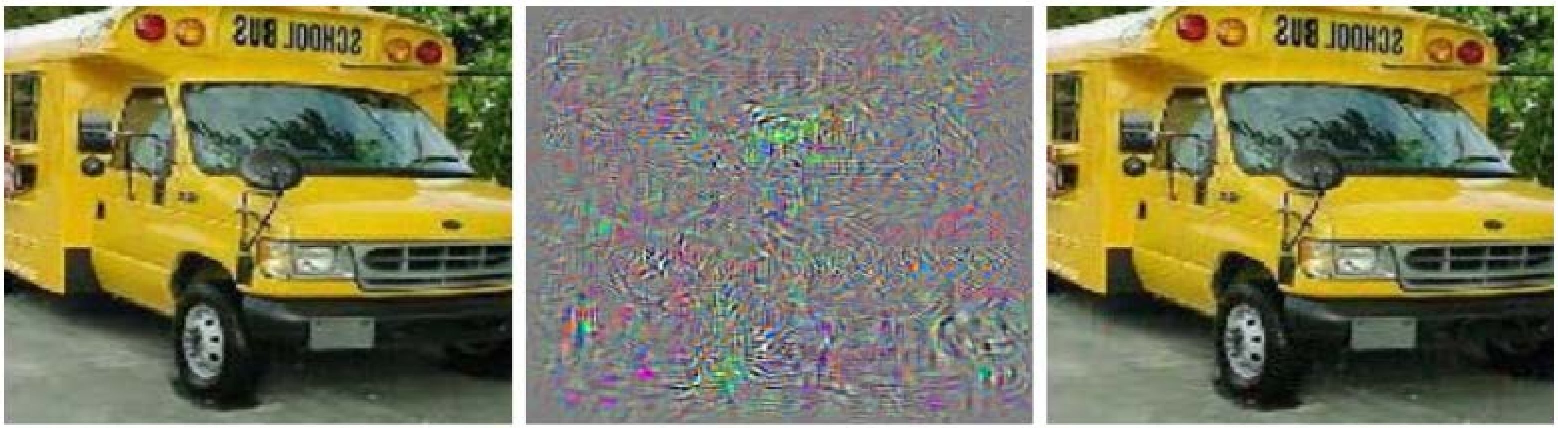
Original image + **P**erturbation



1.Bus
2. …

Untargeted adversarial examples

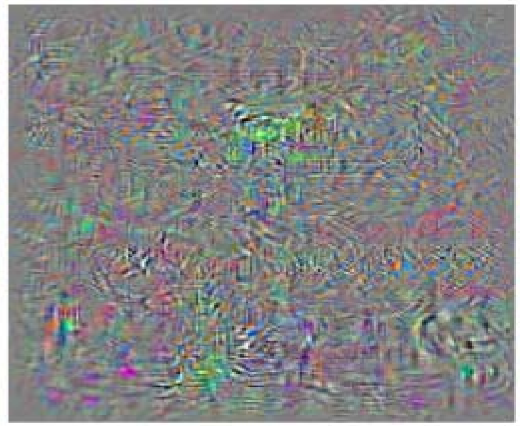
Original image + Perturbation = Perturbed image



1.Bus
2. …

Untargeted adversarial examples

Original image + **P**erturbation = Perturbed image


1.Bus
2. …

1. Ostrich
2. Bus

Targeted adversarial examples

Given a input (X, C) and a target class T, an input $X' = X + P$ is an targeted adversarial example iff the top prediction is T and P is small according to some metric.

Targeted adversarial examples

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Given a input **(X, C)** and a target class **T**, an input **X'=X + P** is an targeted adversarial example iff the top prediction is T and P is small according to some metric.

Targeted adversarial examples

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Targeted adversarial examples

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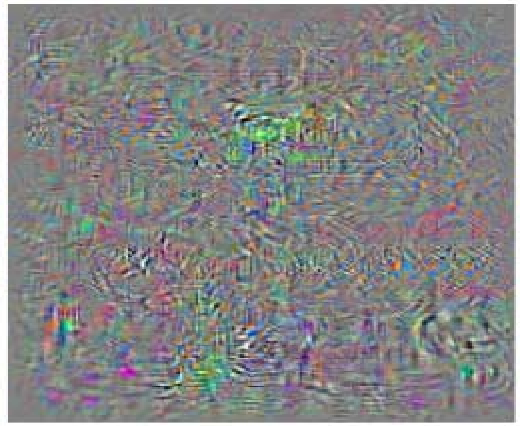
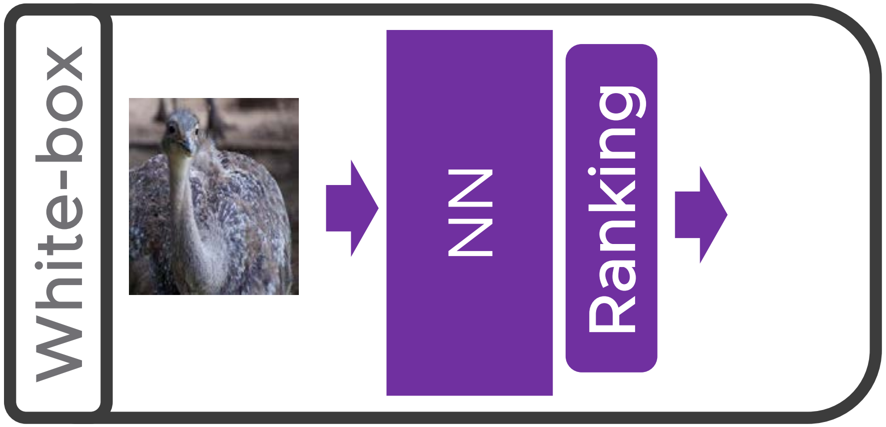


1.Bus
2. …

1. Building
2. Bus

Target: Building

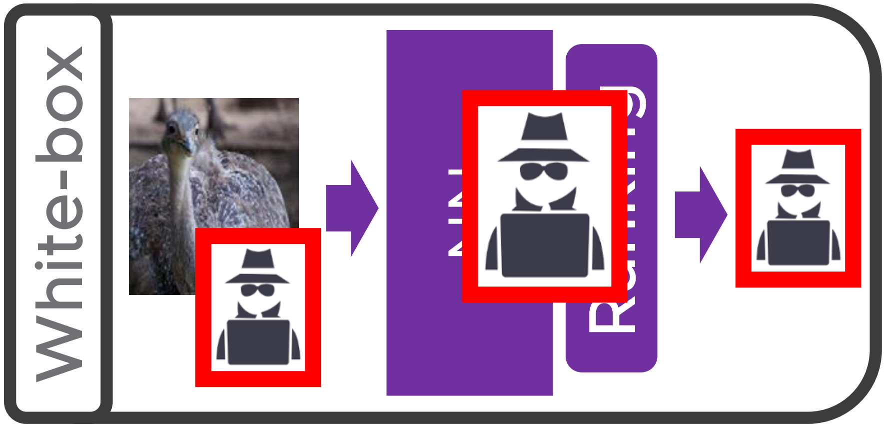
White-box vs Black-box Attacks



[Goodfellow *et al.*, Szegedy *et al.*]

[Papernot *et al.*, 2016a, 2016b]

White-box vs Black-box Attacks



[Goodfellow *et al.*, Szegedy *et al.*]

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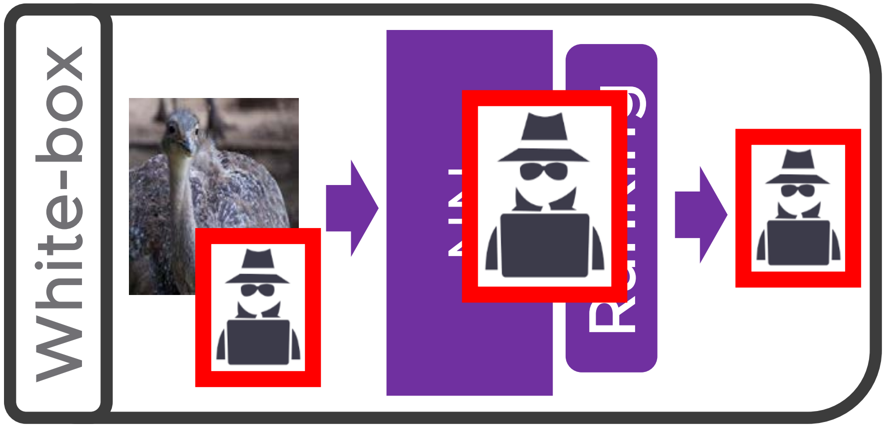
Gradient-based methods that generate adversarial images by perturbing the gradients of the loss function w.r.t. the input image

White-box vs Black-box Attacks



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Gradient-based methods that generate adversarial images by perturbing the gradients of the loss function w.r.t. the input image

- More realistic and applicable model
- Challenging because of weak adversaries: no knowledge of the network architecture
- Previous attacks require 'transferability' assumption on adversarial examples
- GAN based attacks

Are NNs reliable to use in safety-critical application?

Verification of NN

Verification of NN

Verification of Neural Networks

Verification of NN

- Pulina and Tacchella 2010.
  An Abstraction-Refinement Approach to Verification of Artificial Neural Networks.
- Osbert Bastani, Yani Ioannou, Leonidas Lampropoulos, D. Vytiniotis, Aditya Nori, and A. Criminisi.
  Measuring neural net robustness with constraints
- Guy Katz, Clark W. Barrett, David L. Dill, Kyle Julian, and Mykel J. Kochenderfer.
  Reluplex: An efficient SMT solver for verifying deep neural networks.
- Xiaowei Huang, Marta Kwiatkowska, Sen Wang, and Min Wu.
  Safety verification of deep neural networks
- Svyatoslav Korneev, Nina Narodytska, Luca Pulina, Armando Tacchella, N. Bjorner, and M. Sagiv. ***Constrained image generation using binarized neural networks with decision procedures***.
- Nina Narodytska, Shiva Prasad Kasiviswanathan, Leonid Ryzhyk, Mooly Sagiv, and Toby Walsh. ***Verifying properties of binarized deep neural networks***
- Chih-Hong Cheng, Georg Nuhrenberg, and Harald Ruess.
  Maximum resilience of artificial neural networks.
- Chih-Hong Cheng, Georg Nuhrenberg, and Harald Ruess.
  Verification of binarized neural networks.
- Rudiger Ehlers.
  Formal verification of piece-wise linear feed-forward neural networks.
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  Verifying neural networks with mixed integer programming

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- Scalability (size of the network, dimensionality of perturbations)

Verification of NN

	Core Techniques	Workable Layer Types	Running Time on ACAS Xu	Computational Complexity	Applicable to State-of-the-art Networks?	Maximal No. of Layers in Tested DNNs
SHERLOCK	MILP + Local Search	ReLu	No experiment	NP w.r.t. neuron no.	No (~6845 neurons)	6
Reluplex	SMT + LP	ReLu	O(10^4)-O(10^6)	NP w.r.t. neuron no.	No (~ 300 neurons)	6
Planet	SAT + LP	ReLu, maxpooling	O(10^3)	NP w.r.t. neuron no.	No (~ 300 neurons)	6
MIP	MIP	ReLu, maxpooling	O(10^3)	NP w.r.t. neuron no.	No (~ 300 neurons)	6
BaB	MIP + BaB	ReLu, maxpooling	O(10^2)	NP w.r.t. neuron no.	No (~ 300 neurons)	6
DeepGO (this paper)	GO + Lipschitz Continuty	Layer with Lipschitz Continuty (Sigmod, Tanh, max-pooling, ReLu, etc)	O(10^2)	NP w.r.t. changed input dimensions	Yes (millions of neurons)	19

Figure 8: A high-level comparison with state-of-the-art methods: SHERLOCK [10], Reluplex [7], Planet [26], MIP [11, 9] and BaB [12].

IJCAI'18:
Reachability Analysis of Deep Neural Networks with Provable Guarantees
Wenjie Ruan, Xiaowei Huang, Marta Kwiatkowska

Verification of NN

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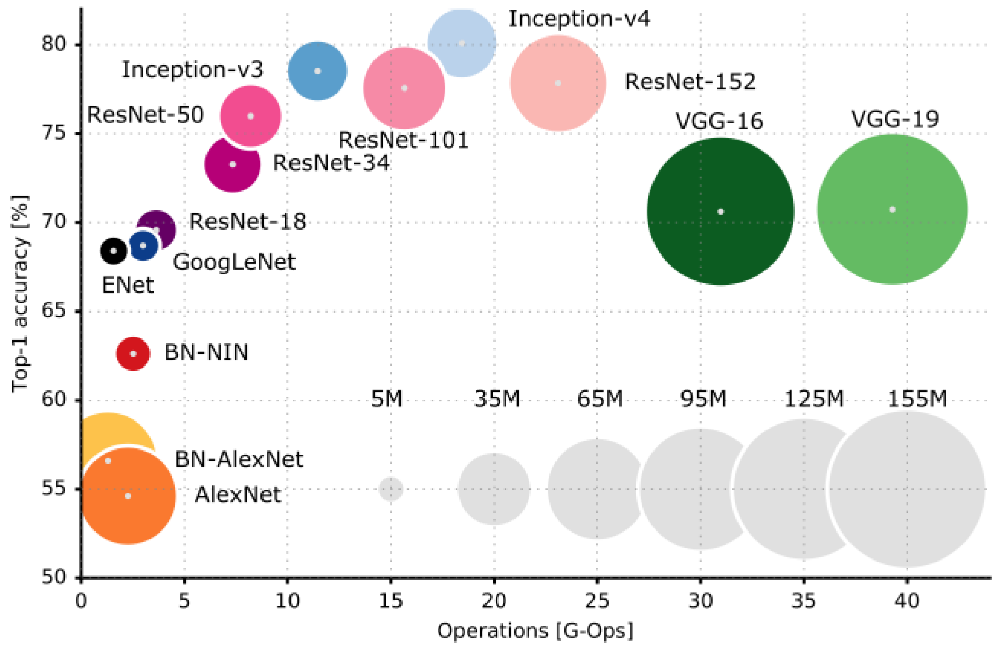
Neural Networks

Neural Networks



[Alfredo Canziani, Adam Paszke, Eugenio Culurciello
An Analysis of Deep Neural Network Models for Practical Applications]

Binarized Neural Networks

Binarized Neural Networks: Training Deep Neural Networks with Weights and Activations Constrained to +1 or -1
Matthieu Courbariaux, Itay Hubara, Daniel Soudry, Ran El-Yaniv, Yoshua Bengio

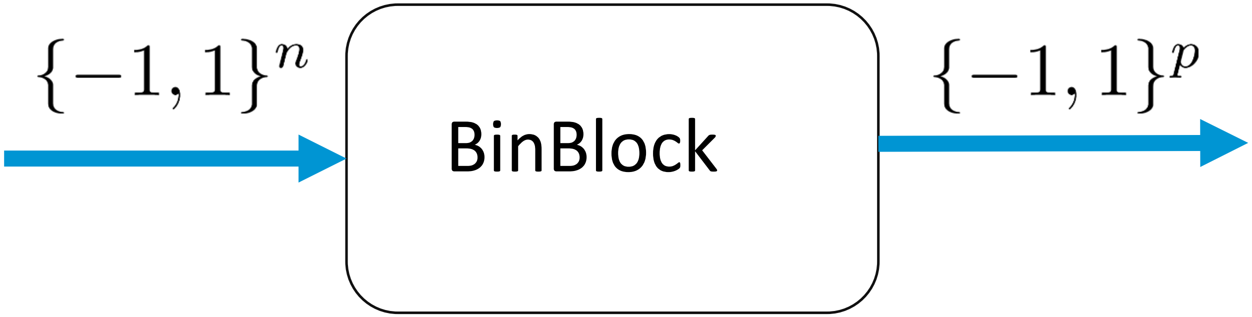
Why Binarized Neural Networks

- special class of NN, where most parameters are binary {-1,1}
- allows fast binary matrix multiplication (7x speed up on a GPU).
- produces smaller size models as most parameters are binary

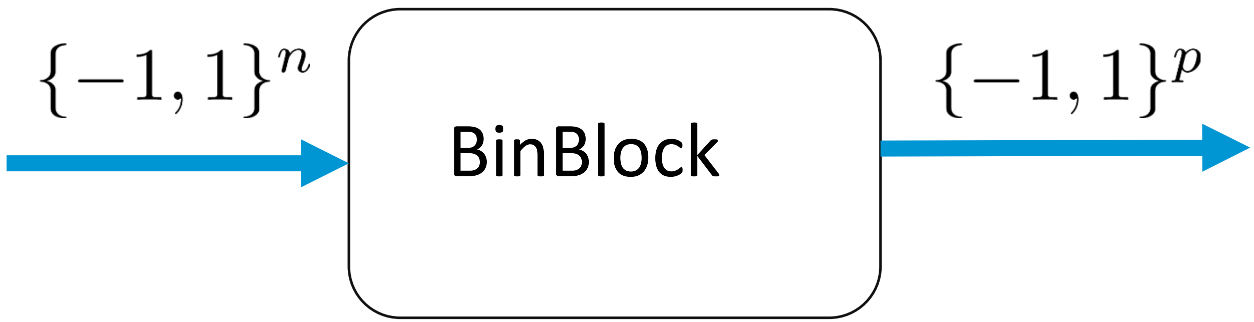
Binarized neural networks I Hubara, M Courbariaux, D Soudry, R El-Yaniv, Y Bengio Advances in Neural Information Processing Systems, 4107-4115	634 *	2016
Binaryconnect: Training deep neural networks with binary weights during propagations M Courbariaux, Y Bengio, JP David Advances in neural information processing systems, 3123-3131	483	2015

vmware

Binarized Building Block

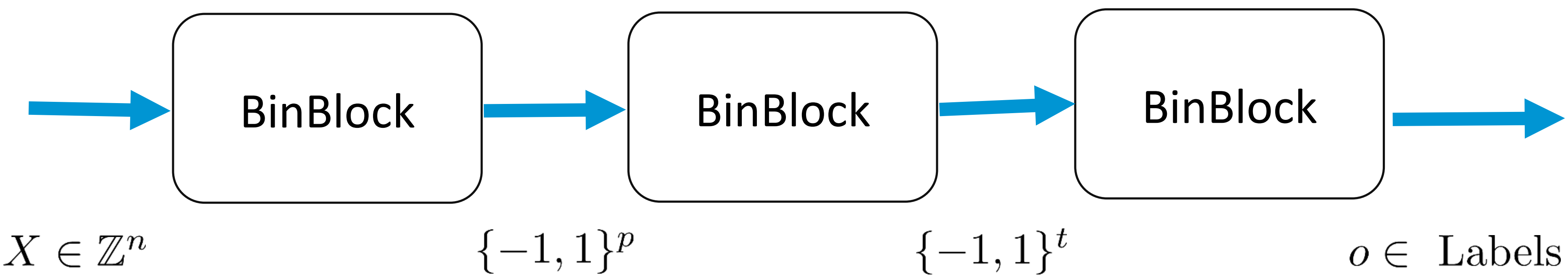


Binarized Building Block



A block can be encoded as SAT

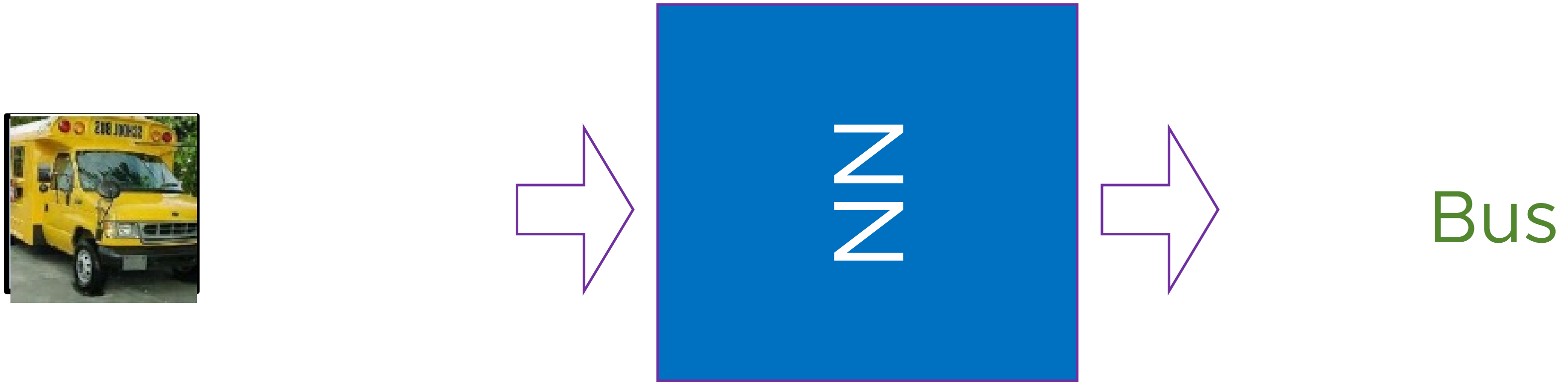
Binarized Building Block

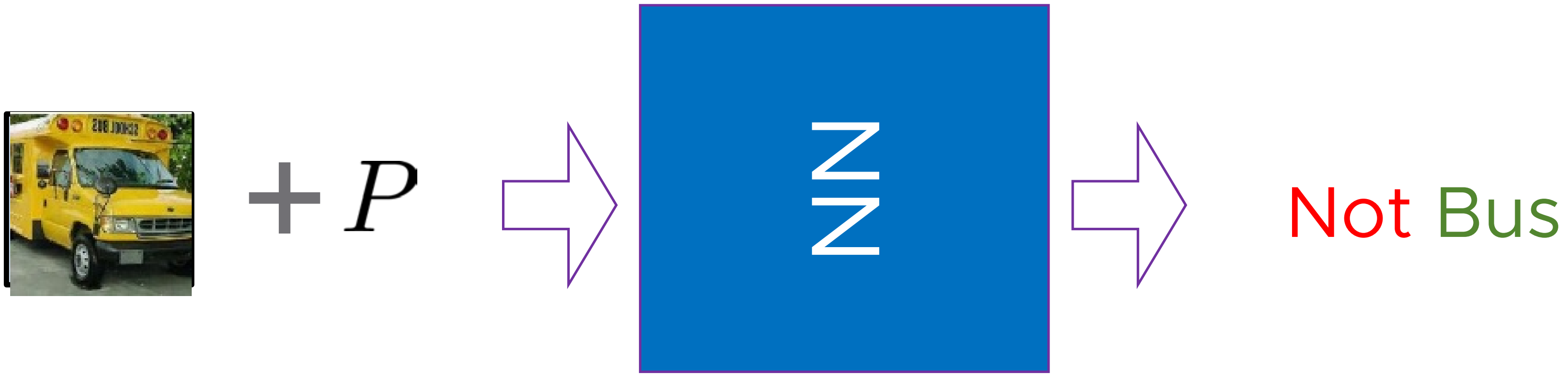


SAT-based approach to adversarial examples

Verifying Properties of Binarized Deep Neural Networks

N.Narodytska, with S. Kasiviswanathan, L. Ryzhyk, M. Sagiv, T. Walsh


NN
Bus


+ P
NN
Not Bus

Boolean encoding

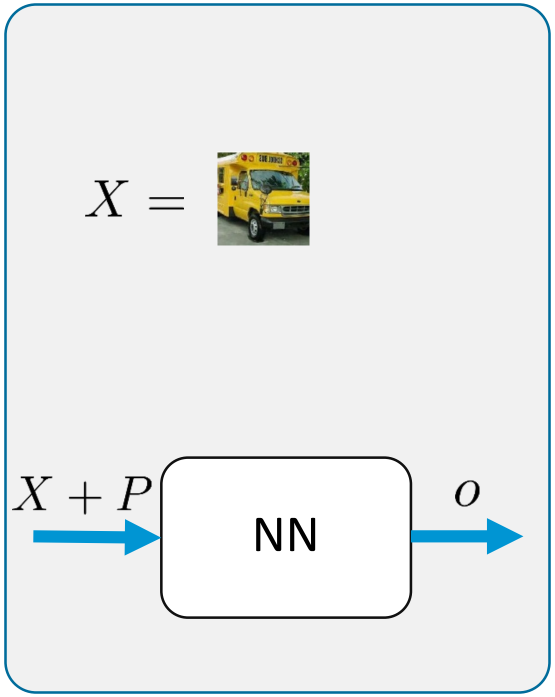
Network function

- Adversarial goal
- Constraints on perturbation

Step 1

Step 2

Boolean encoding

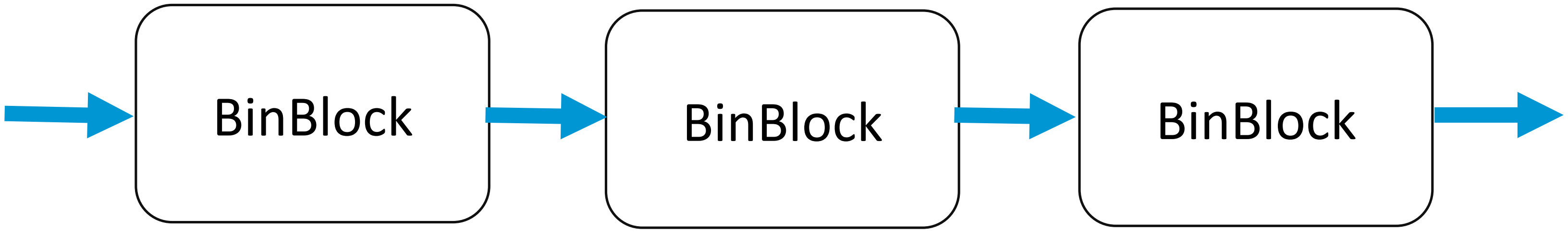


- Adversarial goal
- Constraints on perturbation

Step 1

Step 2

Block-wise BNN encoding



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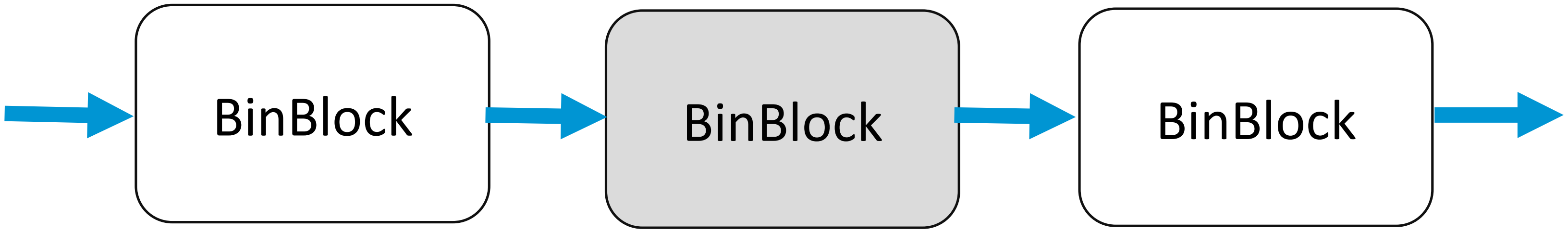

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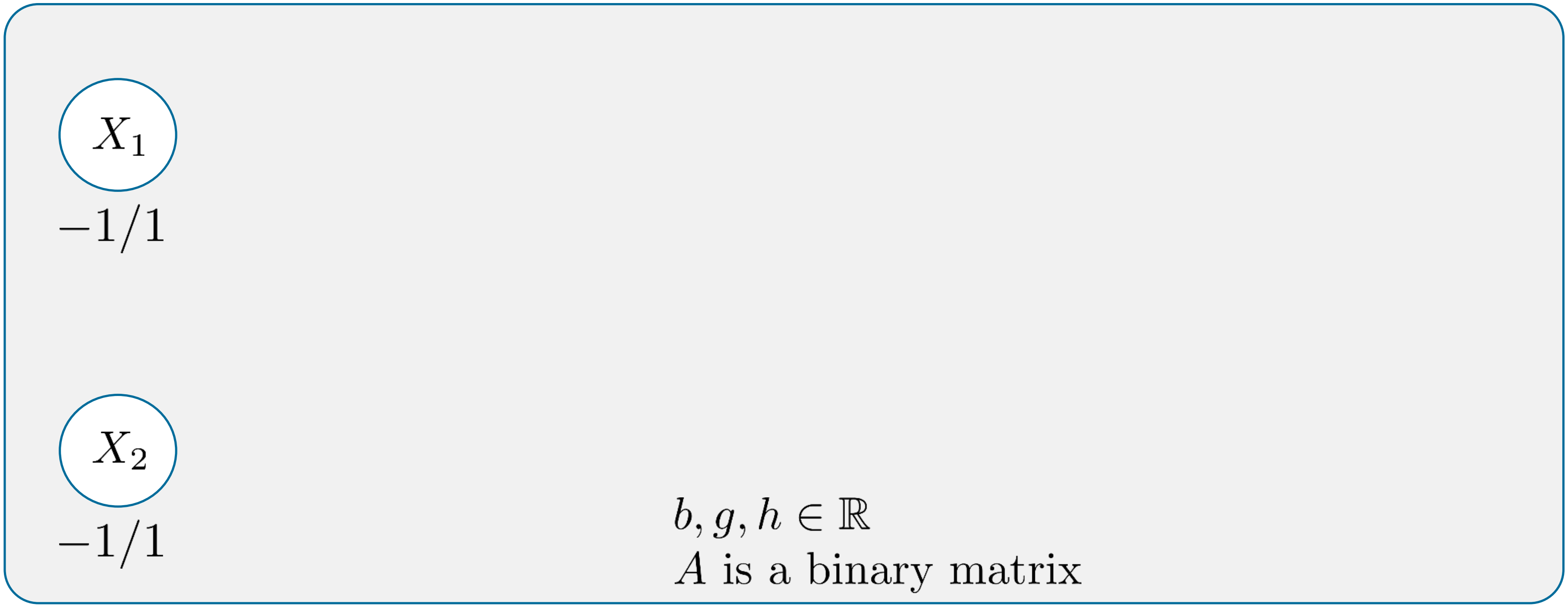

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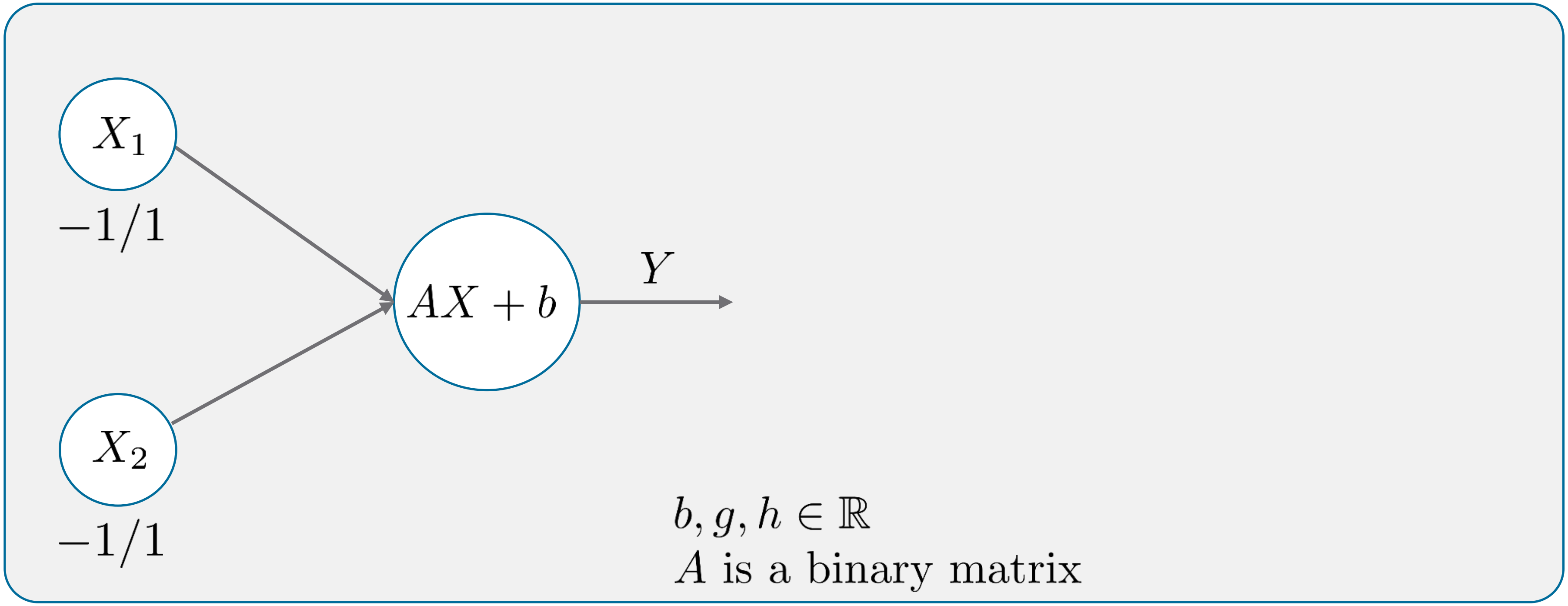
Block-wise BNN encoding



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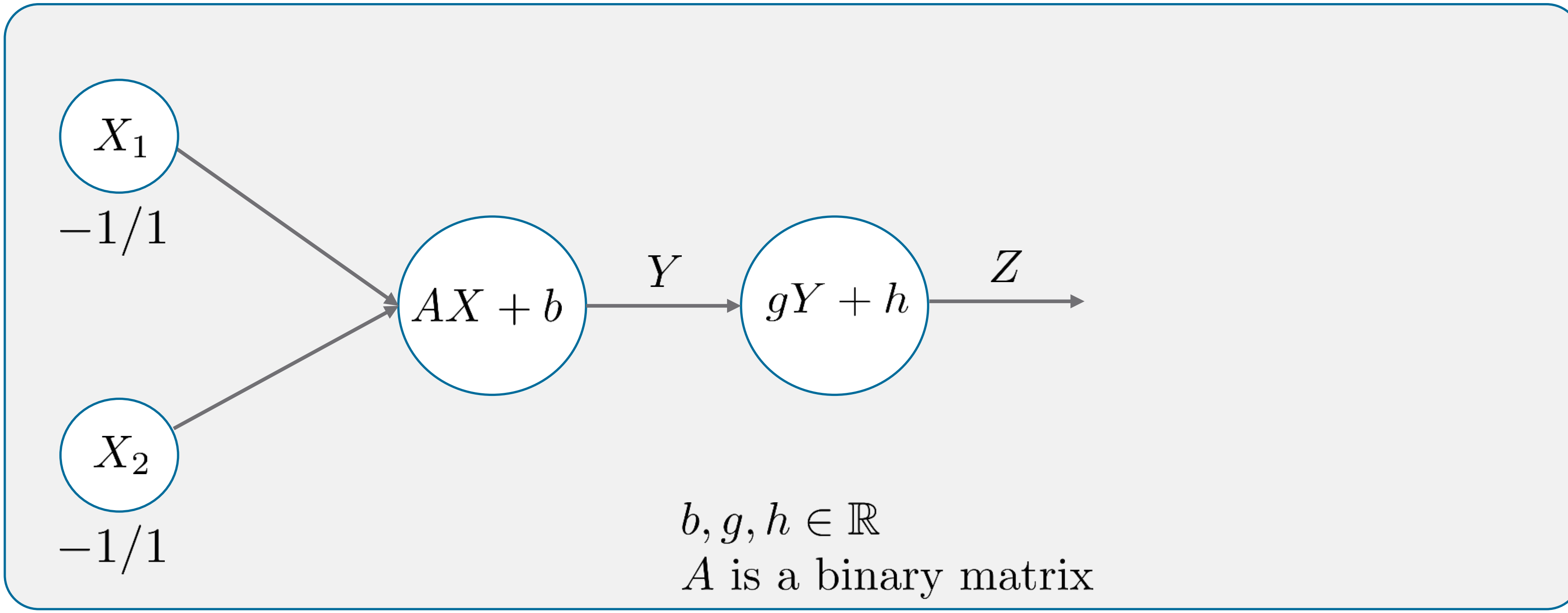

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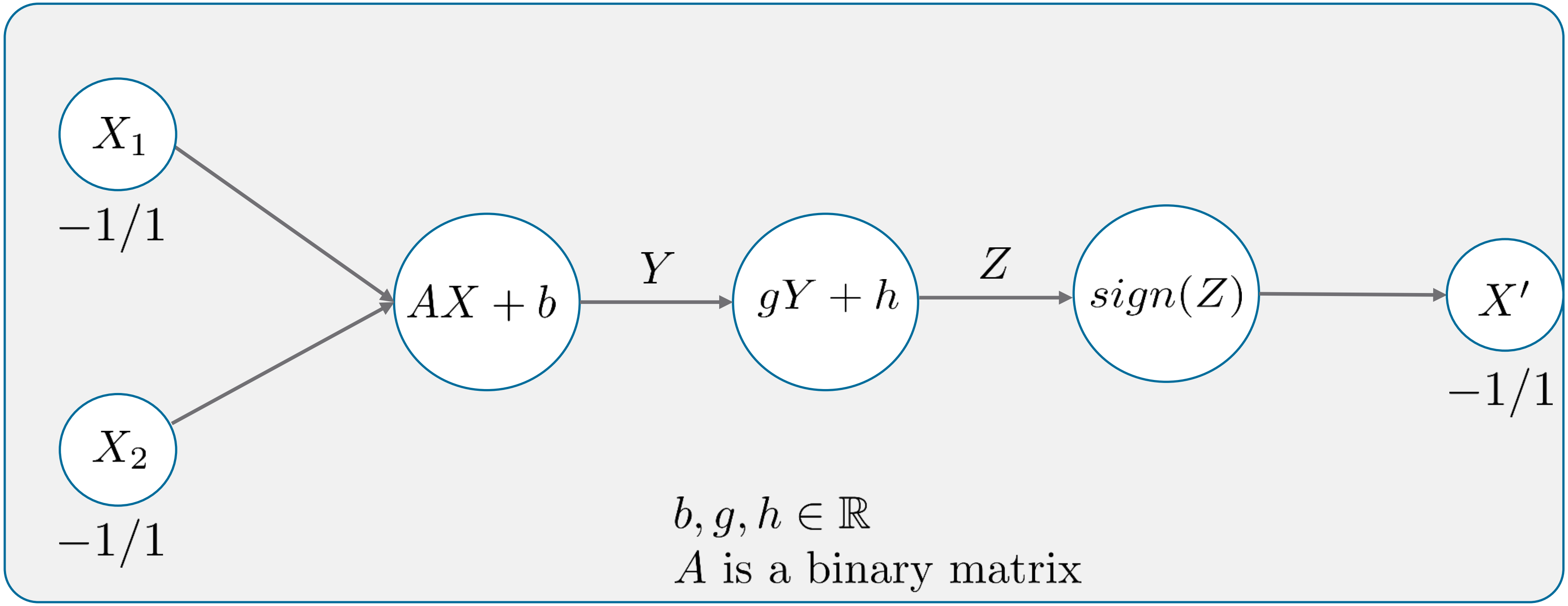

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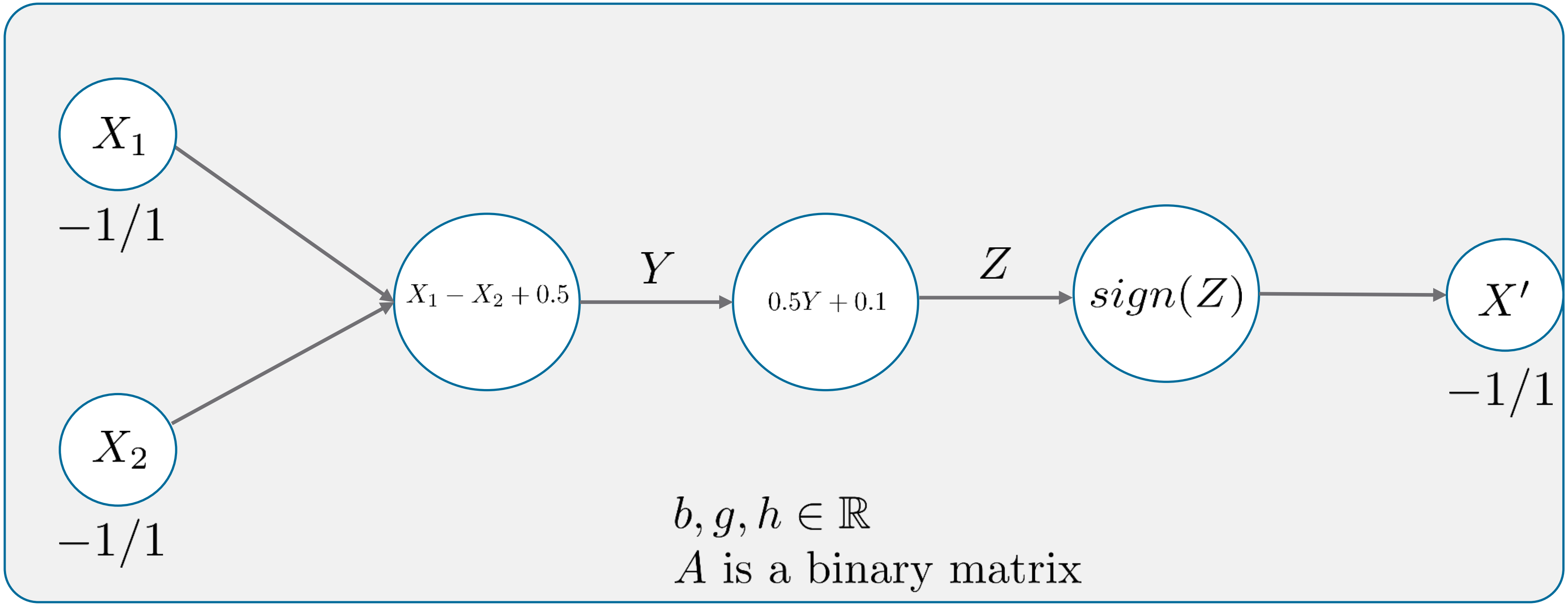

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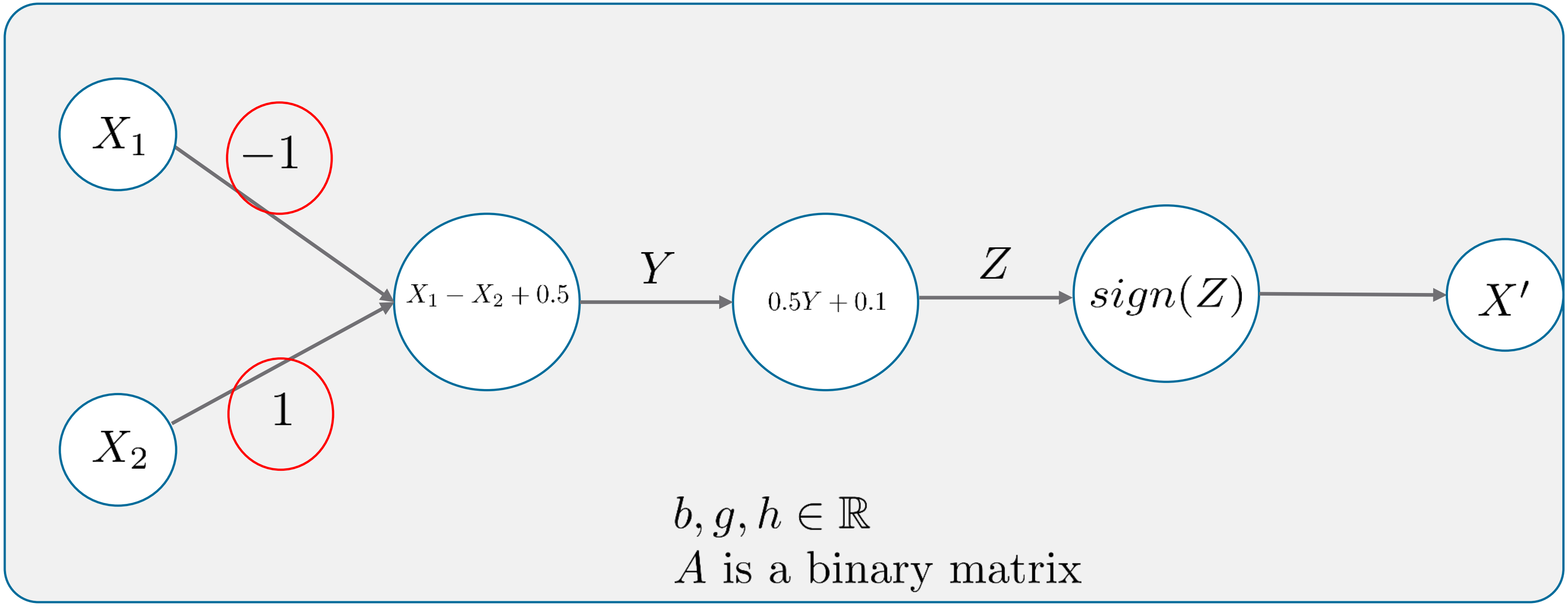

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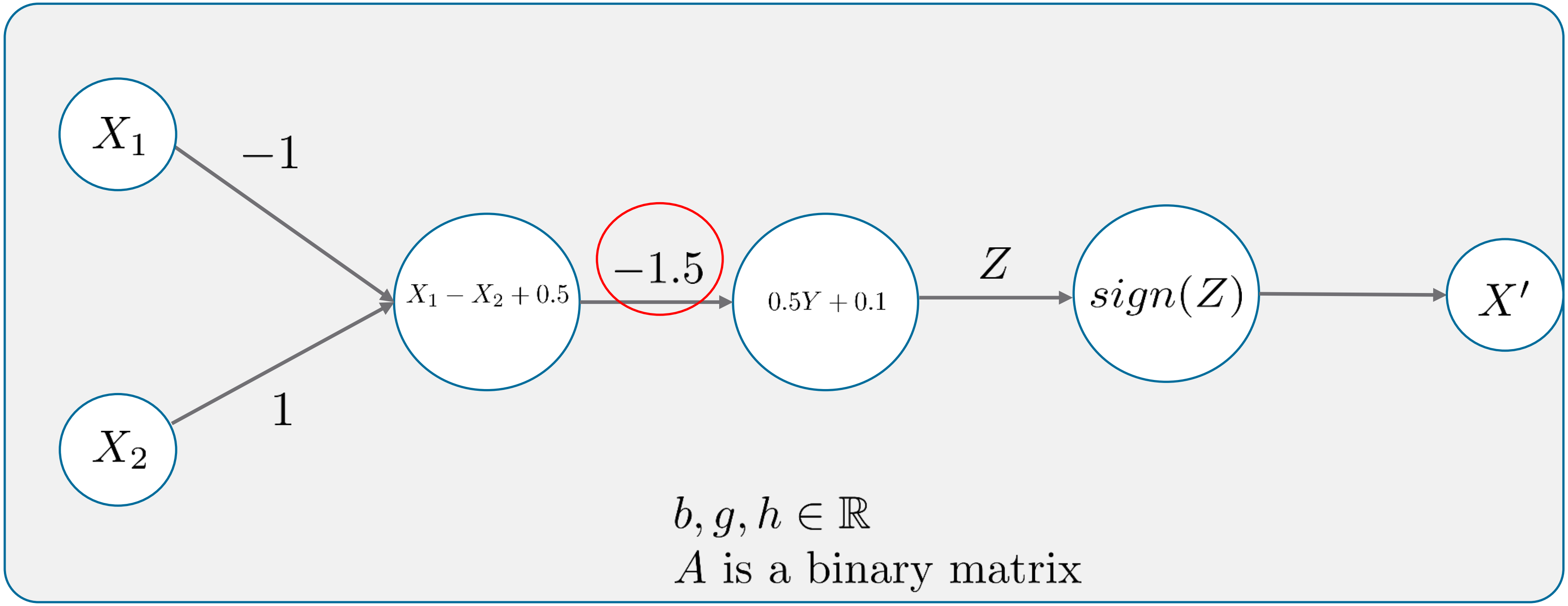

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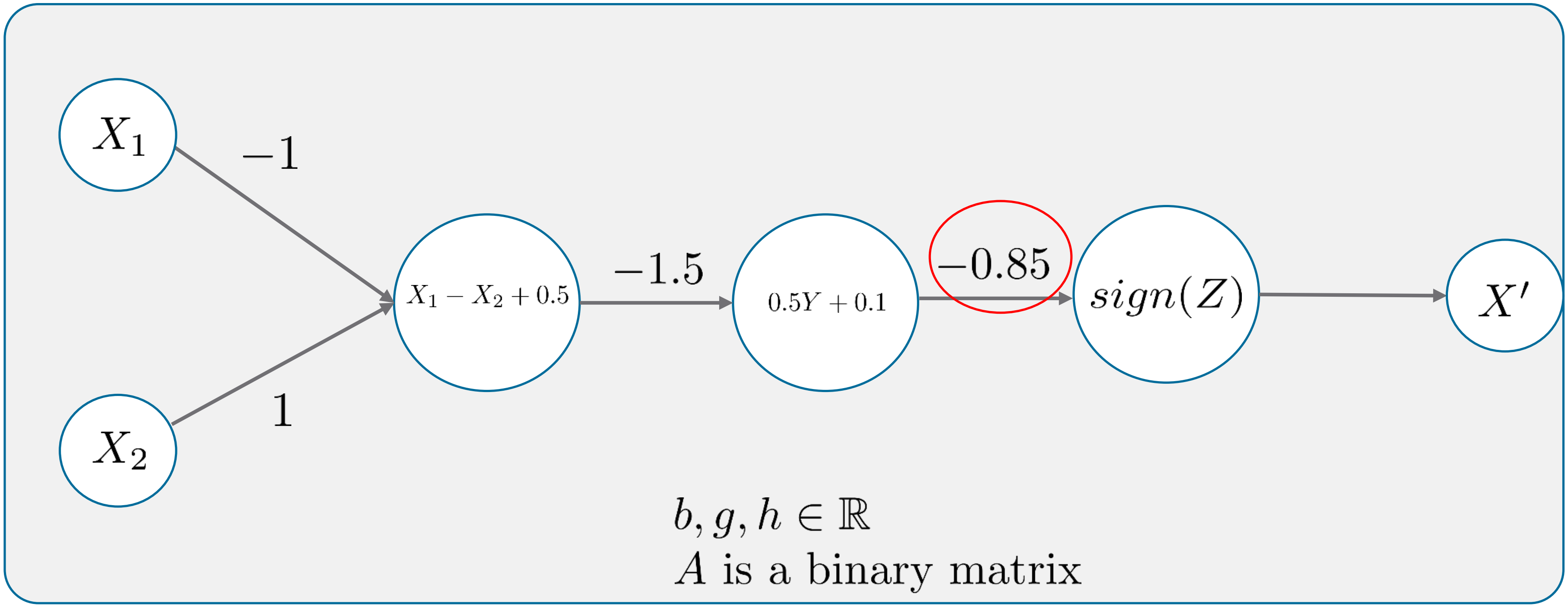

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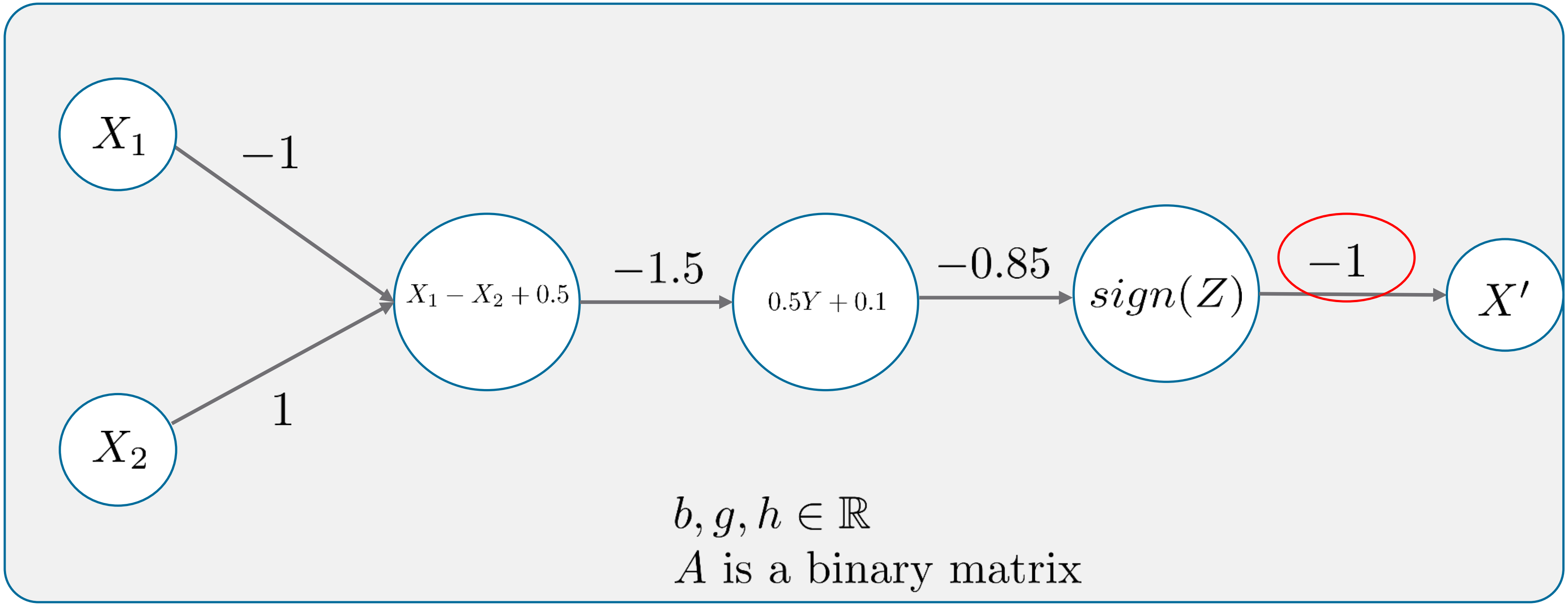

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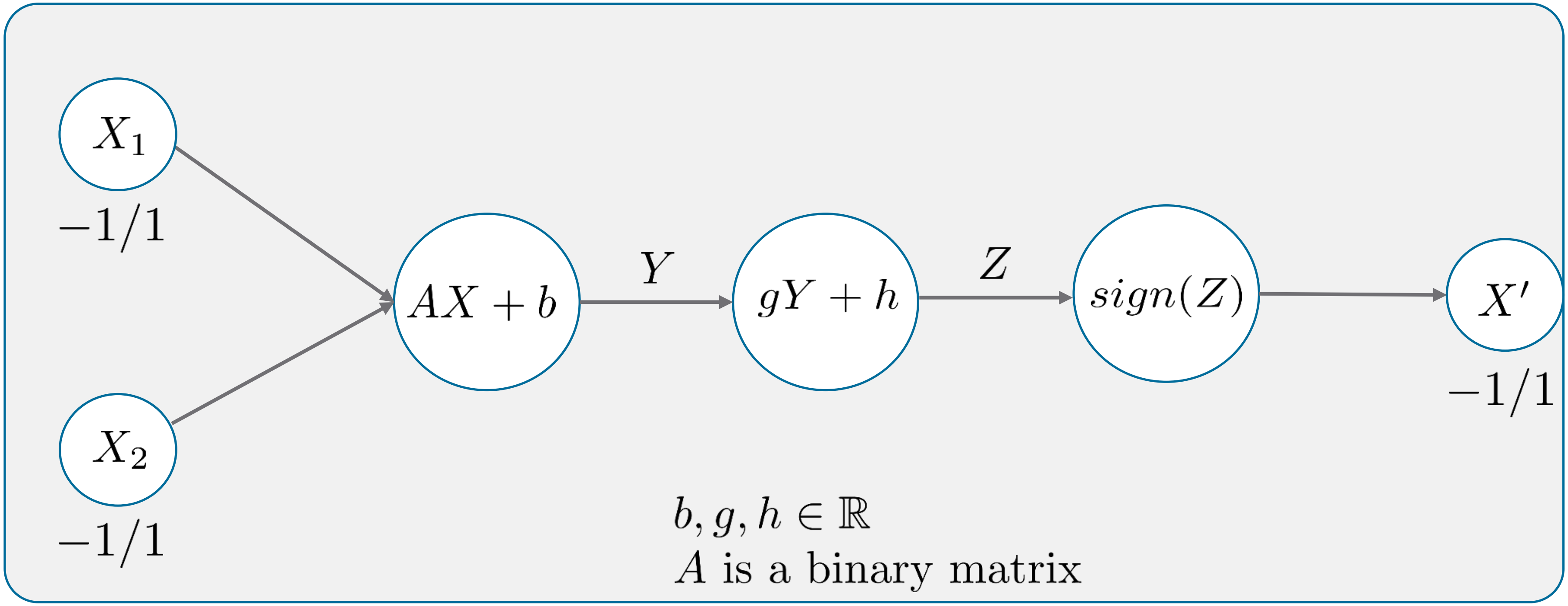

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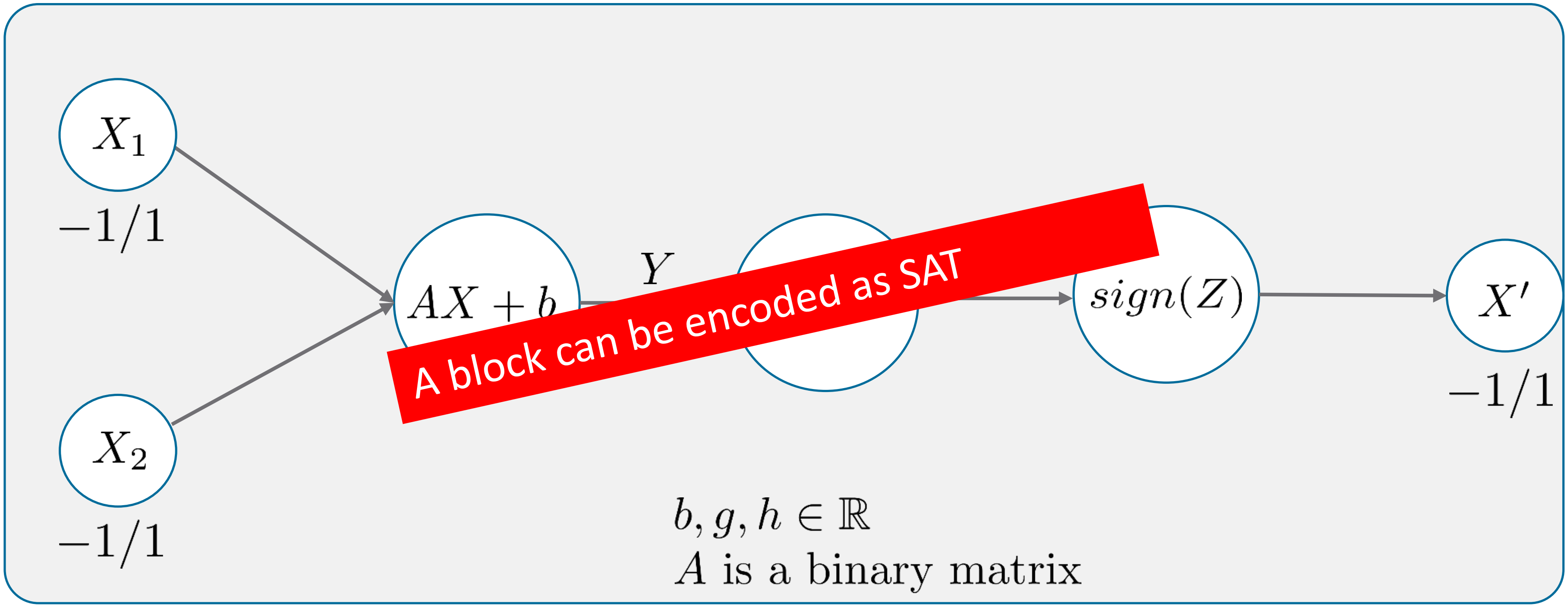

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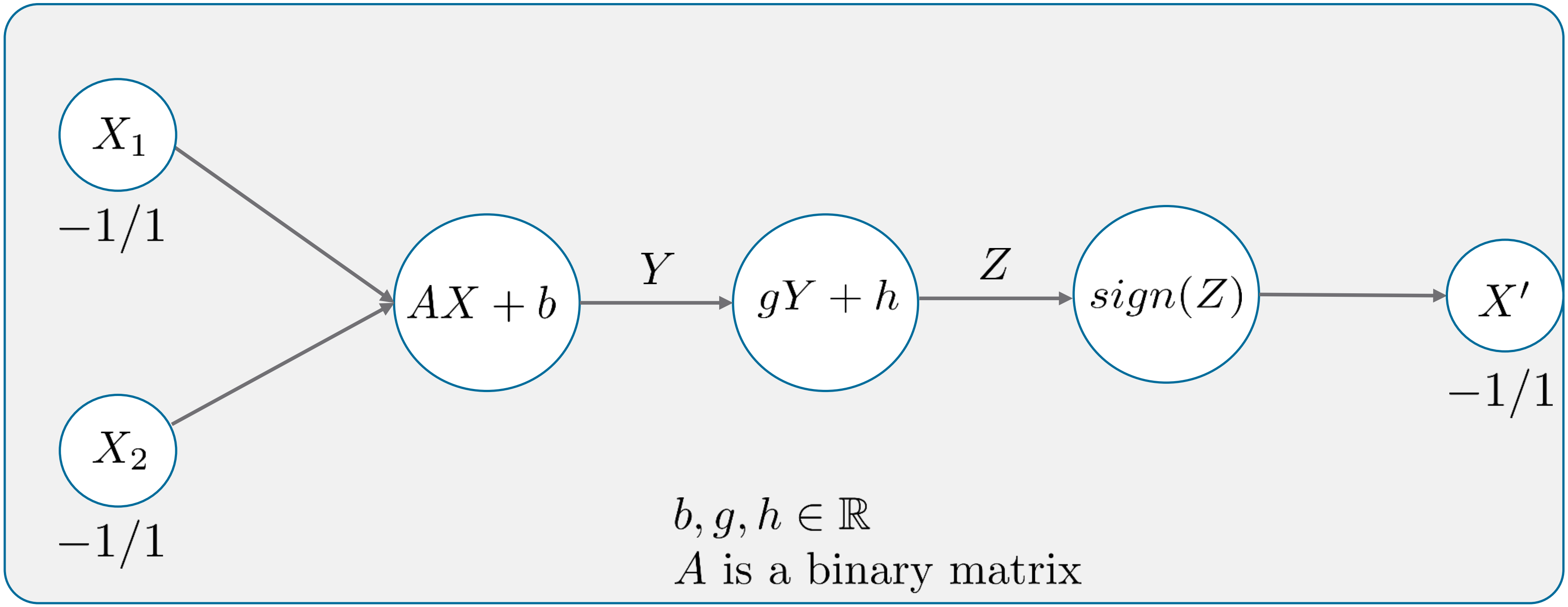

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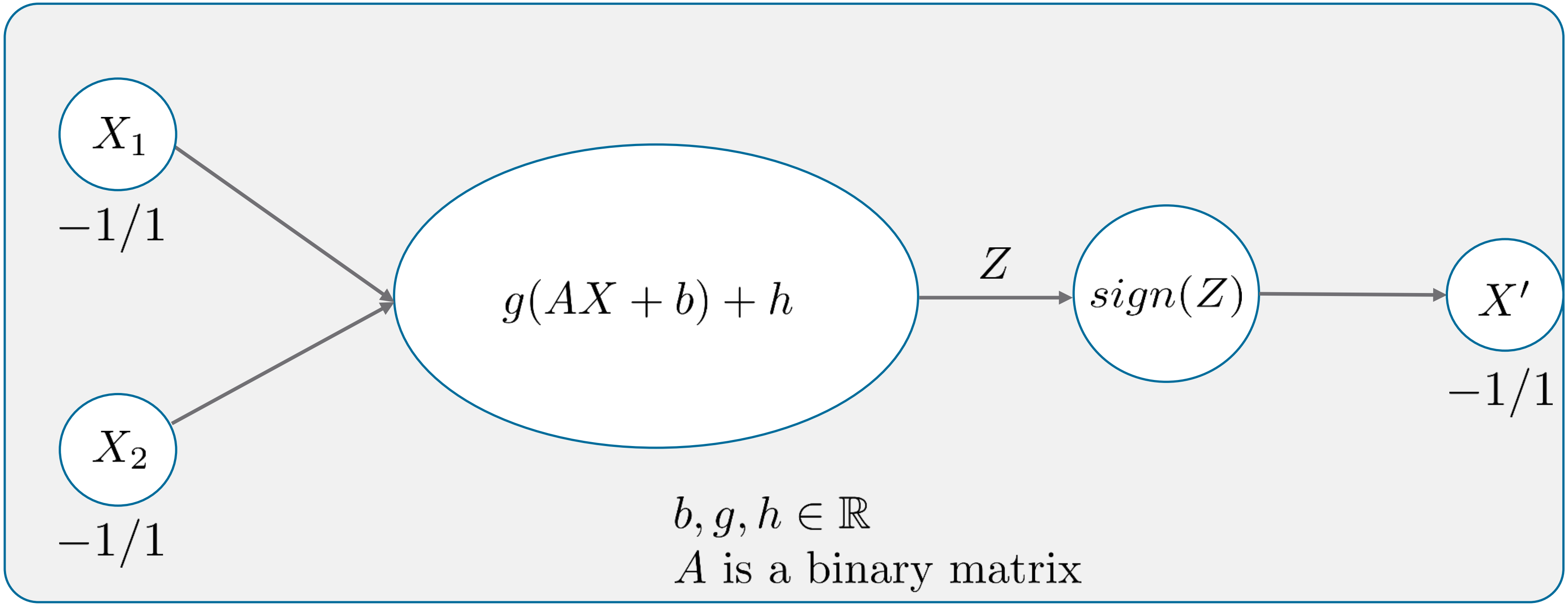

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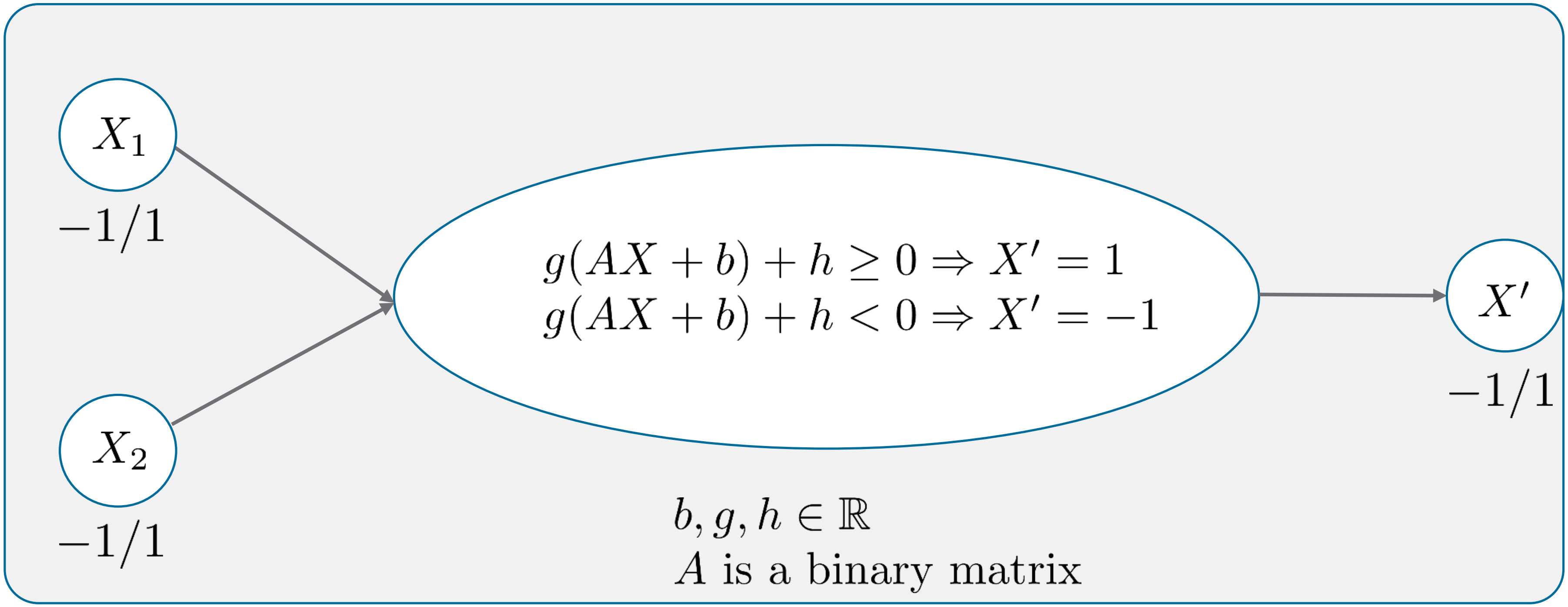

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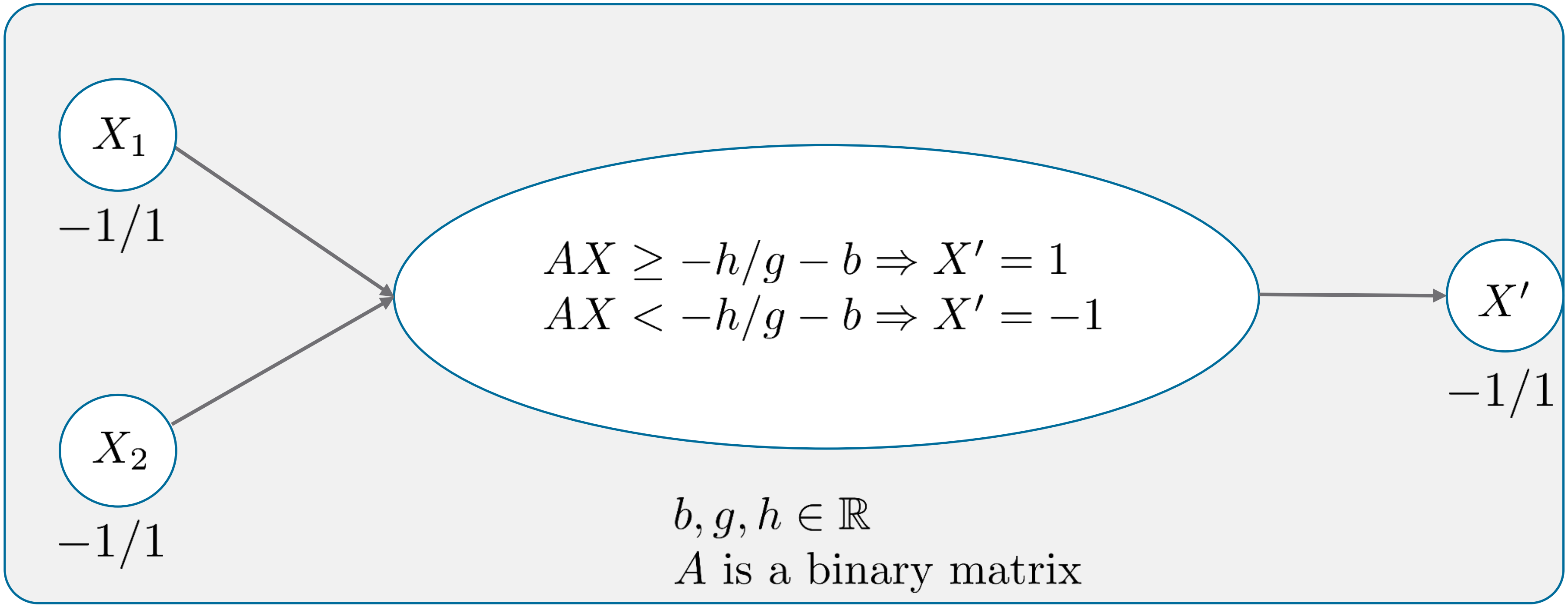

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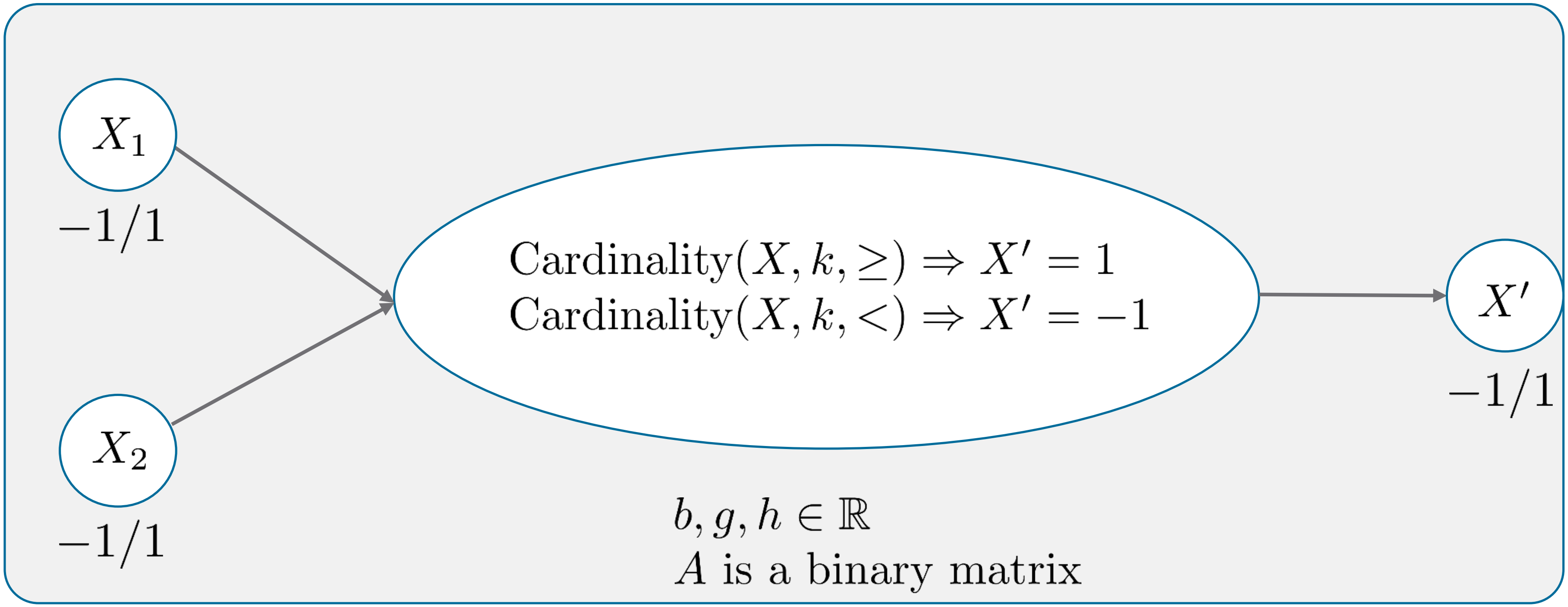

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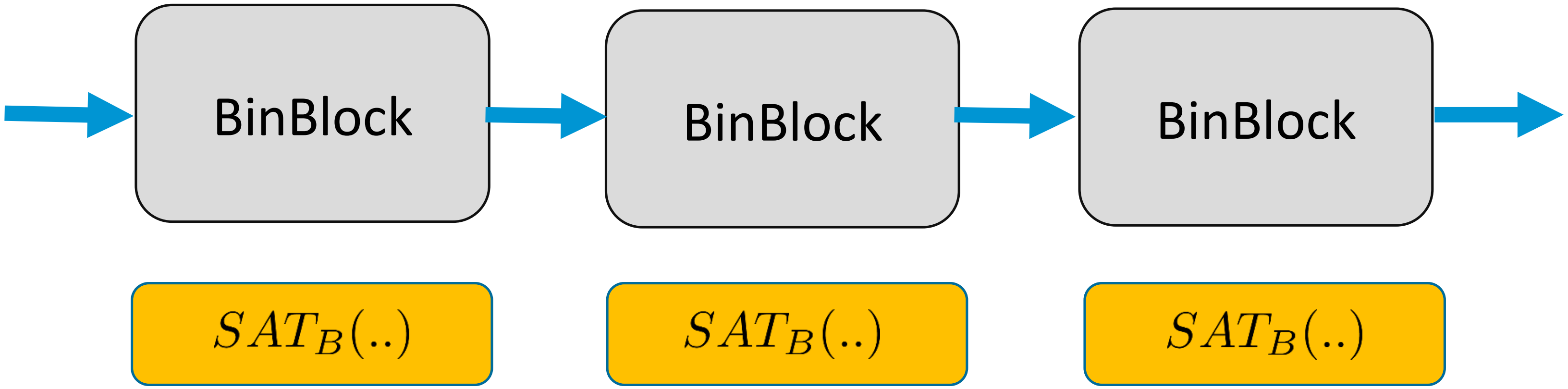

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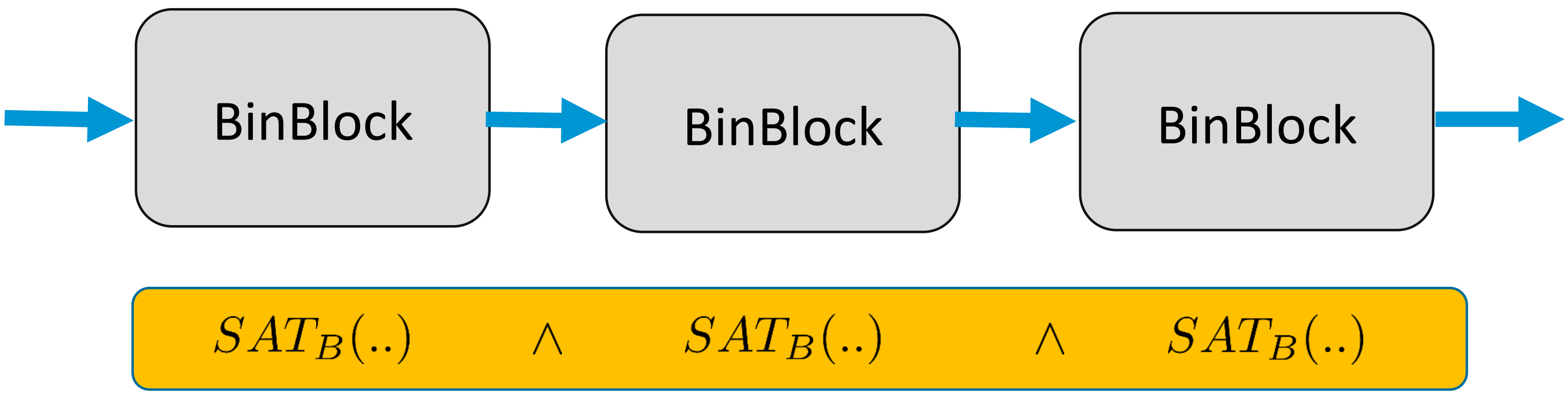
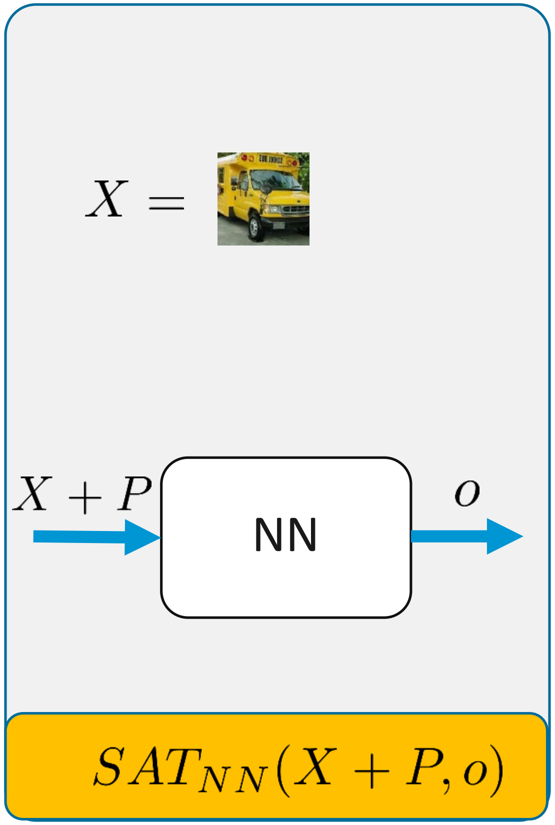


Boolean encoding



Step 1

Step 2

Boolean encoding

$X =$ 

$X + P \rightarrow$ NN $\rightarrow o$

$SAT_{NN}(X + P, o)$

Step 1

- Adversarial goal
- Constraints on perturbation

Step 2

Boolean encoding

$X =$ 

$X + P \rightarrow$ NN $\xrightarrow{o}$

$SAT_{NN}(X + P, o)$

Step 1

$o \neq \text{bus}$

$\max(P) < \epsilon$

Step 2

Boolean encoding

$X =$ 

$X + P \rightarrow \boxed{\text{NN}} \rightarrow o$

$SAT_{NN}(X + P, o)$

Step 1

$o \neq \text{bus}$

$\max(P) < \epsilon$

$SAT_{Ad}(P, bus, o)$

Step 2

Boolean encoding

$X =$ 

$$X + P \rightarrow \boxed{\text{NN}} \rightarrow o$$

$$SAT_{NN}(X + P, o)$$

Step 1

$$o \neq \text{ bus}$$

$$\max(P) < \epsilon$$

$$SAT_{Ad}(P, bus, o)$$

Step 2

Boolean encoding

$$SAT_{NN}(X + P, o)$$

$$SAT_{Ad}(P, bus, o)$$

Search procedure

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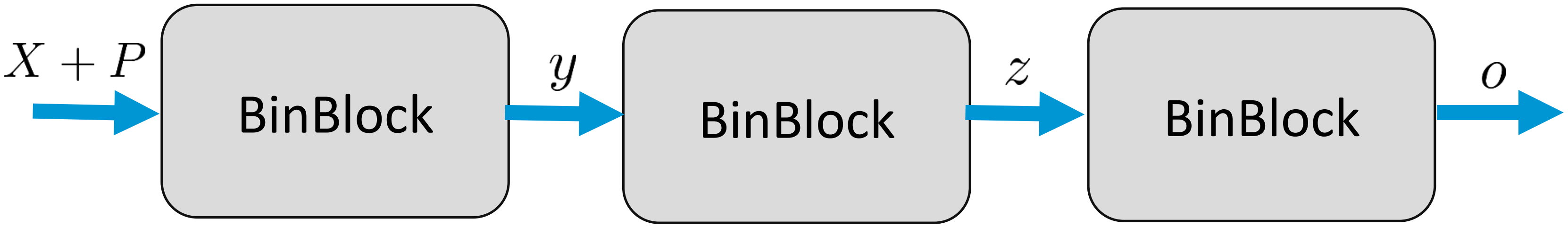

$$Init(P) \wedge SAT_B(X+P, y) \wedge SAT_B(y, z) \wedge SAT_B(z, o) \wedge Ad(o)$$

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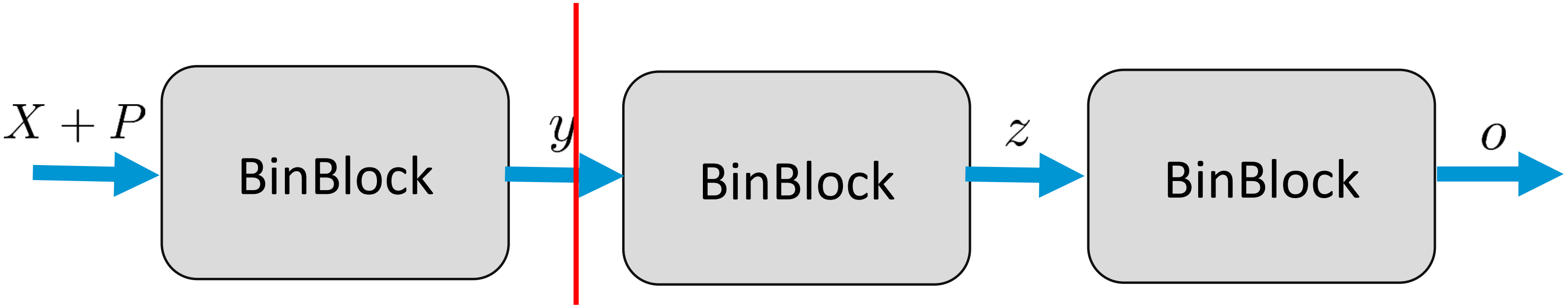

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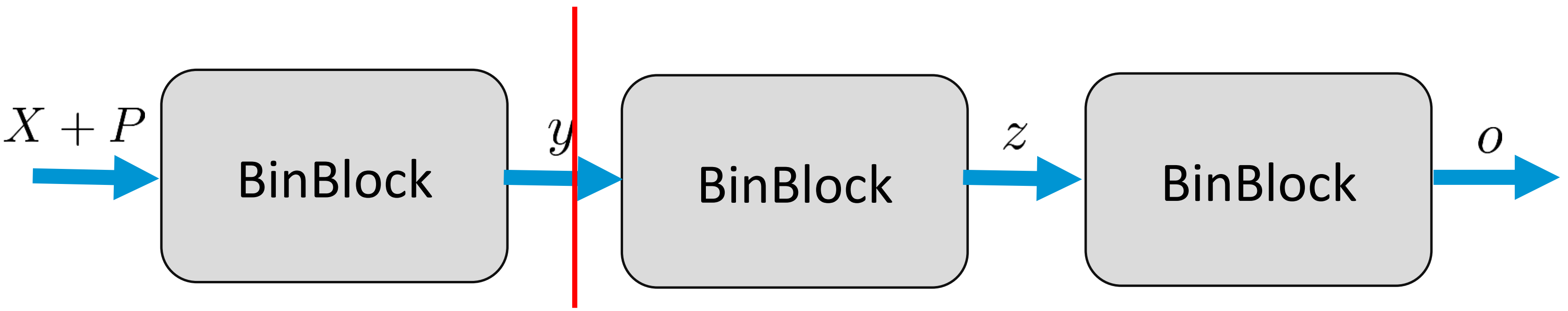

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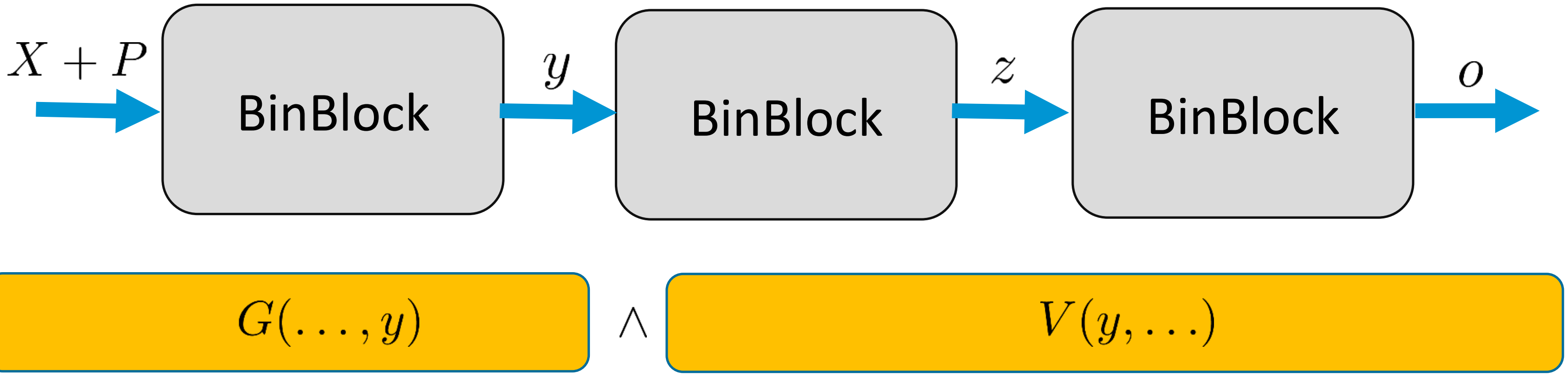

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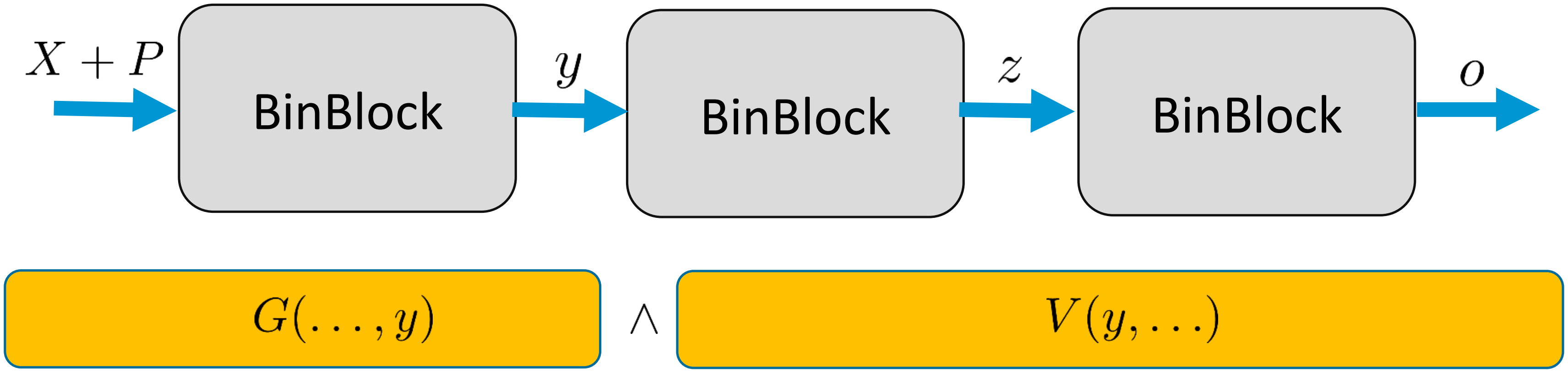
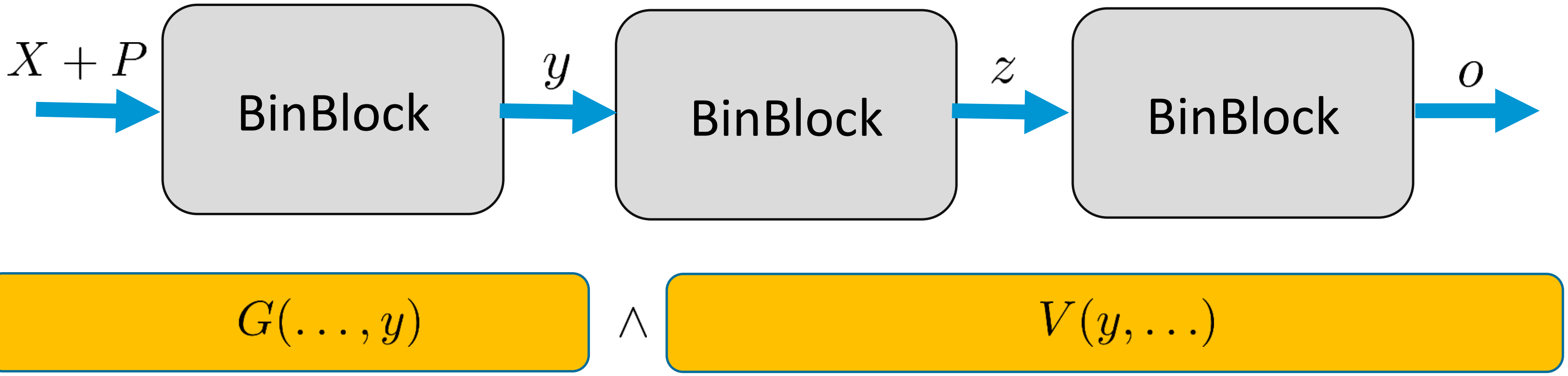


$$G(\ldots, y) \cap V(y, \ldots)$$

Search procedure



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Craig interpolants

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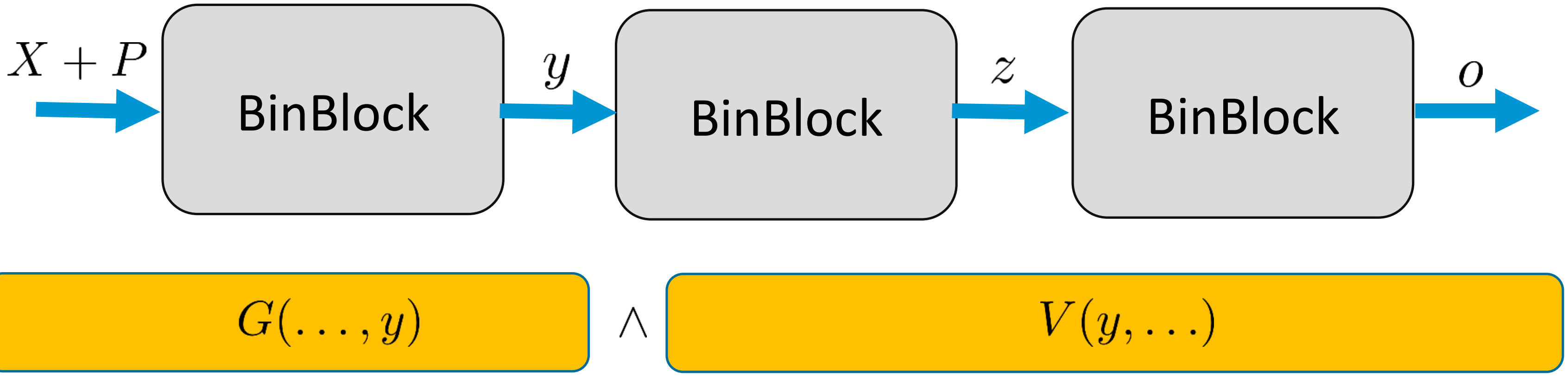

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Search procedure

$G(\ldots, y)$ $\wedge$ $V(y, \ldots)$

$$Solve(G(y)) \xrightarrow{\hat{y}}$$

Search procedure

$G(\ldots, y)$ $\wedge$ $V(y, \ldots)$

$$Solve(G(y)) \xrightarrow{\hat{y}} Solve(V(y), y = \hat{y})$$

✗ $\rightarrow$ $Compute(I(y))$

✓ $\rightarrow$ $returnP$

Search procedure

$G(\ldots, y) \wedge V(y, \ldots)$

$$Solve(G(y)) \xrightarrow{\hat{y}} Solve(V(y), y = \hat{y})$$

$$returnP$$

$$G(y) = G(y) \wedge \neg I(y) \xleftarrow{I(y)} Compute(I(y))$$

Search procedure

$$G(\dots, y) \quad \land \quad V(y, \dots)$$

$$\begin{tikzcd}
Solve(G(y)) \arrow[r, "\hat{y}"] & Solve(V(y), y = \hat{y}) \arrow[d] \arrow[r] & returnP \\
G(y) = G(y) \land \neg I(y) \arrow[u] & Compute(I(y)) \arrow[l, "I(y)"'] &
\end{tikzcd}$$

Experiments

Experiments

Dataset: MNIST, MNIST-ROT, MNIST-BACK
Network: BNN with FC layers
Problem: Untargeted adversarial examples
Encodings: SAT, ILP, CEG-SAT
+ few simplifications, e.g. un-normalized and binarized inputs

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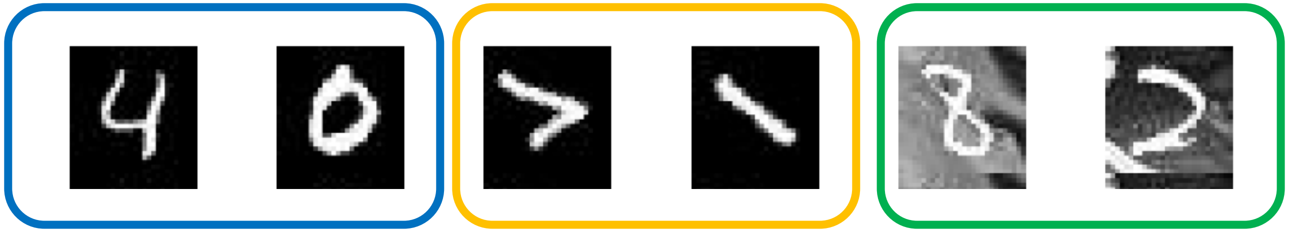

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Vary:

- the value of maximum perturbation ε

Untargeted adversarial examples

Input: ( , 4)

Untargeted adversarial examples

Input: ( , 4)

Goal:

Untargeted adversarial examples

Input: ( , 4)

Goal:

Adversarial X’ =  + P,

Untargeted adversarial examples

Input: ( , 4)

Goal:

Adversarial X’ =  + P,

$$\max(P_1 \ldots P_n) < \varepsilon$$

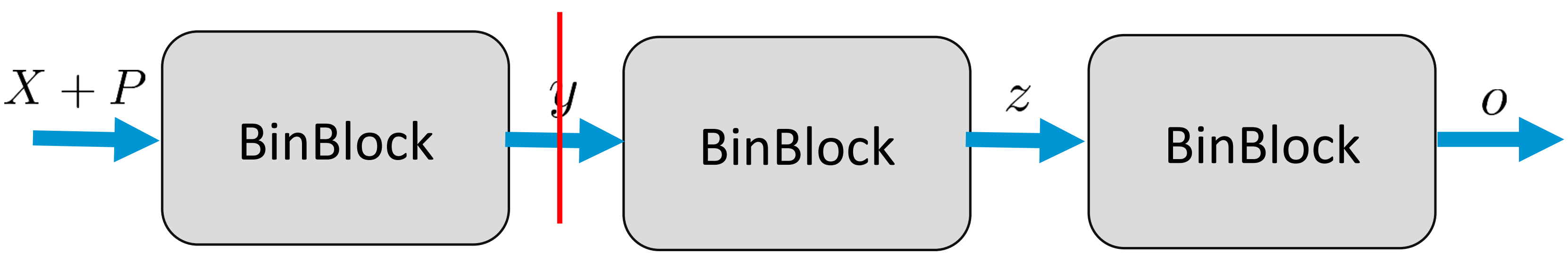
Untargeted adversarial examples

Input: ( , 4)

Goal:

Adversarial X’ =  + P,

max(P_1… P_n) < ε



MNIST

	Solved instances (out of 200)									Certifiably ϵ-robust		
	MNIST			MNIST-rot			MNIST-back-image					
	SAT	ILP	CEG	SAT	ILP	CEG	SAT	ILP	CEG	SAT	ILP	CEG
	#solved (t)	#solved (t)	#solved (t)	#solved (t)	#solved (t)	#solved (t)	#solved (t)	#solved (t)	#solved (t)	#	#	#
$\epsilon = 1$	180 (77.3)	130 (31.5)	171 (34.1)	179 (57.4)	125 (10.9)	197 (13.5)	191 (18.3)	143 (40.8)	191 (12.8)	138	96	138
$\epsilon = 3$	187 (77.6)	148 (29.0)	181 (35.1)	193 (61.5)	155 (9.3)	198 (13.7)	107 (43.8)	67 (52.7)	119 (44.6)	20	5	21
$\epsilon = 5$	191 (79.5)	165 (29.1)	188 (36.3)	196(62.7)	170(11.3)	198(13.7)	104 (48.8)	70 (53.8)	116 (47.4)	3	–	4

Table 2: Results on MNIST, MNIST-rot and MNIST-back-image datasets.

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MNIST

	Solved instances (out of 200)									Certifiably ϵ-robust		
	MNIST			MNIST-rot			MNIST-back-image					
	SAT	ILP	CEG	SAT	ILP	CEG	SAT	ILP	CEG	SAT	ILP	CEG
	#solved (t)	#solved (t)	#solved (t)	#solved (t)	#solved (t)	#solved (t)	#solved (t)	#solved (t)	#solved (t)	#	#	#
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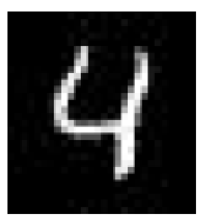

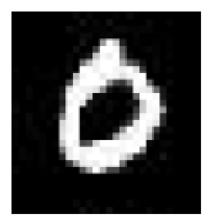

MNIST-ROT

	Solved instances (out of 200)									Certifiably ϵ-robust		
	MNIST			MNIST-rot			MNIST-back-image					
	SAT	ILP	CEG	SAT	ILP	CEG	SAT	ILP	CEG	SAT	ILP	CEG
	#solved (t)	#solved (t)	#solved (t)	#solved (t)	#solved (t)	#solved (t)	#solved (t)	#solved (t)	#solved (t)	#	#	#
$\epsilon = 1$	180 (77.3)	130 (31.5)	171 (34.1)	179 (57.4)	125 (10.9)	197 (13.5)	191 (18.3)	143 (40.8)	191 (12.8)	138	96	138
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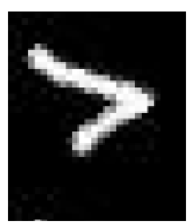

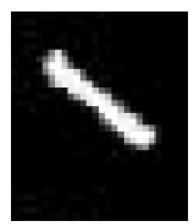

MNIST-BACK

	Solved instances (out of 200)									Certifiably ϵ-robust		
	MNIST			MNIST-rot			MNIST-back-image					
	SAT	ILP	CEG	SAT	ILP	CEG	SAT	ILP	CEG	SAT	ILP	CEG
	#solved (t)	#solved (t)	#solved (t)	#solved (t)	#solved (t)	#solved (t)	#solved (t)	#solved (t)	#solved (t)	#	#	#
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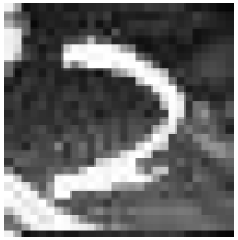

MNIST-BACK

	Solved instances (out of 200)									Certifiably ϵ-robust		
	MNIST			MNIST-rot			MNIST-back-image					
	SAT	ILP	CEG	SAT	ILP	CEG	SAT	ILP	CEG	SAT	ILP	CEG
	#solved (t)	#solved (t)	#solved (t)	#solved (t)	#solved (t)	#solved (t)	#solved (t)	#solved (t)	#solved (t)	#	#	#
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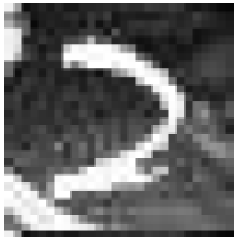

Few observations on properties

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- Most papers focus on robustness property

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- Network equivalence
- Invertibility of the network

Why robustness property?

$$y_1 = 1 \times max(0, 10 \times x_1 + x_2)$$

$$y_2 = 2 \times max(0, -5 \times x_1 + x_2)$$

$$x_1 \in [0.9, 1]$$
$$x_2 \in [-1, 1]$$
$$y_i > 1, i = 1, 2$$

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Few observations on networks

Few observations on networks

- Most papers focus on classification problems
- Generative adversarial networks
- Reinforcement learning

Summary

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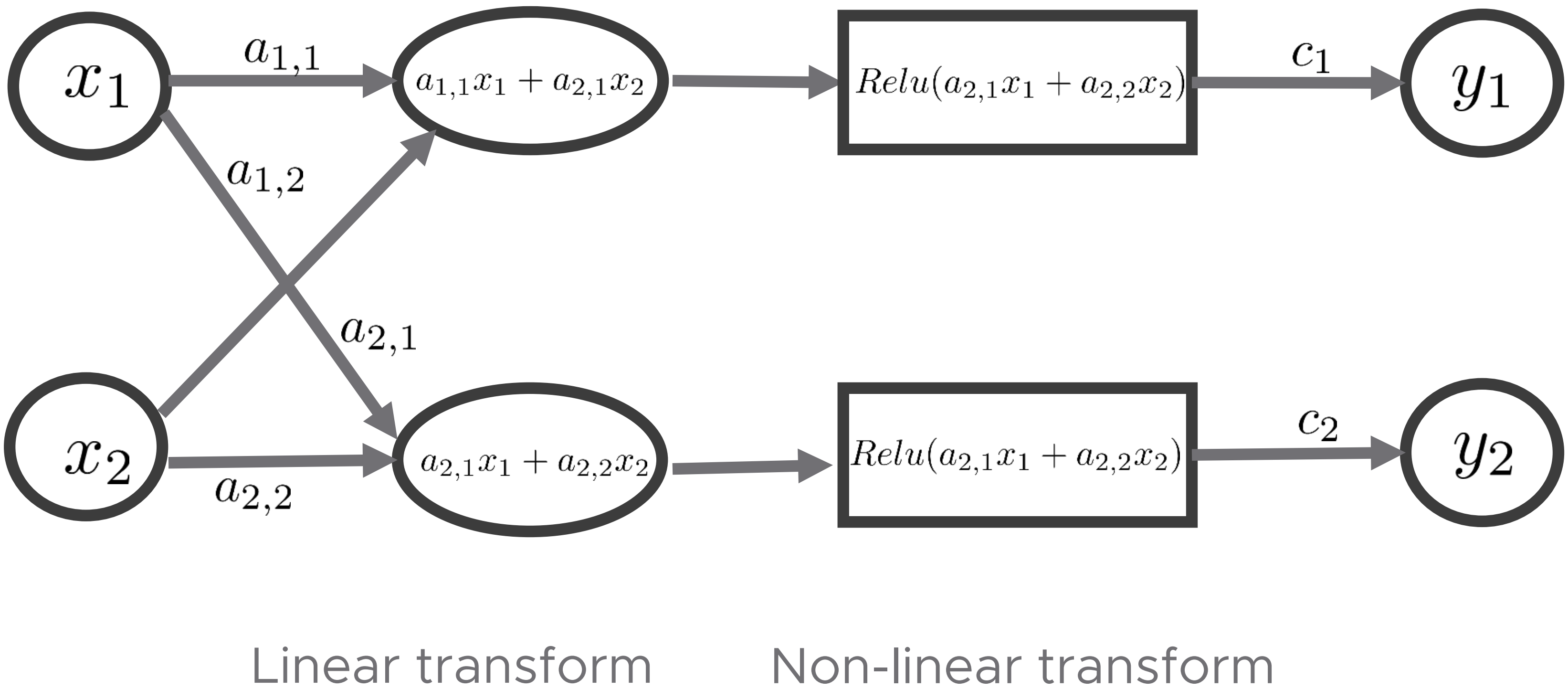
- Scalability remains the main issue
- We need to look beyond robustness

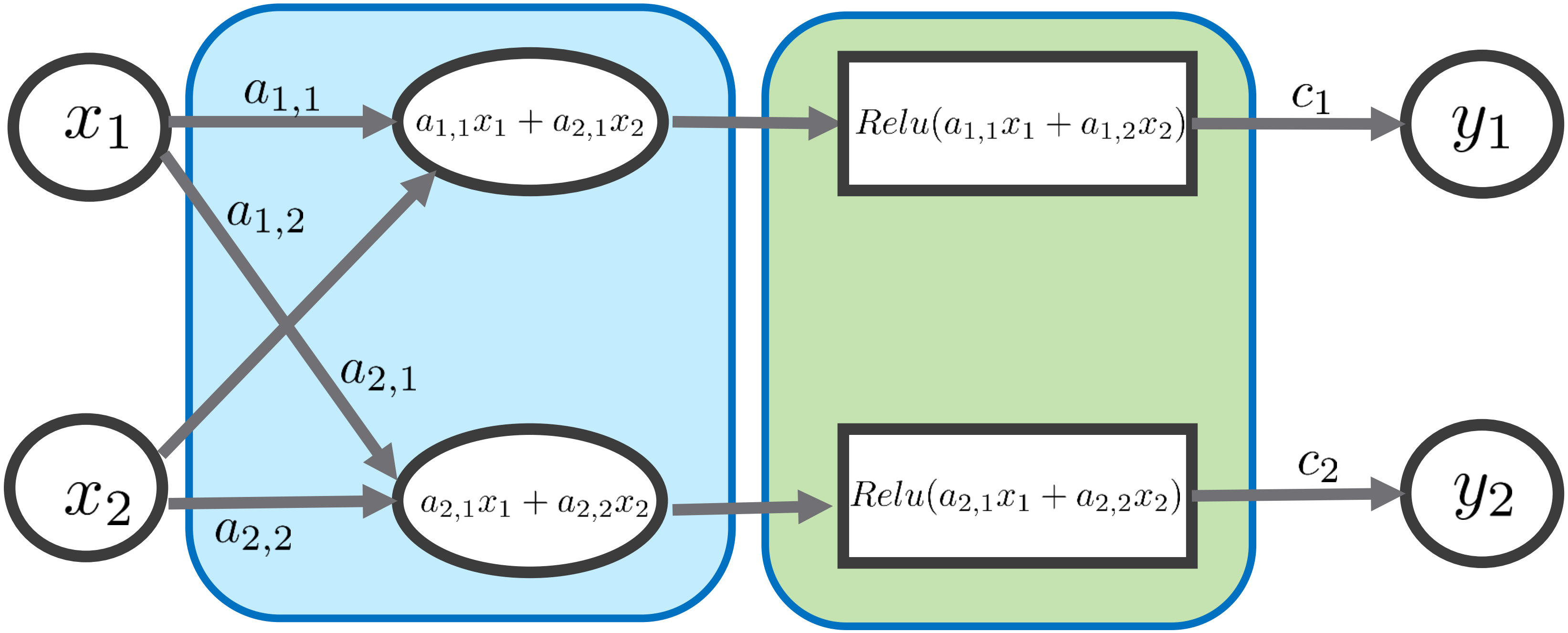
Verification of Neural Networks is an emerging exciting area!

Thanks!

High-level structure



High-level structure



Linear transform Non-linear transform

Network formula

$$y_1 = c_1 Relu(a_{1,1}x_1 + a_{1,2}x_2)$$
$$y_2 = c_2 Relu(a_{2,1}x_1 + a_{2,2}x_2)$$

Decision (robustness) problem

$$y_1 = c_1 Relu(a_{1,1}x_1 + a_{1,2}x_2)$$
$$y_2 = c_2 Relu(a_{2,1}x_1 + a_{2,2}x_2)$$

$$x_i \in [w_1, w_2], i = 1, 2$$
$$y_i > q, i = 1, 2$$

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