MAX-SAT for Temporal Logics

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Banff International Research Station for Mathematical Innovation and Discovery

Casa Mathematica Theoretical Foundations of SAT Solving Workshop
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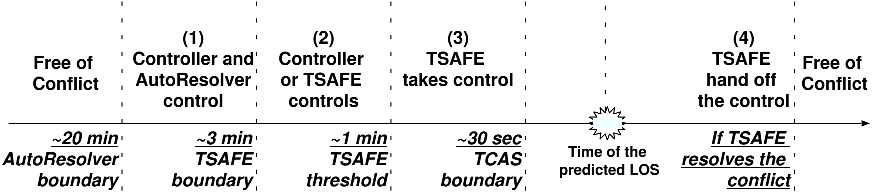
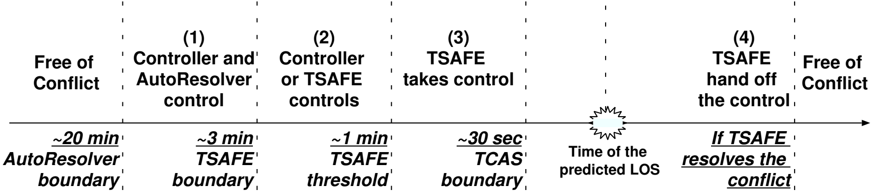
AAC Operational Concept[1]



[1] H Erzberger, K Heere. "Algorithm and operational concept for resolving short-range conflicts." Proc. IMechE G J. Aerosp. Eng. 224 (2) (2010) 225–243.

AAC Operational Concept[2]

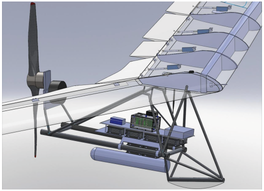


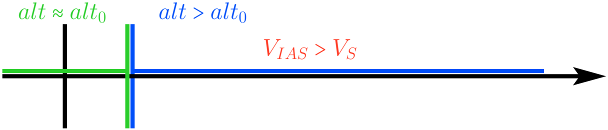
Formal verification triggered system design changes[1]

[1] Y. Zhao and K.Y. Rozier. "Formal Specification and Verification of a Coordination Protocol for an Automated Air Traffic Control System." SCP Journal, vol-96, no-3, pg 337-353, 2014.

[2] H Erzberger, K Heere. "Algorithm and operational concept for resolving short-range conflicts." Proc. IMechE G J. Aerosp. Eng. 224 (2) (2010) 225–243.

Operational Concept for the Swift UAS





Whenever the Swift UAS is in the air, its indicated airspeed (V_{IAS}) must be greater than its stall speed V_S. The UAS is considered to be air-bound when its altitude alt is larger than that of the runway alt_0.[3]

[3] T. Reinbacher, K.Y. Rozier, J. Schumann. "Temporal-Logic Based Runtime Observer Pairs for System Health Management of Real-Time Systems." TACAS 2014.

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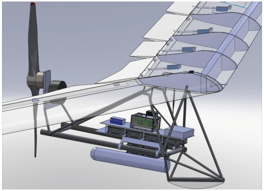

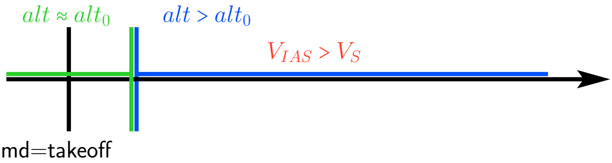

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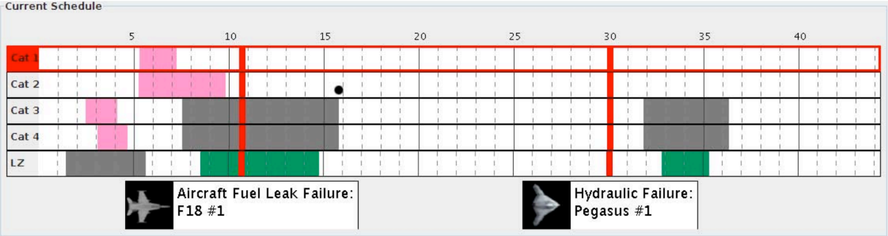
$$\text{ALWAYS}((alt > alt_0) \rightarrow (V_{IAS} > V_S))$$

[3] T. Reinbacher, K.Y. Rozier, J. Schumann. "Temporal-Logic Based Runtime Observer Pairs for System Health Management of Real-Time Systems." TACAS 2014.

There is a Pattern Here. . .



Air Force aircraft carrier deck scheduling: deck resource timeline displaying three failures[4]

[4] J.C.Ryan, M.L.Cummings, N.Roy, A Banerjee, A.Schulte. "Designing an Interactive Local and Global Decision Support System for Aircraft Carrier Deck Scheduling." AIAA Infotech, 2011.

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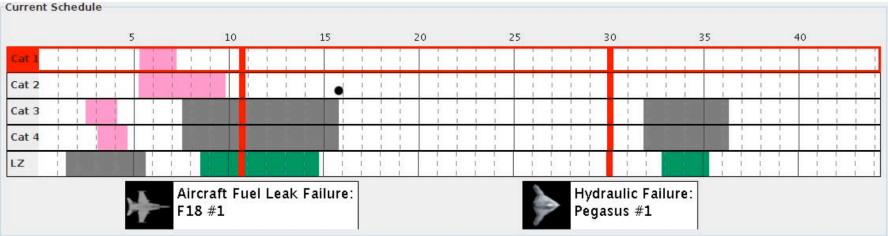


Air Force aircraft carrier deck scheduling: deck resource timeline displaying three failures[4]

Aerospace Operational Concepts Are Often Specified With Timelines

[4] J.C.Ryan, M.L.Cummings, N.Roy, A Banerjee, A.Schulte. "Designing an Interactive Local and Global Decision Support System for Aircraft Carrier Deck Scheduling." AIAA Infotech, 2011.

A Natural Logic for Operational Timelines: Linear Temporal Logic

Linear Temporal Logic (LTL) formulas reason about linear timelines:

- finite set of atomic propositions {p q}
- Boolean connectives: $\neg$, $\wedge$, $\vee$, and $\rightarrow$
- temporal connectives:

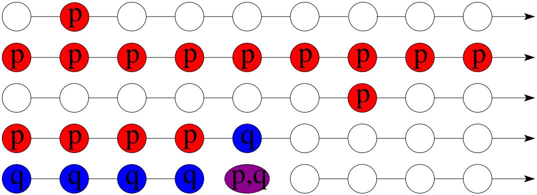
$\mathcal{X}p$ NEXT TIME

$\Box p$ ALWAYS

$\Diamond p$ EVENTUALLY

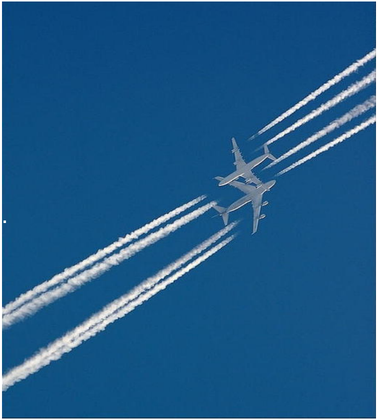
$p\mathcal{U}q$ UNTIL

$p\mathcal{R}q$ RELEASE



Formal Verification Via Model Checking

1. Describe system requirements in a formal specification, φ.
2. Create a system model with formal semantics, M.
3. Check that M satisfies φ.



Model checking finds disagreements between the system model and the formal specification.

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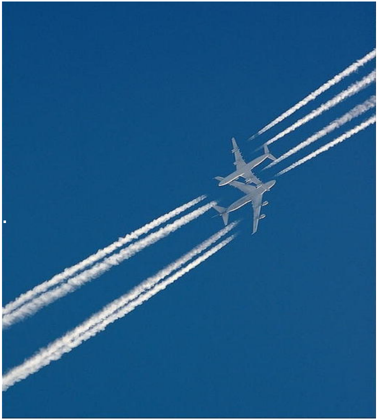

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Formal Verification Via LTL Model Checking

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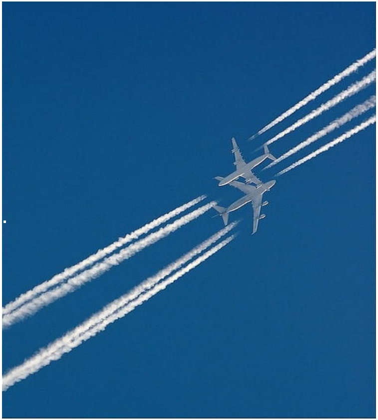
   - Graph-search-based
   - BDD-based
   - BMC-based
   - IC3-based



Model checking finds disagreements between the system model and the formal specification.

Formal Verification Via LTL Model Checking

1. Describe system requirements in a formal LTL specification, φ.
   - Only works if the formula is correct!
2. Create a system model with formal semantics, M.
3. Check that M satisfies φ.
   - Graph-search-based
   - BDD-based
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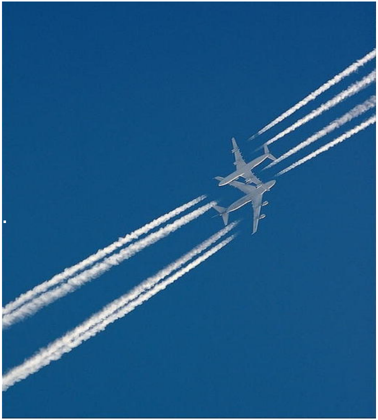

Model checking finds disagreements between the system model and the formal specification.

Property Assurance: We Propose Satisfiability Checking

$M \models \varphi$ may not mean the system has the intended behavior

Recall that a property φ is *valid* iff $\neg\varphi$ is *unsatisfiable*.

If $\neg\varphi$ is not satisfiable, then

- There can never be a counterexample.
- Model checkers will always return "success."
- φ is probably wrong.

Property Assurance: We Propose Satisfiability Checking

$M \models \varphi$ may not mean the system has the intended behavior

$M \not\models \varphi$ may not mean the system does not have the intended behavior

Recall that a property φ is *valid* iff $\neg\varphi$ is *unsatisfiable*.

If $\neg\varphi$ is not satisfiable, then

- There can never be a counterexample.
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If φ is not satisfiable, then

- There is always a counterexample.
- Model checkers will always return "failure."
- φ is probably wrong.

Specification Debugging: LTL Satisfiability Checking

For each property φ and $\neg\varphi$ we should check for satisfiability.

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LTL-to-Automaton Complexity

- LTL property f of size $|\varphi|$
- System model M of size $|M|$
- LTL satisfiability checking takes time $|M| \cdot 2^{\mathcal{O}(|\varphi|)}$.

LTL Satisfiability Checking is PSPACE-Complete!

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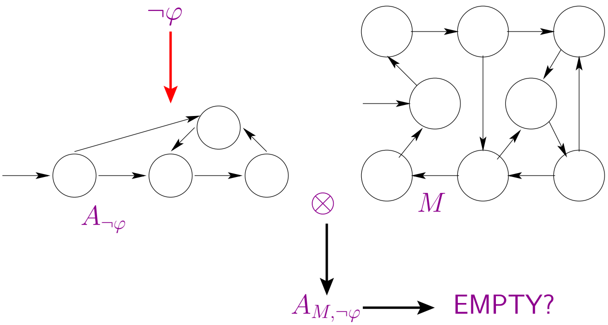
LTL Satisfiability Checking is PSPACE-Complete!

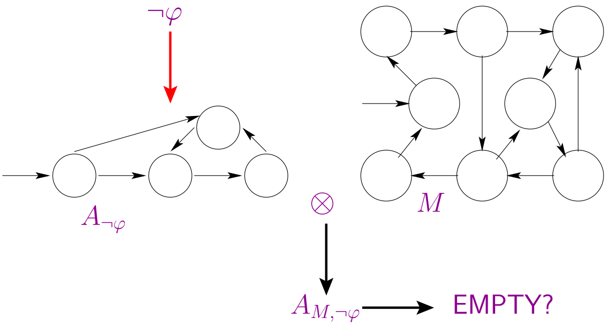
We have to be smart about encoding the problem!

Ex: Automata-Theoretic Approach to Model Checking: One of the PSPACE-Complete Algorithms for LTL-SAT

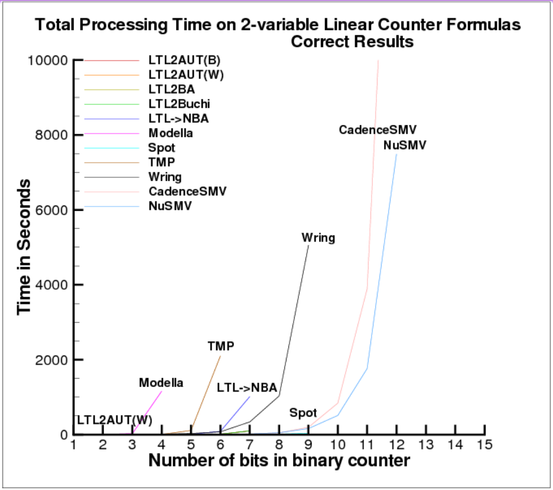


Ex: Automata-Theoretic Approach to Model Checking: One of the PSPACE-Complete Algorithms for LTL-SAT

Requires efficient LTL-to-automaton translation.



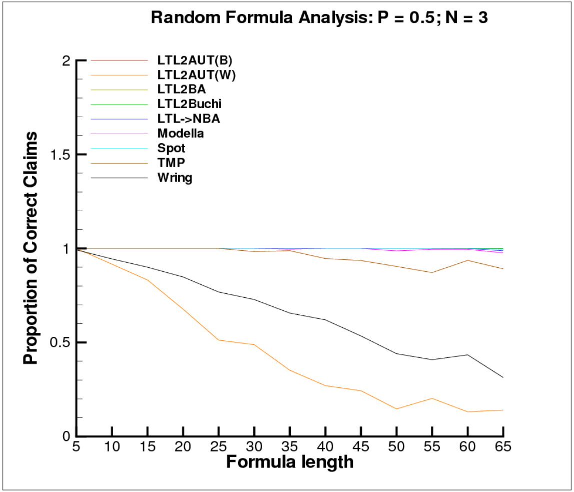
LTL Satisfiability is Hard to Scale[5]



Many tools cannot check 8-bit binary counter formulas

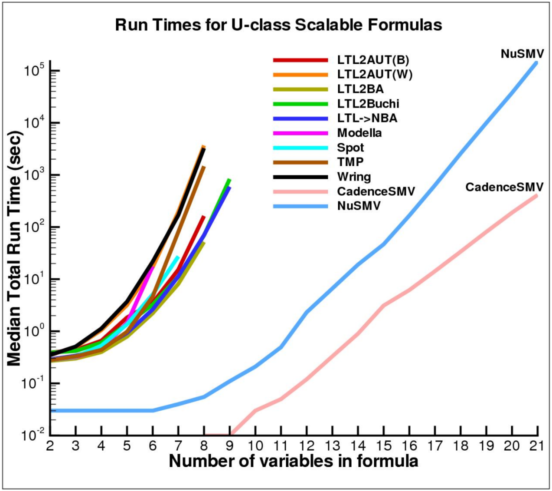
[5] K.Y.Rozier, M.Y.Vardi. "LTL Satisfiability Checking." STTT Journal, pg. 123–137, 2010.

LTL Satisfiability is Hard to Code Correctly[6]



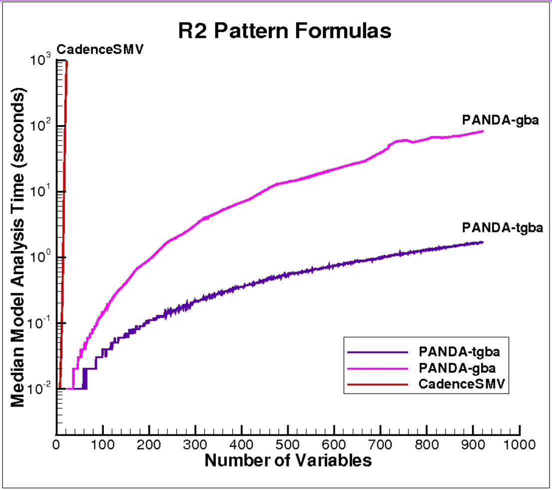
6 K.Y.Rozier, M.Y.Vardi. "LTL Satisfiability Checking." STTT Journal, pg. 123–137, 2010.

Implementation is Hugely Influential[7]



[7] K.Y.Rozier, M.Y.Vardi. "LTL Satisfiability Checking." STTT Journal, pg. 123–137, 2010.

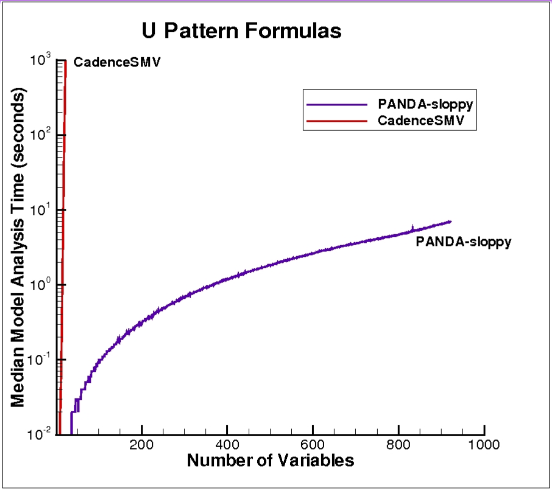
Better Encoding Can Lead to Exponential Improvement! [8]



$$R_2(n) = (..(p_1 \mathcal{R}\ p_2)\ \mathcal{R}\ \ldots)\ \mathcal{R}\ p_n.$$

[8] K.Y. Rozier and M.Y. Vardi. "A Multi-Encoding Approach for LTL Symbolic Satisfiability Checking." FM'11.

Even for Very Hard Formulas! [9]



$$U(n) = (\ldots (p_1 \,\mathcal{U}\, p_2) \,\mathcal{U}\, \ldots) \,\mathcal{U}\, p_n.$$

9 K.Y. Rozier and M.Y. Vardi. "A Multi-Encoding Approach for LTL Symbolic Satisfiability Checking." FM'11.

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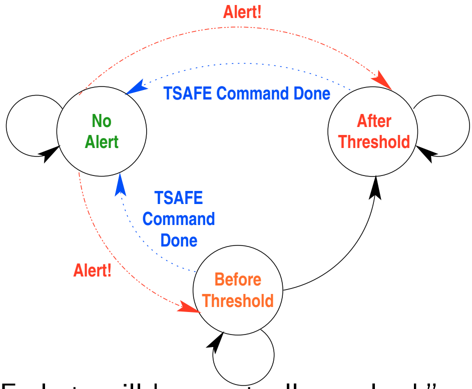
Is this actually required in real life?

LTL Satisfiability Checking Found A Specification Bug

LTL safety requirement φ_0
LTL fairness constraint φ_1

ALWAYS EVENTUALLY $\varphi_1 \rightarrow \varphi_0$

An overstrict φ_1 can effectively cause φ_0 to be valid!



Example:

Safety Requirement: "All TSAFE alerts will be eventually resolved."
Fairness Constraint: Progress between TSAFE alerts

Wrong: `FAIRNESS (TSAFE_Alert = Non);`
Right: `FAIRNESS (TSAFE_Alert != AT);`

Problem Overview

- **Specification Debugging:** If the conjunction of all properties is not satisfiable, where is the problem?

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- **Specification Debugging:** If the conjunction of all properties is not satisfiable, where is the problem?
- **Requirements Engineering:** If the conjunction of all requirements is UNSAT, how many can I have? What's the closest you can give me to what I want?
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These are all MAX-SAT!

Problem Overview

- **Specification Debugging:** If the conjunction of all properties is not satisfiable, where is the problem?
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- **XAI:** "I could not solve this because . . . This (smallest subset of) requirement(s) is not compatible with the rest of the set"

These are all MAX-SAT!

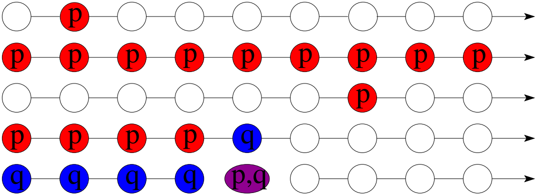
But SAT for LTL is already hard!

Linear Temporal Logic: Reasons Over Infinite Traces

Linear Temporal Logic (LTL) formulas reason about linear timelines:

- finite set of atomic propositions {p q}
- Boolean connectives: $\neg$, $\wedge$, $\vee$, and $\rightarrow$
- temporal connectives:

$\mathcal{X}p$	NEXT TIME
$\Box p$	ALWAYS
$\Diamond p$	EVENTUALLY
$p\mathcal{U}q$	UNTIL
$p\mathcal{R}q$	RELEASE



LTLf: Linear Temporal Logic on Finite Traces[10]

LTLf formulas reason about *finite* linear timelines *terminating at Tail*:

- finite set of atomic propositions {p q}
- Boolean connectives: $\neg$, $\wedge$, $\vee$, and $\rightarrow$
- temporal connectives:

$\mathcal{X} p$	NEXT TIME
$\Box p$	ALWAYS
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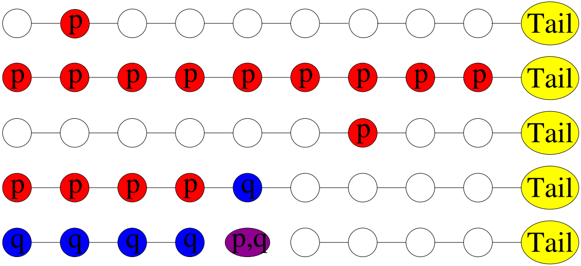
$p \mathcal{U} q$	UNTIL
$p \mathcal{R} q$	RELEASE



[10] G. De Giacomo, M.Y. Vardi. "Linear temporal logic and linear dynamic logic on finite traces." IJCAI 2013.

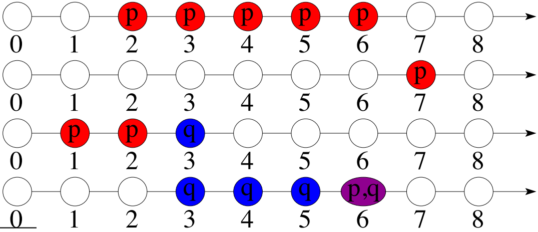
Mission-Bounded Linear Temporal Logic [11]

Mission-Time Temporal Logic (MLTL) reasons about *integer-bounded* timelines:

- finite set of atomic propositions {p q}
- Boolean connectives: ¬, ∧, ∨, and →
- temporal connectives *with time bounds*:

Symbol	Operator
$\Box_{[2,6]} p$	ALWAYS$_{[2,6]}$
$\Diamond_{[0,7]} p$	EVENTUALLY$_{[0,7]}$
$p \mathcal{U}_{[1,5]} q$	UNTIL$_{[1,5]}$
$p \mathcal{R}_{[3,8]} q$	RELEASE$_{[3,8]}$

Timeline



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MLTL Runtime Benchmark Generation: An Easier Problem[12]

Time:										
	0	1	2	3	4	5	6	7	8	9

MLTL formula φ evaluated over system trace π:

$$\forall i : 0 \leq i \leq \text{MissionTime } \pi, i \models \varphi.$$

An MLTL Runtime Benchmark is a 3-tuple:

- Input stream, or computation, π
- MLTL formula, φ, over n propositional variables
- Oracle $\mathcal{O}$, of $\langle time, verdict \rangle$

[12] J.Walling and K.Y.Rozier. "Generating System-Agnostic Runtime Verification Benchmarks from MLTL Formulas via SAT." Under Submission, 2018.

MLTL Runtime Benchmark Generation: An Example[13]

Time:	a	¬a	¬a	a	a	a	a	a	a	a
	0	1	2	3	4	5	6	7	8	9

MLTL formula φ evaluated over system trace π:

$$\forall i : 0 \leq i \leq \text{MissionTime}\ \pi, i \vDash \varphi.$$

MLTL Runtime Benchmark Example:

- $\pi = a, \neg a, \neg a, a, a, a, a, a, a, a$
- $\varphi = \text{ALWAYS}_{[5]}(a)$
- $\mathcal{O} = \langle 0, F\rangle, \langle 1, F\rangle, \langle 2, F\rangle, \langle 3, T\rangle, \langle 4, T\rangle, \ldots$

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Time:	a	¬a	¬a	a	a	a	a	a	a	a
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MLTL Runtime Benchmark Example:

- $\pi = a, \neg a, \neg a, a, a, a, a, a, a, a$
- $\varphi = \textsc{Always}_{[5]}(a)$
- $\mathcal{O} = \langle 0, F \rangle, \langle 1, F \rangle, \langle 2, F \rangle, \langle 3, T \rangle, \langle 4, T \rangle, \ldots$

A SAT Encoding:

Assign a_i to a at time i.

Iteratively conjunct the satisfying assignment from i to the formula for $i + 1$. Record UNSAT as $\mathcal{O} = \langle i, F \rangle$; otherwise $\langle i, T \rangle$

13 J.Walling and K.Y.Rozier. "Generating System-Agnostic Runtime Verification Benchmarks from MLTL Formulas via SAT." Under Submission, 2018.

Open Questions

- How can we design (more) efficient MAX-SAT for MLTL?
- Can we design a MAX-SAT solver for LTL? For LTLf?
- Can we develop heuristics specific to MAX-SAT for temporal logics?
- Can we take advantage of the intuitions inherent to this domain?