Multi-Stage Stochastic Optimization for Clean Energy Transition

◇

Mixing Time Blocks and Price/Resource Decomposition Methods Application to Long Term Battery Management

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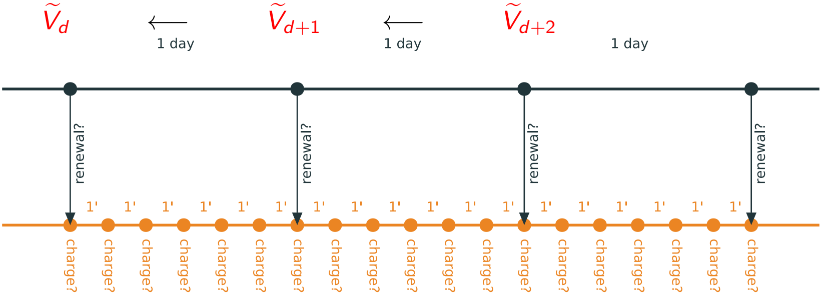
Battery management involves short time control and long term renewal, hence two time scales

- When to renew a battery (long term decision)?
- How to optimally control the battery (short time decision)?
  $\longrightarrow$ impact on aging?

$$\underbrace{10,512,000}_{\text{stages}} = \underbrace{7300}_{\text{days}} \times \underbrace{1440}_{\text{minutes}}$$

We will decompose the battery management problem according to control/renewal scales



- Under what assumptions is there a Bellman Equation day by day?
- How to compute the one day Bellman operator,
  which involves an optimization problem at minute time scale?

Lecture outline

Background on two time scales decomposition

Mixing time blocks and price/resource decompositions

- Two time scales battery management problem statement
- Intraday time block and resource decomposition algorithm
- Intraday time block and price decomposition algorithm
- Producing minute scale policies

Numerical results

Conclusion

Outline of the presentation

We introduce notations for two time scales

Time is described by to indices $(d, m) \in \mathbb{T}$

$$\mathbb{T} = \{0, \dots, D\} \times \{0, \dots, M\} \cup \{(D + 1, 0)\}$$

1. Battery charge/discharge, decision every minute $m \in \{0, \dots, M\}$ of every day $d \in \{0, \dots, D\}$
   $\rightarrow$ Minutes in day d are $(d, 0)$, $(d, 1)$,..., (d, M)
2. Renewal of the battery, decision every day $d \in \{0, \dots, D + 1\}$
   $\rightarrow$ Start of days are $(0, 0)$,..., $(d + 1, 0)$,..., $(D + 1, 0)$
3. Compatibility between days: $\big((d, M + 1) = (d + 1, 0)\big)$

$\mathbb{T}$ is a totally ordered set when equipped with the *lexicographical order*

$$(d, m) < (d', m') \iff (d < d') \vee \big(d = d' \wedge m < m'\big)$$

Bellman Operators and Dynamic Programming

We introduce Bellman functions V_t for $t \in \mathbb{T}$,
solution of the Bellman or dynamic programming equation with history

- Bellman operator at time t: $\varphi \in \mathbb{L}^0_+(\mathbb{H}_{t+1}, \mathcal{H}_{t+1})$ and $h_t \in \mathbb{H}_t$,

$$\big(\mathcal{B}_{t+1:t}\varphi\big)(h_t) = \inf_{u_t \in \mathbb{U}_t} \int_{\mathbb{W}_{t+1}} \varphi(h_t, u_t, w_{t+1}) \rho_{t:t+1}(h_t, dw_{t+1})$$

- Bellman equations

$$\begin{aligned} V_T &= j \ , \\ V_t &= \mathcal{B}_{t+1:t} V_{t+1} \ , \quad \text{for } \ t \in \mathbb{T} \end{aligned}$$

$\rightarrow$ State reduction at times $(d, 0)$ for $d \in \{0, \ldots, D+1\}$

Graphical representation of state reduction

- The triplet $(\theta_r, \theta_t, f_{(d,0):(d+1,0)})$ is a state reduction across $((d,0):(d+1,0))$ if the following diagram is commutative

$$
\begin{array}{ccccc}
\mathbb{H}_{(d,0)} & \times & \mathbb{H}_{(d,1):(d,M+1)} & \xrightarrow{\;I_d\;} & \mathbb{H}_{(d+1,0)} \\
\Big\downarrow \theta_{(d,0)} & & \Big\downarrow I_d & & \Big\downarrow \theta_{(d+1,0)} \\
\mathbb{X}_{(d,0)} & \times & \mathbb{H}_{(d,1):(d,M+1)} & \xrightarrow{\;f_{(d,0):(d+1,0)}\;} & \mathbb{X}_{(d+1,0)}
\end{array}
$$

- Compatibility with kernels $p \in \{1, \ldots, M\}$

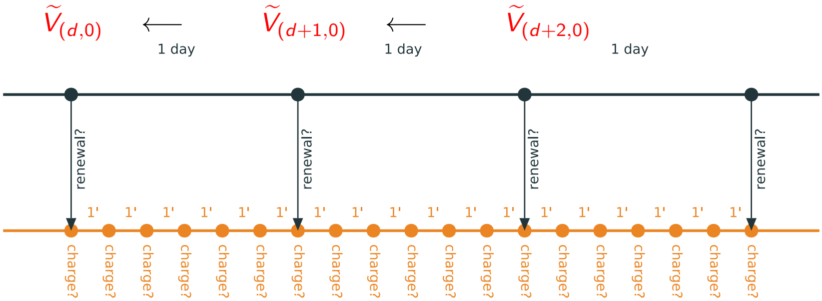
$$
\begin{array}{ccccc}
\mathbb{H}_{(d,0)} & \times & \mathbb{H}_{(d,1):(d,p-1)} & \xrightarrow{\;\rho_{(d,p-1):(d,p)}\;} & \Delta(\mathbb{W}_{(d,p)}) \\
\Big\downarrow \theta_{(d,0)} & & \Big\downarrow I_d & \nearrow \widetilde{\rho}_{(d,p-1):(d,p)} & \\
\mathbb{X}_{(d,0)} & \times & \mathbb{H}_{(d,1):(d,p-1)} & &
\end{array}
$$

Application of Time Blocks Dynamic Programming

We will now present an application
to a two time-scales optimization problem

- optimize long-term investment decisions (slow time-scale)
  — here the renewal of batteries in an energy system
- but the optimal long-term decisions highly depend
  on short-term operating decisions (fast time-scale)
  — here the way the battery is operated in real-time.

We will decompose the scales (day and minutes)



We propose numerical schemes that provide upper and lower bounds on the family of reduced value functions $\left\{ \widetilde{V}_{(d,0)} \right\}_{d=0,\dots,D}$

- Assuming between days independence assumption enables time scale decomposition
- Within a day, the fast time scale uncertainties can be dependent, and we will resort to other decomposition principles: resource/price decomposition techniques to solve day by day problems.

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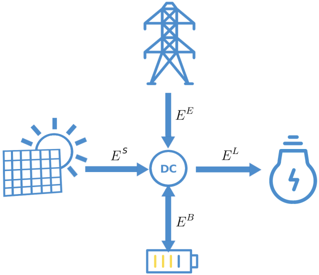
Physical model: a home with load, solar panels and storage



- Two time scales uncertainties
  - $\boldsymbol{E}^L_{d,m}$: Uncertain demand
  - $\boldsymbol{E}^S_{d,m}$: Uncertain solar electricity
  - $\boldsymbol{P}^b_d$: Uncertain storage cost
- Two time scales controls
  - $\boldsymbol{E}^E_{d,m}$: National grid import
  - $\boldsymbol{E}^B_{d,m}$: Storage charge/discharge
  - $\boldsymbol{R}_d$: Storage renewal
- Two time scales states
  - $\boldsymbol{B}_{d,m}$: Storage state of charge
  - $\boldsymbol{H}_{d,m}$: Storage health
  - $\boldsymbol{C}_d$: Storage capacity
- Balance equation:

  $$\boldsymbol{E}^E_{d,m} + \boldsymbol{E}^S_{d,m} = \boldsymbol{E}^B_{d,m} + \boldsymbol{E}^L_{d,m}$$
- Battery dynamic:

  $$\boldsymbol{B}_{d,m+1} = \boldsymbol{B}_{d,m} - \frac{1}{\rho_d}\boldsymbol{E}^{B-}_{d,m} + \frac{1}{\rho_d}\rho_c\boldsymbol{E}^{B+}_{d,m}$$

Two time scales dynamics: aging and renewal model

- At the end of every day d, we can buy a new battery at cost $\boldsymbol{P}^b_d \times \boldsymbol{R}_d$

$$\text{Storage capacity: } \boldsymbol{C}_{d+1} = \begin{cases} \boldsymbol{R}_d \ , & \text{if } \boldsymbol{R}_d > 0 \\ \boldsymbol{C}_d \ , & \text{otherwise} \end{cases}$$

- A new battery can make a maximum number of cycles $N_c(\boldsymbol{R}_d)$:

$$\text{Storage health: } \boldsymbol{H}_{d+1,0} = \begin{cases} 2 \times N_c(\boldsymbol{R}_d) \times \boldsymbol{R}_d \ , & \text{if } \boldsymbol{R}_d > 0 \\ \boldsymbol{H}_{d,M} \ , & \text{otherwise} \end{cases}$$

$\boldsymbol{H}_{d,m}$ is the amount of exchangeable energy day d, minute m

$$\boldsymbol{H}_{d,m+1} = \boldsymbol{H}_{d,m} - \frac{1}{\rho_d} \boldsymbol{E}^{B-}_{d,m} - \rho_c \boldsymbol{E}^{B+}_{d,m}$$

- A new battery is empty

$$\text{Storage state of charge: } \boldsymbol{B}_{d+1,0} = \begin{cases} \underline{B} \times \boldsymbol{R}_d \ , & \text{if } \boldsymbol{R}_d > 0 \\ \boldsymbol{B}_{d,M} \ , & \text{otherwise} \end{cases}$$

We build a non standard SOC problem

- Objective to be minimized

$$\mathbb{E}\left(\sum_{d=0}^{D} \Big(\underbrace{\boldsymbol{P}_d^b \times \boldsymbol{R}_d}_{\text{renewal}} + \sum_{m=0}^{M-1} \underbrace{p_{d,m}^e}_{\textit{price}} \times \big(\underbrace{\boldsymbol{E}_{d,m}^B + \boldsymbol{E}_{d,m+1}^L - \boldsymbol{E}_{d,m+1}^S}_{\text{national grid energy consumption}} \big) \Big) \right)$$

- Controls

$$\boldsymbol{U}_d = (\boldsymbol{E}_{d,0}^B \dots, \boldsymbol{E}_{d,m}^B, \dots, \boldsymbol{E}_{d,M-1}^B, \boldsymbol{R}_d)$$

- Uncertainties

$$\boldsymbol{W}_d = \left(\begin{pmatrix} \boldsymbol{E}_{d,1}^S \\ \boldsymbol{E}_{d,1}^L \end{pmatrix}, \dots, \begin{pmatrix} \boldsymbol{E}_{d,m}^S \\ \boldsymbol{E}_{d,m}^L \end{pmatrix}, \dots, \begin{pmatrix} \boldsymbol{E}_{d,M-1}^S \\ \boldsymbol{E}_{d,M-1}^L \end{pmatrix}, \begin{pmatrix} \boldsymbol{E}_{d,M}^S \\ \boldsymbol{E}_{d,M}^L \\ \boldsymbol{P}_d^b \end{pmatrix} \right)$$

- States and dynamics

$$\boldsymbol{X}_d = \begin{pmatrix} \boldsymbol{C}_d \\ \boldsymbol{B}_{d,0} \\ \boldsymbol{H}_{d,0} \end{pmatrix} \text{ and } \boldsymbol{X}_{d+1} = f_d(\boldsymbol{X}_d, \boldsymbol{U}_d, \boldsymbol{W}_d)$$

Two time scales stochastic optimal control problem

$$
\begin{aligned}
\mathcal{P}: \quad \widetilde{V}_0 = \min_{\boldsymbol{X}_{0:D+1},\ \boldsymbol{U}_{0:D}} \ & \mathbb{E}\left(\sum_{d=0}^{D} L_d(\boldsymbol{X}_d, \boldsymbol{U}_d, \boldsymbol{W}_d) + K(\boldsymbol{X}_{D+1}) \right) , \\
\text{s.t } & \boldsymbol{X}_{d+1} = f_d(\boldsymbol{X}_d, \boldsymbol{U}_d, \boldsymbol{W}_d) , \\
& \boldsymbol{U}_d = (\boldsymbol{U}_{d,0}, \dots, \boldsymbol{U}_{d,m}, \dots, \boldsymbol{U}_{d,M}) \\
& \boldsymbol{W}_d = (\boldsymbol{W}_{d,0}, \dots, \boldsymbol{W}_{d,m}, \dots, \boldsymbol{W}_{d,M}) \\
& \sigma(\boldsymbol{U}_{d,m}) \subset \sigma\big(\boldsymbol{W}_{d',m'}; (d', m') \leq (d, m)\big)
\end{aligned}
$$

Two time scales because of the nonanticipativity constraint
written every minute!

- Intraday time stages: $M = 24 * 60 = 1440$ minutes
- Daily time stages: $D = 365 * 20 = 7300$ days
- $D \times M = 10,512,000$ stages!

We write a Bellman equation with daily time blocks

Daily Independence Assumption

$\{\boldsymbol{W}_d\}_{d=0,\dots,D}$ is a sequence of independent random variables

We set $\widetilde{V}_{(D+1,0)} = K$ and

$$\widetilde{V}_{(d,0)}(x) = \min_{\boldsymbol{X}_{d+1},\boldsymbol{U}_d} \mathbb{E}\left[L_d(x, \boldsymbol{U}_d, \boldsymbol{W}_d) + \widetilde{V}_{(d+1,0)}(\boldsymbol{X}_{d+1})\right]$$

$$\text{s.t } \boldsymbol{X}_{d+1} = f_d(x, \boldsymbol{U}_d, \boldsymbol{W}_d)$$

$$\sigma(\boldsymbol{U}_{d,m}) \subset \sigma(\boldsymbol{W}_{d,0:m})$$

where $\boldsymbol{W}_{d,0:m} = (\boldsymbol{W}_{d,0}, \dots, \boldsymbol{W}_{d,m})$
is possibly made of non independent random variables within a day

Proposition

Under Daily Independence Assumption, $\widetilde{V}_{(0,0)}$ is the value of problem $\mathcal{P}$

Independence assumption at the day scale
is the key to enable stochastic kernels reduction (commutative diagram)

We introduce price/resource daily decompositions

We present two efficient daily decomposition algorithms to compute upper and lower bounds of the daily value functions $\left\{ \widetilde{V}_{(d,0)} \right\}_{d=0,\dots,D}$

1. resource (targets) decomposition gives an upper bound

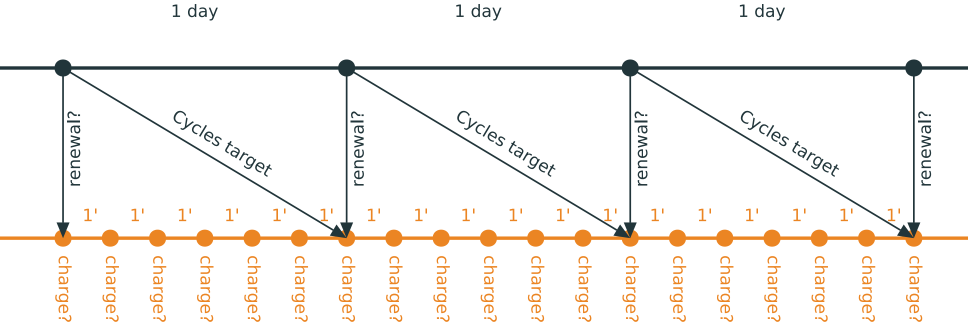
$$\underbrace{\boldsymbol{X}_{d+1} = \boldsymbol{X}}_{\text{resource decomposition}} \quad , \quad f_d(x, \boldsymbol{U}_d, \boldsymbol{W}_d) = \boldsymbol{X}$$

2. price (weights) decomposition gives a lower bound

$$\underbrace{\left\langle \lambda_d \, , \boldsymbol{X}_{d+1} - f_d(x, \boldsymbol{U}_d, \boldsymbol{W}_d) \right\rangle}_{\text{price decomposition}}$$

Outline of the presentation

Decomposing by imposing targets



Stochastic target decomposition

We introduce the stochastic target intraday problem

$$\phi_{(d,=)}\big(x_d, \boldsymbol{X}_{d+1}\big) = \min_{\boldsymbol{U}_d} \quad \mathbb{E}\Big[L_d(x, \boldsymbol{U}_d, \boldsymbol{W}_d)\Big]$$

$$\text{s.t } f_d(x, \boldsymbol{U}_d, \boldsymbol{W}_d) = \boldsymbol{X}_{d+1}$$

$$\sigma(\boldsymbol{U}_{d,m}) \subset \sigma(\boldsymbol{W}_{d,0:m})$$

Proposition

Under Daily Independence Assumption, $\widetilde{V}_d := \widetilde{V}_{(d,0)}$ satisfies

$$\widetilde{V}_d(x) = \min_{\boldsymbol{X} \in L^0(\Omega,\mathcal{F},\mathbb{P};\mathbb{X}_{d+1})} \Big(\phi_{(d,=)}\big(x, \boldsymbol{X}\big) + \mathbb{E}\big[\widetilde{V}_{d+1}(\boldsymbol{X})\big]\Big)$$

$$\text{s.t } \sigma(\boldsymbol{X}) \subset \sigma(\boldsymbol{W}_d)$$

Relaxed stochastic targets decomposition

We introduce a relaxed target intraday problem

$$\phi_{(d,\geq)}\big(x_d, \boldsymbol{X}_{d+1}\big) = \min_{\boldsymbol{U}_d} \quad \mathbb{E}\Big[L_d(x, \boldsymbol{U}_d, \boldsymbol{W}_d)\Big]$$
$$\text{s.t } f_d(x, \boldsymbol{U}_d, \boldsymbol{W}_d) \geq \boldsymbol{X}_{d+1}$$
$$\sigma(\boldsymbol{U}_{d,m}) \subset \sigma(\boldsymbol{W}_{d,0:m})$$

A relaxed daily value function

$$\widetilde{V}_{(d,\geq)}(x) = \min_{\boldsymbol{X} \in L^0(\Omega,\mathcal{F},\mathbb{P};\mathbb{X}_{d+1})} \Big(\phi_{(d,\geq)}\big(x, \boldsymbol{X}\big) + \mathbb{E}\big[\widetilde{V}_{(d+1,\geq)}(\boldsymbol{X})\big]\Big)$$
$$\text{s.t } \sigma(\boldsymbol{X}) \subset \sigma(\boldsymbol{W}_d)$$

Because of relaxation, we have $\widetilde{V}_{(d,\geq)} \leq \widetilde{V}_d$
but $\widetilde{V}_{(d,\geq)}$ is hard to compute due to the stochastic targets

Relaxed deterministic targets decomposition

Now we can define value functions with deterministic targets:

$$\widetilde{V}_{(d,\geq,\mathbb{X}_{d+1})}(x) = \min_{X \in \mathbb{X}_{d+1}} \Big(\phi_{(d,\geq)}\big(x, X\big) + \widetilde{V}_{(d+1,\geq,\mathbb{X}_{d+1})}(X) \Big)$$

Monotonicity Assumption

The daily value functions $\widetilde{V}_d$ are nonincreasing

Theorem

Under Monotonicity Assumption

- $\widetilde{V}_{(d,\geq)} = \widetilde{V}_d$
- $\widetilde{V}_{(d,\geq,\mathbb{X}_{d+1})} \geq \widetilde{V}_{(d,\geq)} = \widetilde{V}_d$

There are efficient ways to compute the upper bounds $\widetilde{V}_{(d,\geq,\mathbb{X}_{d+1})}$

Numerical efficiency of deterministic targets decomposition

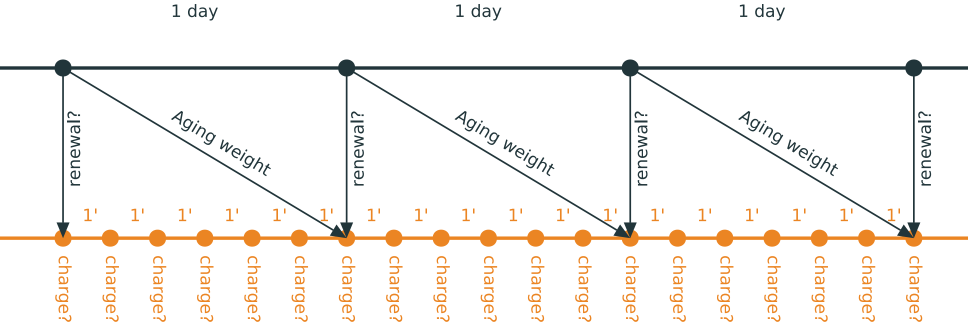
$$\overbrace{\widetilde{V}_{(d,\geq,\mathbb{X}_{d+1})}(x) = \min_{X \in \mathbb{X}_{d+1}} \Big(\underbrace{\phi_{(d,\geq)}(x, X)}_{\text{Hard to compute}} + \widetilde{V}_{(d+1,\geq,\mathbb{X}_{d+1})}(X) \Big)}^{\text{Easy to compute by dynamic programming}}$$

It is challenging to compute $\phi_{(d,\geq)}(x, X)$
for each couple (x, X) and each day d but

- We can exploit periodicity of the problem, e.g $\phi_{(d,\geq)} = \phi_{(0,\geq)}$
- In some cases $\phi_{(d,\geq)}(x, X) = \phi_{(d,\geq)}(x - X, 0)$
- We can parallelize the computation of $\phi_{(d,\geq)}$ on day d
- We can use any suitable method to solve the multistage intraday problems $\phi_{(d,\geq)}$ (SDP, scenario tree based SP, . . .)

Outline of the presentation

Decomposing by sending weights



Stochastic weights decomposition

We introduce the dualized intraday problems

$$\psi_{(d,\star)}(x_d, \boldsymbol{\lambda}_{d+1}) = \min_{\boldsymbol{U}_d} \mathbb{E}\Big[L_d(x_d, \boldsymbol{U}_d, \boldsymbol{W}_d) + \langle \boldsymbol{\lambda}_{d+1}, f_d(x_d, \boldsymbol{U}_d, \boldsymbol{W}_d)\rangle\Big]$$

$$\text{s.t } \sigma(\boldsymbol{U}_{d,m}) \subset \sigma(\boldsymbol{W}_{d,0:m})$$

Stochastic weights daily value function

$$V_{(d,\star)}(x_d) = \sup_{\boldsymbol{\lambda}_{d+1} \in L^q(\Omega,\mathcal{F},\mathbb{P};\Lambda_{d+1})} \psi_{(d,\star)}(x_d, \boldsymbol{\lambda}_{d+1}) - \Big(\mathbb{E}V_{(d+1,\star)}\Big)^\star(\boldsymbol{\lambda}_{d+1})$$

$$\text{s.t } \sigma(\boldsymbol{\lambda}_{d+1}) \subset \sigma(\boldsymbol{X}_{d+1})$$

where $\Big(\mathbb{E}V\Big)^\star(\boldsymbol{\lambda}_{d+1}) = \sup_{\boldsymbol{X} \in L^p(\Omega,\mathcal{F},\mathbb{P};\mathbb{X}_{d+1})} \langle \boldsymbol{\lambda}_{d+1}, \boldsymbol{X}\rangle - \mathbb{E}\big[V(\boldsymbol{X})\big]$
is the Fenchel transform of $\mathbb{E}V$

Deterministic weights decomposition

We define value functions with deterministic weights

$$V_{(d,\star,\mathbb{E})}(x_d) = \sup_{\lambda_{d+1} \in \Lambda_{d+1}} \psi_{(d,\star)}(x_d, \lambda_{d+1}) - V^*_{(d+1,\star,\mathbb{E})}(\lambda_{d+1})$$

By weak duality and restriction, we get

$$V_{(d,\star,\mathbb{E})} \leq \underbrace{V_{(d,\star)} \leq \widetilde{V}_d}_{\text{weak duality}}$$

Theorem

If $ri\Big(dom(\psi_{(d,\star)}(x_d,\cdot)) - dom(\mathbb{E}\widetilde{V}_{d+1}(\cdot))\Big) \neq \emptyset$ and $\mathcal{P}$ is convex, then we have

$$V_{(d,\star,\mathbb{E})} \leq V_{(d,\star)} = \widetilde{V}_d$$

There are efficient ways to compute the lower bounds $V_{(d,\star,\mathbb{E})}$

Numerical efficiency of deterministic weights decomposition

$$\overbrace{V_{(d,\star,\mathbb{E})}(x_d) = \sup_{\lambda_{d+1}\in\Lambda_{d+1}} \underbrace{\psi_{(d,\star)}(x_d,\lambda_{d+1})}_{\text{Hard to compute}} - V^{*}_{(d+1,\star,\mathbb{E})}(\lambda_{d+1})}^{\text{Easy to compute by dynamic programming}}$$

It is challenging to compute $\psi_{(d,\star)}(x,\lambda)$ for each couple (x,λ) and each day d but

- Under Monotonicity Assumption,
  we can restrict to positive weights $\lambda \geq 0$
- We can exploit periodicity of the problem $\psi_{(d,\star)} = \psi_{(0,\star)}$
- We can parallelize the computation of $\psi_{(d,\star)}$ on day d

Outline of the presentation

Back to daily intraday problems with final costs

We obtained two bounds

$$\widetilde{V}_{(d,\star,\mathbb{E})} \leq \widetilde{V}_d \leq \widetilde{V}_{(d,\geq,\mathbb{X}_{d+1})}$$

Now we can solve all intraday problems with a final cost

$$\begin{aligned}\min_{\boldsymbol{X}_{d+1},\boldsymbol{U}_d} \ & \mathbb{E}\left[L_d(x,\boldsymbol{U}_d,\boldsymbol{W}_d) + \widetilde{V}_{d+1}(\boldsymbol{X}_{d+1})\right] \\ \text{s.t } & \boldsymbol{X}_{d+1} = f_d(x,\boldsymbol{U}_d,\boldsymbol{W}_d) \\ & \sigma(\boldsymbol{U}_{d,m}) \subset \sigma(\boldsymbol{W}_{d,0:m})\end{aligned}$$

with $\widetilde{V}_{d+1} = \widetilde{V}_{(d,\geq,\mathbb{X}_{d+1})}$ or $\widetilde{V}_{d+1} = \widetilde{V}_{(d,\star,\mathbb{E})}$
We obtain one targets and one weights minute scale policies

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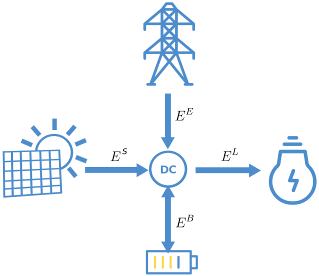
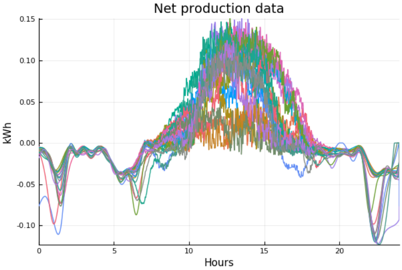
We present numerical results associated to two use cases

Common data: load/production from a house with solar panels

1. Managing a given battery charge and health on 5 days
   to compare our algorithms to references on a "small" instance
2. Managing batteries purchases, charge and health on 7300 days
   to show that targets decomposition scales





Application 1
Managing charge and aging of a battery

We control a battery

- capacity $c_0 = 13$ kWh
- $h_{0,0} = 100$ kWh of exchangeable energy (4 cycles remaining)
- over $D = 5$ days or $D \times M = 7200$ minutes
- with 1 day periodicity

We compare 4 algorithms

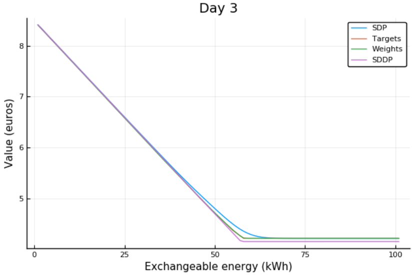
1. Stochastic dynamic programming (that is, SDP alone)
2. Stochastic dual dynamic programming (that is, SDDP alone)
3. Targets decomposition (+ SDDP for intraday problems)
4. Weights decomposition (+ SDP for intraday problems)

Decomposition algorithms + S(D)DP provide tighter bounds than S(D)DP alone

We know that

- $\widetilde{V}_d^{sddp} \leq \widetilde{V}_d \leq \widetilde{V}_d^{sdp}$
- $\widetilde{V}_{(d,\star,\mathbb{E})} \leq \widetilde{V}_d \leq \widetilde{V}_{(d,\geq,\mathbb{X}_{d+1})}$

We observe that $V_d^{sddp} \leq \widetilde{V}_{(d,\star,\mathbb{E})} \leq \widetilde{V}_{(d,\geq,\mathbb{X}_{d+1})} \leq \widetilde{V}_d^{sdp}$



We beat SDP and SDDP (that cannot fully handle 7200 stages)

Computation times and convergence

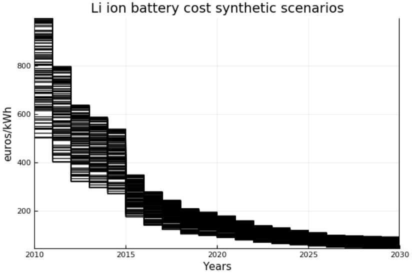
	SDP	Weights	SDDP	Targets
Total time (with parallelization)	22.5 min	5.0 min	3.6 min	0.41 min
Gap ($200 \times \frac{mc-v}{mc+v}$)	0.91 %	0.32 %	0.90 %	0.28 %

The Gap is between Monte Carlo simulation (upper bound)
and value functions at time 0

- Decomposition algorithms display smaller gaps
- Targets decompositon + SDDP is faster than SDDP
- Weights decomposition + SDP is faster than SDP

Application 2
Managing batteries purchases, charge and aging

- 20 years, $10,512,000$ minutes, 1 day periodicity
- Battery capacity between 0 and 20 kWh
- Scenarios for batteries prices



SDP and SDDP fail to solve such a problem over $10,512,000$ stages!

Target decomposed SDDP can handle **10, 512, 000** stages problems

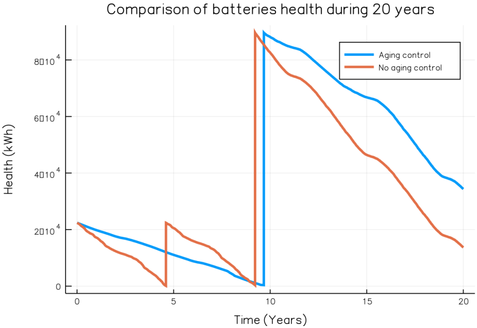
$$\overbrace{\widetilde{V}_{(d,\geq,\mathbb{X}_{d+1})}(x) = \min_{X \in \mathbb{X}_{d+1}} \Big(\underbrace{\phi_{(d,\geq)}\big(x, X\big)}_{\text{Computing } \phi_{(d,\geq)}(\cdot,\cdot) \text{ with SDDP takes 60 min}} + \widetilde{V}_{(d+1,\geq,\mathbb{X}_{d+1})}(X) \Big)}^{\text{Computing daily value functions by dynamic programming takes 45 min}}$$

Complexity: 45 min + $D \times$ 60 min

- Periodicity: 45 min + $N \times$ 60 min with $N << D$
- Parallelization: 45 min + 60 min

Does it pay to control aging?

We draw one battery prices scenario and one solar/demand scenario over $10,512,000$ minutes and simulate the policy of targets algorithm



We make a simulation of $10,512,000$ decisions in 45 minutes

We compare to a policy that does not control aging

- Without aging control: 3 battery purchases
- With aging control: 2 battery purchases

It pays to control aging with targets decomposition!

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1. We have solved problems with millions of time steps using targets decomposed SDDP
2. We have designed control strategies for sizing/charging/aging/investment of batteries
3. We have used our algorithms to improve results obtained with algorithms that are sensitive to the number of time steps (SDP, SDDP)