Multi-Stage Stochastic Optimization for Clean Energy Transition

◇

An Overview of Decomposition/Coordination Methods for Multistage Stochastic Optimization Problems

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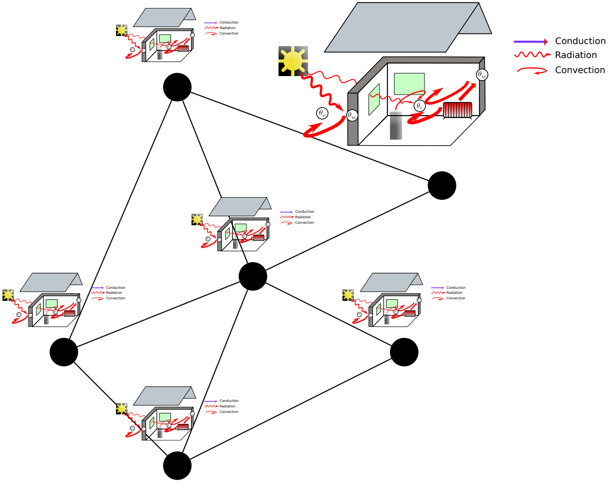
Motivation



Lecture outline

Decomposition and coordination

- The three dimensions of stochastic optimization problems
- A bird's eye view of decomposition methods: the cube

A brief insight into three decomposition methods

- Scenario decomposition methods
- Spatial (price/resource) decomposition methods
- Time decomposition methods

Summary and research agenda

Outline of the presentation

Decomposition and coordination

A brief insight into three decomposition methods

Summary and research agenda

Outline of the presentation

Temporal, scenario and spatial structures in multistage stochastic optimization problems

In multistage stochastic optimization problems,
we consider that the control variable

$$\mathbf{U}_t^i(\omega)$$

is indexed by

- Time/stages $t \in \mathbb{T}$ $(= \{0, \ldots, T-1\})$
- Scenarios $\omega \in \Omega$
- Space/units/agents $i \in \mathbb{I}$ $(= \{1, \ldots, N\})$

The letter U comes from the Russian word *upravlenie* for control

Let us fix problem and notations

$$\min_{\mathbf{U},\mathbf{X}} \overbrace{\mathbb{E}\Big(\sum_{i\in\mathbb{I}}\sum_{t\in\mathbb{T}} L_t^i(\mathbf{X}_t^i,\mathbf{U}_t^i,\mathbf{W}_{t+1})\Big)}^{\text{additive costs}} \qquad \text{subject to}$$

dynamics constraints

$$\underbrace{\mathbf{X}_{t+1}^i}_{\text{state}} = g_t^i(\mathbf{X}_t^i,\mathbf{U}_t^i,\ \underbrace{\mathbf{W}_{t+1}}_{\text{uncertainty}}\ )\ ,\ \ \mathbf{X}_0^i = g_{-1}^i(\mathbf{W}_0)$$

measurability constraints (nonanticipativity of the control $\mathbf{U}_t^i$)

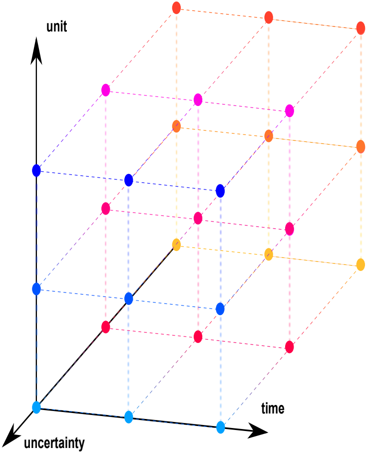
$$\sigma\big(\mathbf{U}_t^i\big) \subset \sigma(\mathbf{W}_0,\ldots,\mathbf{W}_t) \iff \mathbf{U}_t^i = \mathbb{E}\big(\mathbf{U}_t^i \mid \mathbf{W}_0,\ldots,\mathbf{W}_t\big)$$

spatially coupling constraints

$$\sum_{i\in\mathbb{I}} \Theta_t^i(\mathbf{X}_t^i,\mathbf{U}_t^i) = 0$$

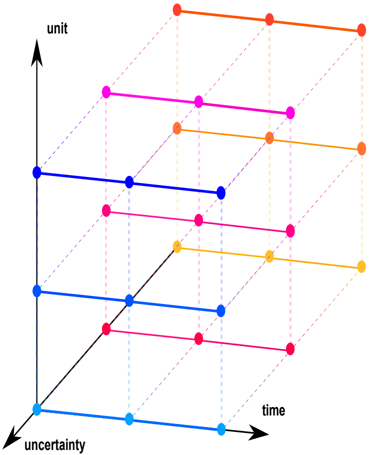
Outline of the presentation

Couplings for stochastic problems



$$\min \quad \mathbb{E}\Big(\sum_i \sum_t L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) \Big)$$

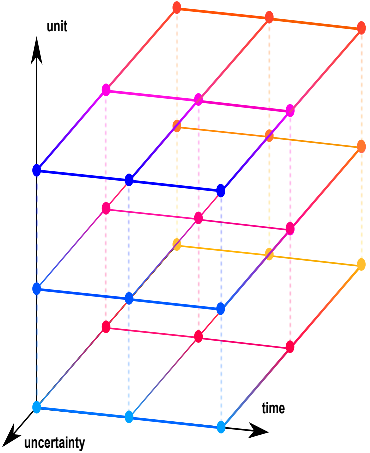
Couplings for stochastic problems: in time



$$\min \mathbb{E}\Big(\sum_i \sum_t L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) \Big)$$

$$\text{s.t.} \quad \mathbf{X}_{t+1}^i = g_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1})$$

Couplings for stochastic problems: in uncertainty

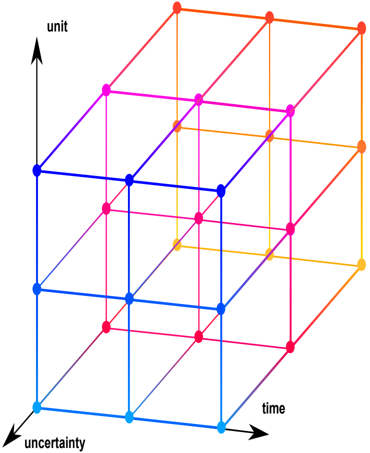


$$\min \mathbb{E}\Big(\sum_i \sum_t L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) \Big)$$

s.t. $\mathbf{X}_{t+1}^i = g_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1})$

$$\mathbf{U}_t^i = \mathbb{E}\big(\mathbf{U}_t^i \mid \mathbf{W}_0, \ldots, \mathbf{W}_t\big)$$

Couplings for stochastic problems: in space



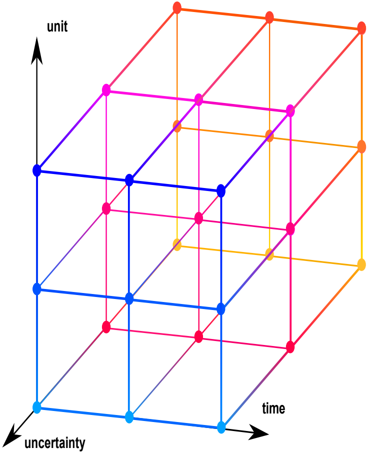
$$\min \mathbb{E}\Big(\sum_i \sum_t L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) \Big)$$

$$\text{s.t. } \mathbf{X}_{t+1}^i = g_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1})$$

$$\mathbf{U}_t^i = \mathbb{E}\big(\mathbf{U}_t^i \mid \mathbf{W}_0, \ldots, \mathbf{W}_t\big)$$

$$\sum_i \Theta_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i) = 0$$

Can we decouple stochastic optimization problems?



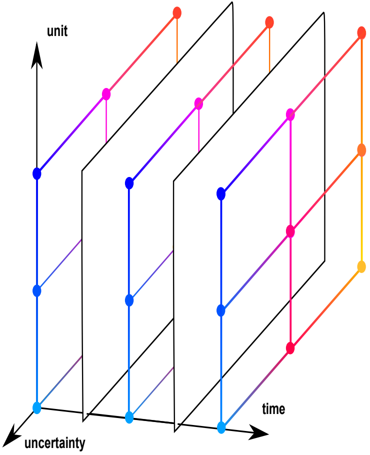
$$\min \mathbb{E}\Big(\sum_i \sum_t L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) \Big)$$

s.t. $$\mathbf{X}_{t+1}^i = g_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1})$$

$$\mathbf{U}_t^i = \mathbb{E}\big(\mathbf{U}_t^i \mid \mathbf{W}_0, \ldots, \mathbf{W}_t\big)$$

$$\sum_i \Theta_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i) = 0$$

Sequential decomposition in time



$$\min \mathbb{E}\Big(\sum_i \sum_t L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) \Big)$$

$$\text{s.t.} \quad \mathbf{X}_{t+1}^i = g_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1})$$

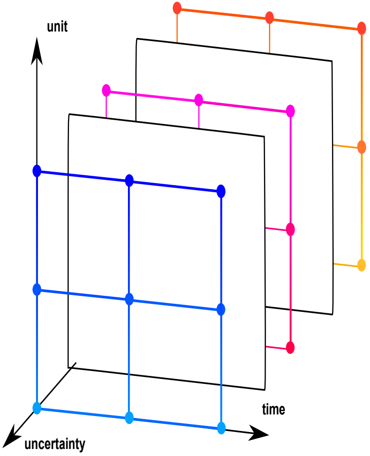
$$\mathbf{U}_t^i = \mathbb{E}\big(\mathbf{U}_t^i \mid \mathbf{W}_0, \ldots, \mathbf{W}_t\big)$$

$$\sum_i \Theta_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i) = 0$$

Dynamic Programming (DP)

Bellman (56)

Parallel decomposition in uncertainty/scenarios



$$\min \mathbb{E}\Big(\sum_i \sum_t L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) \Big)$$

$$\text{s.t. } \mathbf{X}_{t+1}^i = g_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1})$$

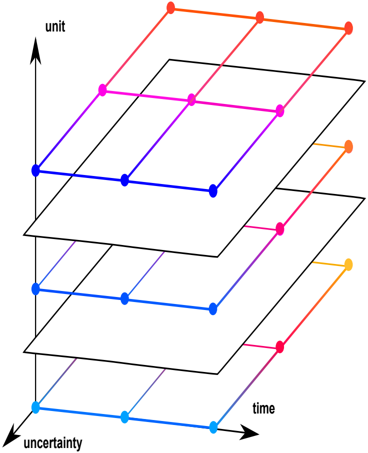
$$\mathbf{U}_t^i = \mathbb{E}\big(\mathbf{U}_t^i \mid \mathbf{W}_0, \dots, \mathbf{W}_t\big)$$

$$\sum_i \Theta_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i) = 0$$

Progressive Hedging

Rockafellar-Wets (91)

Parallel decomposition in space/units



$$\min \mathbb{E}\Big(\sum_i \sum_t L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) \Big)$$

$$\text{s.t.} \quad \mathbf{X}_{t+1}^i = g_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1})$$

$$\mathbf{U}_t^i = \mathbb{E}\big(\mathbf{U}_t^i \mid \mathbf{W}_0, \ldots, \mathbf{W}_t\big)$$

$$\sum_i \Theta_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i) = 0$$

Price and Resource Decompositions

Decomposition-coordination: divide and conquer

- Temporal decomposition
  - A state is an information summary
  - Time coordination realized through Dynamic Programming, by value functions (of the state)
  - Hard nonanticipativity constraints
- Scenario decomposition
  - Along each scenario, subproblems are deterministic (powerful algorithms)
  - Scenario coordination realized through Progressive Hedging, by updating nonanticipativity multipliers
  - Soft nonanticipativity constraints
- Spatial decomposition
  - By prices (multipliers of the spatial coupling constraint)
  - By resources (splitting the spatial coupling constraint)

Outline of the presentation

Decomposition and coordination

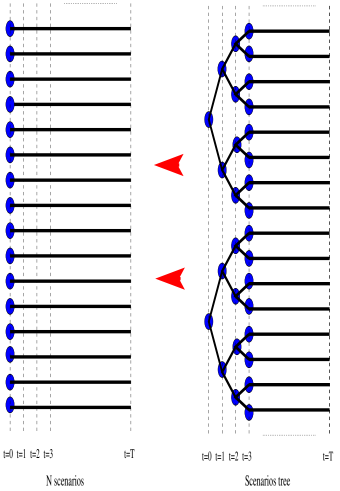
A brief insight into three decomposition methods

Summary and research agenda

Outline of the presentation

Moving from tree to fan (and scenarios)

Equivalent formulations of the nonanticipativity constraints



- On a (scenario) tree,
  the nonanticipativity constraints

  $$\sigma\big(\mathbf{U}_t\big) \subset \sigma(\mathbf{W}_0, \ldots, \mathbf{W}_t)$$

  are "hardwired"
- On a fan,
  the nonanticipativity constraints
  write as linear equality constraints

  $$\mathbf{U}_t = \mathbb{E}\big(\mathbf{U}_t \mid \mathbf{W}_0, \ldots, \mathbf{W}_t\big)$$

Progressive Hedging stands as a scenario decomposition method

Rockafellar-Wets (91) dualize the nonanticipativity constraints

$$\mathbf{U}_t = \mathbb{E}\big(\mathbf{U}_t \mid \mathbf{W}_0, \ldots, \mathbf{W}_t\big)$$

- When the criterion is strongly convex,
  one uses a Lagrangian relaxation (algorithm "à la Uzawa")
  to obtain a scenario decomposition
- When the criterion is linear, Rockafellar-Wets (91)
  propose to use an augmented Lagrangian,
  and obtain the Progressive Hedging algorithm

Data: step $\rho > 0$, initial multipliers $\left\{\lambda_s^{(0)}\right\}_{s\in\mathbb{S}}$ and mean first decision $\overline{\mathbf{u}}^{(0)}$;

Result: optimal first decision $\mathbf{u}$;

repeat

forall *scenarios* $s \in \mathbb{S}$ **do**

Solve the deterministic minimization problem for scenario s,

with a penalization $+\lambda_s^{(k)}\left(\mathbf{u}_s^{(k+1)} - \overline{\mathbf{u}}^{(k)}\right)$,

and obtain optimal first decision $\mathbf{u}_s^{(k+1)}$;

Update the mean first decisions

$$\overline{\mathbf{u}}^{(k+1)} = \sum_{s\in\mathbb{S}} \pi_s \mathbf{u}_s^{(k+1)} ;$$

Update the multiplier by

$$\lambda_s^{(k+1)} = \lambda_s^{(k)} + \rho\Big(\mathbf{u}_s^{(k+1)} - \overline{\mathbf{u}}^{(k+1)}\Big) , \quad \forall s \in \mathbb{S} ;$$

until $\mathbf{u}_s^{(k+1)} - \sum_{s'\in\mathbb{S}} \pi_{s'} \mathbf{u}_{s'}^{(k+1)} = 0 , \quad \forall s \in \mathbb{S}$;

Outline of the presentation

We consider an additive model

Consider the following minimization problem

$$\min_{u \in \mathcal{U}_{\mathrm{ad}} \subset \mathcal{U}} J(u) \quad \text{subject to} \quad \Theta(u) - \theta = 0 \in \mathcal{V}$$

for which exists a decomposition of the space $\mathcal{U} = \mathcal{U}^1 \times \ldots \times \mathcal{U}^N$, so that $u \in \mathcal{U}$ writes $u = \big(u^1, \ldots, u^N\big)$ with $u^i \in \mathcal{U}^i$, and also

- $\mathcal{U}_{\mathrm{ad}} = \mathcal{U}_{\mathrm{ad}}^1 \times \cdots \times \mathcal{U}_{\mathrm{ad}}^N$ $\qquad \mathcal{U}_{\mathrm{ad}}^i \subset \mathcal{U}^i$
- $J(u) = J^1(u^1) + \cdots + J^N(u^N)$ $\qquad u^i \in \mathcal{U}^i$
- $\Theta(u) = \Theta^1(u^1) + \cdots + \Theta^N(u^N)$ $\qquad u^i \in \mathcal{U}^i$

Then the problem displays the following additive structure

$$\min_{\substack{u^1 \in \mathcal{U}_{\mathrm{ad}}^1 \\ \vdots \\ u^N \in \mathcal{U}_{\mathrm{ad}}^N}} \sum_{i=1}^{N} J^i(u^i) \quad \text{subject to} \quad \sum_{i=1}^{N} \Theta^i(u^i) - \theta = 0$$

Additive model — Price decomposition I

$$\min_{u \in \mathcal{U}_{\mathrm{ad}}} \sum_{i=1}^{N} J^i(u^i) \quad \text{subject to} \quad \sum_{i=1}^{N} \Theta^i(u^i) - \theta = 0$$

1. Form the Lagrangian of the problem
   We assume that a saddle point exists,
   so that solving the initial problem is equivalent to

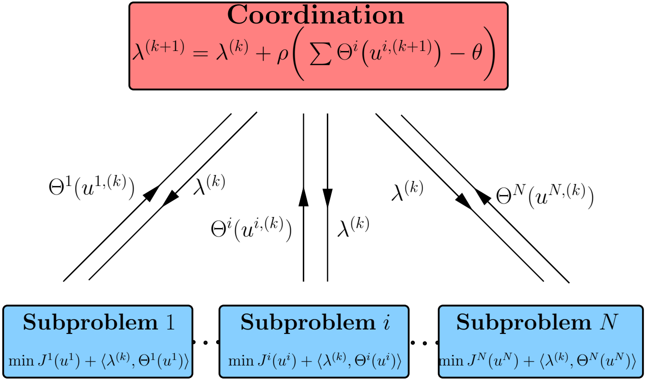
$$\max_{\lambda \in \mathcal{V}} \min_{u \in \mathcal{U}_{\mathrm{ad}}} \sum_{i=1}^{N} \Big(J^i(u^i) + \big\langle \lambda \,, \Theta^i(u^i) \big\rangle \Big) - \big\langle \lambda \,, \theta \big\rangle$$

2. Solve this problem by the Uzawa algorithm

$$u^{i,(k+1)} \in \operatorname*{arg\,min}_{u^i \in \mathcal{U}_{\mathrm{ad}}^i} J^i(u^i) + \big\langle \lambda^{(k)} \,, \Theta^i(u^i) \big\rangle \,, \quad i = 1 \ldots, N$$

$$\lambda^{(k+1)} = \lambda^{(k)} + \rho \bigg(\sum_{i=1}^{N} \Theta^i \Big(u^{i,(k+1)} \Big) - \theta \bigg)$$

Additive model — Price decomposition II



Additive model — Resource allocation I

$$\min_{u \in \mathcal{U}_{\mathrm{ad}}} \sum_{i=1}^{N} J^i(u^i) \quad \text{subject to} \quad \sum_{i=1}^{N} \Theta^i(u^i) - \theta = 0$$

1. Write the constraint in a equivalent manner by introducing new variables $v = (v^1, \ldots, v^N)$ (the so-called "allocation")

$$\sum_{i=1}^{N} \Theta^i(u^i) - \theta = 0 \quad \Leftrightarrow \quad \Theta^i(u^i) - v^i = 0 \text{ and } \sum_{i=1}^{N} v^i = \theta$$

and minimize the criterion w.r.t. u and v

$$\min_{v \in \mathcal{V}^N} \sum_{i=1}^{N} \Big(\min_{u^i \in \mathcal{U}^i_{\mathrm{ad}}} J^i(u^i) \text{ s.t. } \Theta^i(u^i) - v^i = 0 \Big) \text{ s.t. } \sum_{i=1}^{N} v^i = \theta$$

Additive model — Resource allocation II

$$\min_{v\in\mathcal{V}^N} \sum_{i=1}^{N} \Bigg(\underbrace{\min_{u^i\in\mathcal{U}^i_{\mathrm{ad}}} J^i(u^i) \ \text{ s.t. } \ \Theta^i(u^i) - v^i = 0}_{G^i(v^i)} \Bigg) \ \text{ s.t. } \ \sum_{i=1}^{N} v^i = \theta$$

$$\Updownarrow$$

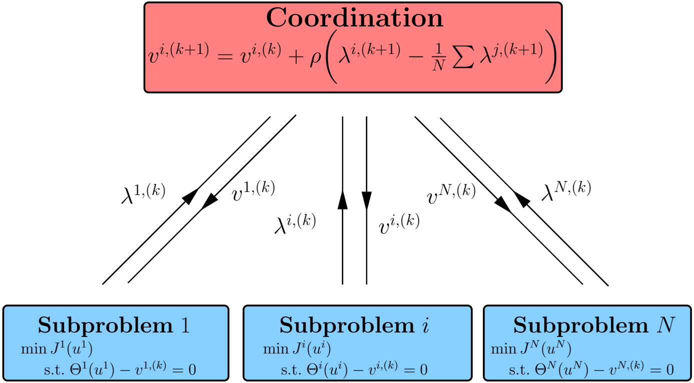
$$\min_{v\in\mathcal{V}^N} \sum_{i=1}^{N} G^i(v^i) \quad \text{s.t.} \quad \sum_{i=1}^{N} v^i = \theta$$

2. Solve the last problem using a projected gradient method

$$G^i(v^{i,(k)}) = \min_{u^i\in\mathcal{U}^i_{\mathrm{ad}}} J^i(u^i) \ \text{ s.t. } \ \Theta^i(u^i) - v^{i,(k)} = 0 \quad \rightsquigarrow \quad \lambda^{i,(k+1)}$$

$$v^{i,(k+1)} = v^{i,(k)} + \rho\Bigg(\lambda^{i,(k+1)} - \frac{1}{N}\sum_{j=1}^{N} \lambda^{j,(k+1)}\Bigg)$$

Additive model — Resource allocation III



Preparing Pierre Carpentier's talk

We can also use price/resource decomposition to bound a minimization problem

$$V_0^\star = \inf_{u^1 \in \mathbb{U}^1_{\mathrm{ad}}, \cdots, u^N \in \mathbb{U}^N_{\mathrm{ad}}} \sum_{i=1}^N J^i(u^i)$$

$$\text{s.t.} \quad \underbrace{\big(\Theta^1(u^1), \cdots, \Theta^N(u^N)\big) \in S}_{\text{coupling constraint}}$$

- $u^i \in \mathbb{U}^i$ be a local decision variable
- $J^i : \mathbb{U}^i \to \mathbb{R}, \; i \in [\![1, N]\!]$ be a local objective function
- $\mathbb{U}^i_{\mathrm{ad}}$ be a subset of the local decision set $\mathbb{U}^i$
- $\Theta^i : \mathbb{U}^i \to \mathcal{C}^i$ be a local constraint mapping
- S be a subset of $\mathcal{C} = \mathcal{C}^1 \times \cdots \times \mathcal{C}^N$

We denote by S^o the polar cone of S

$$S^o = \big\{ p \in \mathcal{C}^\star \mid \langle p \,, r \rangle \leq 0 \,, \;\; \forall r \in S \big\}$$

Price and resource local value functions

For each $i \in [\![1, N]\!]$,

- for any price $p^i \in (\mathcal{C}^i)^\star$, we define the local price value

$$\underline{V}_0^i[p^i] = \inf_{u^i \in \mathbb{U}^i_{\mathrm{ad}}} J^i(u^i) + \left\langle p^i \,, \Theta^i(u^i) \right\rangle$$

- for any resource $r^i \in \mathcal{C}^i$, we define the local resource value

$$\overline{V}_0^i[r^i] = \inf_{u^i \in \mathbb{U}^i_{\mathrm{ad}}} J^i(u^i) \quad \text{s.t.} \quad \Theta^i(u^i) = r^i$$

Proposition (upper and lower bounds for optimal value)

- *For any admissible price* $p = (p^1, \cdots, p^N) \in S^o$
- *For any admissible resource* $r = (r^1, \cdots, r^N) \in S$

$$\sum_{i=1}^N \underline{V}_0^i[p^i] \;\leq\; V_0^\star \;\leq\; \sum_{i=1}^N \overline{V}_0^i[r^i]$$

Outline of the presentation

Brief literature review on dynamic programming

	Bellman	Puterman	Bertsekas Schreve	Evstignev	Witsenhausen (standard form)
	1957	1994	1996	1976	1973
State	X	X	X	–	$(\omega, U_{1:t-1})$
Dynamics	$f(X, U, W)$	$P^u_{x,x'}$	$f(X, U, W)$	–	$X_t = (X_{t-1}, U_t)$
Uncertainties	Indep.	–	ρ	$(\Omega, \mathcal{F})$	$(\Omega, \mathcal{F})$
Cost	$\sum_t$	$\sum_t$	$\sum_t$	$j(\omega, U)$	$j(\omega, U)$
Controls	$\gamma(X)$	$\gamma(X)$ $\gamma(H)$	$\gamma(X)$ $\gamma(H)$	$\mathcal{F}_t$-meas.	$\gamma(x_t)$ $\mathcal{I}_t$-meas.
History	–	$(X, U, \ldots)_t$	$(W, U, \ldots)_t$	–	X_t

We introduce the history

- The timeline is

$$w_0 \rightsquigarrow u_0 \rightsquigarrow w_1 \rightsquigarrow u_1 \rightsquigarrow \quad \ldots \quad \rightsquigarrow w_{T-1} \rightsquigarrow u_{T-1} \rightsquigarrow w_T$$

- and the history is

$$\overbrace{h_t}^{\text{history}} = (\overbrace{w_0}^{\text{uncertainty}}, \overbrace{u_0}^{\text{control}}, \overbrace{w_1}^{\text{uncertainty}}, u_1, \ldots, u_{t-1}, w_t)$$

$$\in \mathbb{H}_t = \mathbb{W}_0 \times \prod_{s=0}^{t-1} (\underbrace{\mathbb{U}_s}_{\text{control space}} \times \underbrace{\mathbb{W}_{s+1}}_{\text{uncertainty space}})$$

History is the largest state

The history follows the dynamics

$$\begin{aligned} h_{t+1} &= (\overbrace{w_0, u_0, w_1, u_1, \ldots, u_{t-1}, w_t}^{\text{history } h_t}, u_t, w_{t+1}) \\ &= \Big(h_t, \underbrace{u_t}_{\text{control}}, \underbrace{w_{t+1}}_{\text{uncertainty}}\Big) \end{aligned}$$

We formulate a sequence of minimization problems over increasing history spaces

- Once given
  - a criterion $j : \mathbb{H}_T \to \mathbb{R}$
  - a sequence of stochastic kernels $\rho_{t:t+1} : \mathbb{H}_t \to \Delta(\mathbb{W}_{t+1})$
- we define, for any history h_t, a minimization problem

$$V_t(h_t) = \underbrace{\inf_{\gamma_{t:T-1} \in \Gamma_{t:T-1}}}_{\text{history feedbacks}} \int_{\mathbb{H}_T} \overbrace{j(h'_T)}^{\text{criterion}} \underbrace{\rho^{\gamma}_{t:T}(h_t, \mathrm{d}h'_T)}_{\text{controlled stochastic kernel}}$$

There is a Bellman equation involving value functions over increasing history spaces without white noise assumption

$$V_T = j$$
$$V_t = \mathcal{B}_{t+1:t} V_{t+1}$$

with

$$\big(\mathcal{B}_{t+1:t}\varphi\big)(h_t) = \inf_{u_t \in \mathbb{U}_t} \int_{\mathbb{W}_{t+1}} \varphi(h_t, u_t, w_{t+1}) \rho_{t:t+1}(h_t, dw_{t+1})$$

Preparing Jean-Philippe Chancelier's talk

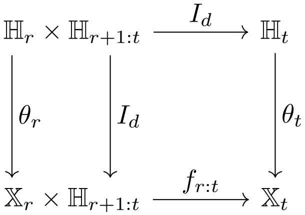
Towards state reduction by time blocks

- History h_t is itself a canonical state variable, which lives in the history space $\mathbb{H}_t = \mathbb{W}_0 \times \prod_{s=0}^{t-1}(\mathbb{U}_s \times \mathbb{W}_{s+1})$
- However the size of this canonical state increases with t, which is a nasty feature for dynamic programming
- We will now
  - introduce "state" spaces $\mathbb{X}_t$
  - and then reduce the history with a mapping $\theta_r : \mathbb{H}_r \to \mathbb{X}_r$
  - to obtain a compressed "state" variable $\theta_t(h_t) = x_t \in \mathbb{X}_t$
  - but only at some specified times $0 = t_0 < t_1 < \cdots < t_N = T$
- As an application, we will handle stochastic independence between time blocks but possible dependence *within* time blocks

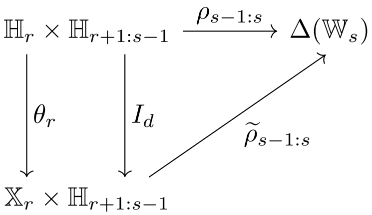
State reduction graphically

The triplet $(\theta_r, \theta_t, f_{r:t})$ is a state reduction across $(r:t)$ if

- the following diagram, for the dynamics, commutes



- the following diagrams, for the stochastic kernels, commute



Bellman operator across $(r:t)$

$\mathcal{B}_{r:t} : \mathbb{L}^0_+(\mathbb{H}_r, \mathcal{H}_r) \to \mathbb{L}^0_+(\mathbb{H}_t, \mathcal{H}_t)$ is defined by

$$\mathcal{B}_{r:t} = \mathcal{B}_{t+1:t} \circ \cdots \circ \mathcal{B}_{r:r-1} \ ,$$

where the one time step operators $\mathcal{B}_{s:s-1}$ are

$$\big(\mathcal{B}_{s:s-1}\varphi\big)(h_{s-1}) = \inf_{u_{s-1} \in \mathbb{U}_{s-1}} \int_{\mathbb{W}_s} \varphi(h_{s-1}, u_{s-1}, w_s) \rho_{s-1:s}(h_{s-1}, dw_s)$$

State reduction and Dynamic Programming

Denoting by $\theta_r^\star : \mathbb{L}_+^0(\mathbb{X}_r, \mathcal{X}_r) \to \mathbb{L}_+^0(\mathbb{H}_r, \mathcal{H}_r)$
the operator defined by

$$\theta_r^\star(\widetilde{\varphi}_r) = \widetilde{\varphi}_r \circ \theta_r \ , \ \ \forall \widetilde{\varphi}_r \in \mathbb{L}_+^0(\mathbb{X}_r, \mathcal{X}_r) \ ,$$

there exists a reduced Bellman operator across $(r\!:\!t)$ such that

$$\theta_t^\star \circ \widetilde{\mathcal{B}}_{r:t} = \mathcal{B}_{r:t} \circ \theta_r^\star \ ,$$

that is, the following diagram is commutative

$$\begin{array}{ccc}
\mathbb{L}_+^0(\mathbb{H}_r, \mathcal{H}_r) & \xrightarrow{\mathcal{B}_{r:t}} & \mathbb{L}_+^0(\mathbb{H}_t, \mathcal{H}_t) \\
\theta_r^\star \Big\uparrow & & \Big\uparrow \theta_t^\star \\
\mathbb{L}_+^0(\mathbb{X}_r, \mathcal{X}_r) & \xrightarrow{\widetilde{\mathcal{B}}_{r:t}} & \mathbb{L}_+^0(\mathbb{X}_t, \mathcal{X}_t)
\end{array}$$

Outline of the presentation

We have sketched three main decomposition methods in multistage stochastic optimization

- time: Dynamic Programming
- scenario: Progressive Hedging
- space: decomposition by prices or by resources

Numerical walls are well-known

- in dynamic programming,
  the bottleneck is the dimension of the state
- in stochastic programming,
  the bottleneck is the number of stages

Here is our research agenda for stochastic decomposition

- Designing risk criteria compatible with decomposition
- Combining different decomposition methods
  - time: Dynamic Programming
  - scenario: Progressive Hedging
  - space: decomposition by prices or by resources
- to produce blends and tackle large scale energy applications
  - time blocks + prices/resources
    (talk of Jean-Philippe Chancelier)
    - dynamic programming across time blocks
      + prices/resources decomposition by time block
    - application to two time scales battery management
  - time + space
    (talk of Pierre Carpentier)
    - nodal decomposition by prices or by resources
      + dynamic programming within node
    - application to large scale microgrid management