Problem Session

Benoit Charbonneau :-

① Spin(7) manifold ex. CY^4

$$\Lambda^2_{21} = \Lambda^{1,1}_0 \oplus \tfrac{1}{2}[[\Lambda^{2,0} \oplus \Lambda^{0,2}]]$$

Spin(7)-instanton eq^n is $\pi_7 F = 0$ or $F \in \Lambda^2_{21}$

HYM equation $F \in \Lambda^{1,1}_0$

Thm [Lewis] If $\begin{matrix} E \\ \downarrow \\ CY^4 \end{matrix}$ admits a HYM connection.

Then if ∇ is a Spin(7)-instanton then it is secretly a HYM instanton. (So it is a fake Spin(7)-instanton).

Problem 1 find a genuine Spin(7) instanton if it doesn't admit HYM instanton).

or

Problem 2 Prove that there are none.

Problem 3 Find a <u>genuine</u> G_2 instanton on $S^1 \times CY^3$. [compact]

Problem 4 What are vol$(B(r))$ growth as $r \to \infty$ for complete, non-compact G_2 manifolds?

<u>Spiro</u> :- What about DUY or K-H correspondence for G_2/Spin(7) manifolds?

Derek Harland

cone over N.K.

C(Nearly Kähler) is G_2 holonomy.

pseudohol. curves $\rightsquigarrow$ associatives

instantons $\rightsquigarrow$ instantons

Problem :- Do not know much about instantons on NK mflds but we know much about pseudoholomorphic curves.

Can we find more examples of instantons on NK?

Problem Analogs of Walpuski, Brendle's results for NK manifolds.

Problem Classify homogenous instantons on NK manifolds. w/ all possible structure group.

Problem Take a twistor space. Suppose $E|_{\text{fibre}}$ is trivial. Are there instantons not pulled back from base?

Problem Are there any smooth instantons on S^6 w/ structure group $SU(2)$?

Henrique Sá Earp :- formulate the right condition of ERP for co-closed G_2 structures.

possible answer :- Gavin Ball

$$d\tau_0 = Q_0(\tau_0, \tau_3) \in \Omega^1_7(M)$$

quadratic

$$d\tau_3 = Q_3(\tau_0, \tau_3) \in \Omega^4_1 \oplus \Omega^4_7 \oplus \Omega^4_{27}$$

do some Rep. Theory.

Problem [Gavin Ball]

find a family of homogenous coclosed G_2 structures φ_t, $t \in [0,1)$ s.t. φ_0 is nearly parallel, φ_t isn't nearly parallel.

Problem Can a G_2 structure of type τ_3 (i.e., only $\tau_3 \neq 0$) be Einstein?

Jesse Madnick

Problem :- Are there any topological obstructions for complete and embedded special Lagrangian 3-folds in $\mathbb{R}^6$, or can every topological type occur?

Spiro Karigiannis

Problem Madsen–Swann example of incomplete G_2 holonomy manifolds.

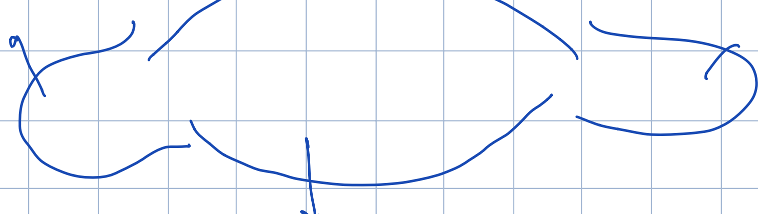
Can we construct compact $\mathrm{Hol} = G_2$ manifolds by gluing blocks that have incomplete metrics ?

Jason Lotay's suggestions :⇝

Gross – Wilson / Hein – Sun – Viaclovsky – Zhang

Tian-Yau −

Tian-Yau +



Incomplete : Ooguri-Vafa in case of Gross / Wilson

$(-T, T) \times N^3$ nilmanifold for H – S – V – Z.

Jason Lotay

$$\Sigma^2 \subseteq \mathbb{R}^7 \text{ real analytic}$$

Harvey-Lawson $\to$ $\exists(!)$ associative $A^3 \supseteq \Sigma^2$.

Problem :- when is Σ the boundary of a compact associative A ?

or

$\Omega \subseteq \mathbb{R}^3$ $\quad u : \partial\Omega \to \mathbb{R}^4$ does $\exists$ associative graph $v : \Omega \to \mathbb{R}^4$ s.t. $v|_{\partial\Omega} = u$?

cf. Harvey-Lawson - pluripotential theory for calibrated manifolds.

- Special Lagrangian boundary value problem has some obstructions [Haskins - Pacini]

Henrique Sá Earp

Study harmonic Spin(7)-structure.

$\xi \in \Gamma(\mathcal{E})$, $\quad y^* \xi_0 = \xi$

$\downarrow$

geometric structure.

$$
\begin{tikzcd}
P = Fr(M) \arrow[r, "H"] \arrow[d] & N \arrow[dl] \\
M &
\end{tikzcd}
\qquad H = Stab(\xi)
$$

$$\xi \longleftrightarrow \sigma$$

$$E(\sigma) = \int_M |d\sigma|^2$$

Critical point $E \iff \boxed{\operatorname{tr} \nabla d^{\nabla} \sigma = 0}$

<u>Def^n</u> σ harmonic

G_2 case :-

$$\operatorname{Crit} E = \{ \operatorname{div} T = 0 \}$$

(Studied by Dwivedi – Gianniotis – Karigiannis & Grigorian).