

**Working Group 2 - If not calculus, then what, and how?**

Discrete mathematics, statistics, mathematics for a general student audience, mathematics for teachers, mathematical modeling with and without calculus, computational mathematics, precalculus. Prerequisites? Full courses vs. multiple learning modules? Training of the future instructors of such courses. Appropriate learning resources.

Group members are Shawn D, Olive, Shannon, Indy, Jana, Jerrod, Miro and Amenda

**Monday August 8 - Teaching math in 2030**

We will discuss the following questions:

● What will course delivery look like in 2030?

● It is easy to claim that, based on the COVID-19 experiences, it will be some kind of a

hybrid - but what would that look like?

● How do we find a framework/strategy that would provide a good balance between the

face-to-face and the online components of learning and teaching?

● How to best address the rising needs for teacher-training of math instructors, and

faculty in general, as this is an essential component of any effective change?

*For Monday’s working group meeting, insert notes here*

***Monday Morning***

* Are we replacing calculus with something else or is this in addition to calculus?
	+ Miro - as a replacement for econ / business / life science (calculus does not provide the required tools)
	+ Jana - also topics in linear algebra that do not match student’s requirements / needs for mathematical tools
* Mathematics is a structural framework of rules and logic - how do we get students to see / work within those frameworks? (Miro/Olive)
* Shawn: a concern - does removing calculus close doors to students to become math majors from econ/arts/etc.?
	+ Miro: no only remove calculus as a *requirement*
	+ Shawn: but will the replacement allow students to take further mathematics courses?
	+ Jana: if you don’t fit the whole calculus stream into your electives, then what? Scale (big/small universities and course offerings?)

***Monday Afternoon***

* Discrete mathematics is a viable option for a first year mathematics course
	+ Logic and proofs and *other topics*
	+ Goal: mature mathematical thinking and algorithmic thinking
	+ *Proofs of “obvious things” are hard for students!*
* Should every course in mathematics be a proofs course?
	+ But we don’t prove things in calculus!
	+ But we could prove things in calculus, little things anyway (when the context makes sense) … focus on logical arguments.
* *Logical reasoning should be in all courses (in first year)!*
	+ Repeat and reiterate throughout curriculum
	+ Why do we care about “proof”? What is “proof”?
* Who is this first-year alternative to calculus course for?
	+ Everyone? Majors and non-majors?
* What is the goal?
	+ Learning that these is something good to mathematics
	+ “CHEM 101” is a little bit of this and that … what is “MATH 101” what would an “Introductory mathematics” course look like?
		- Maybe it is a selection of three topics (team taught)
		- Is it terminal? A pre-requisite?
	+ Students may not know how to think mathematically as they transition from highschool (even after doing 12 years of math)
	+ Learning what mathematics *is*
	+ **Provide an experience in university level mathematics that does not have the barrier/background of 5 years of highschool pre-calculus.**
* What are the barriers?
	+ We start math with **play** … then what happens?
		- Teacher preparation and support - **teachers don’t see the mathematics in play**…
* Would a first-year calculus alternative be terminal?
	+ E.g., what would it prepare you for?
* How do we support students who will identify mathematics later in life as something they see and know about in their lives?
	+ Anecdote: look at that cloud based on my <not math course> it wil be nice this afternoon! … how do we make this math?

**What does Mathematics for Humans look like?**

* No pre-requisites (other than university admission)
	+ Or separate streams based on preparation of students?
* Is not a prerequisite (is an independent course)
* Tag lines
	+ Mathematics that we wish that politicians knew
* What are the topics in a course like this?
	+ Logic < — > Mathematical thinking (somewhere on this spectrum)
		- E.g. logic puzzles, mathematical games, strategy
	+ Proof versus scientific evidence
		- 10 experiments in chemistry versus one counterexample in math
	+ Probability (e.g, Bayes theorem)
	+ Data visualization
		- Math in the media
	+ Voting systems
	+ Finance
		- Mortgage rates, investing, taxes, inflation, time value of money, tuition rates
	+ **Key point: concepts and applications (where it math used?)**
		- Has to be authentic! E.g., can’t just say “it is used in quantum mechanics”
* Who would want to take this course? Why would they want to take it?
* What is the goal of this course?
	+ An idea of what mathematics *is* and what mathematicians *do*?
	+ Think, act and speak as a mathematician does
	+ Learn to be frustrated
	+ Find patterns, generalize and explore – develop mathematical thinking
	+ Develop abstraction as a tool
	+ **Mathematical habits of mind**
		- Tinkering, conjecturing, experimenting, …
* Should this course cover new mathematics or provide context and appliation for mathematics that students already know?
	+ Probably only use grade 10 mathematics

**Maybe we are talking about three courses? (All require “grade 10 math”)**

* **Mathematics for Humans**
	+ Taxes, finance, logic puzzles, reading the newspaper
* **Mathematics for the Working Scientist**
	+ The math that you need to read papers in your field that contain math / stats
* **Introduction to Mathematics**
	+ What *is* mathematics and what do mathematicians *do?*

**Who needs calculus and why do they need it?**

* It is a gate-keeper. Is that okay? Are we okay with it?

*Can we summarize these discussions into one or two slides here* [*https://docs.google.com/presentation/d/1pWaaYWaGT\_VoPbihiVCoOSfDSXb5OYAIL57s0-GLxdw/edit?usp=sharing*](https://docs.google.com/presentation/d/1pWaaYWaGT_VoPbihiVCoOSfDSXb5OYAIL57s0-GLxdw/edit?usp=sharing)*? We will present these slides on Friday’s working group presentation.*

**Tuesday August 9 - First year math courses in 2030**

We will discuss the questions and topics:

● What first-year courses should we/will we teach in 2030?

● Alternative avenues into university mathematics, to reflect the realities of the times

we live in, as well as the diversity of students’ interests and goals. We will investigate

what our colleagues in North America, Europe, and elsewhere, plan to do.

● Increase the breadth of mathematics offerings/ entry points at a first-year level; for

instance, future teachers need courses that are different from those taken by future

scientists, or those taken by mathematics and statistics majors. As well, consider

multiple exit points, to facilitate students’ future plans.

*For Tuesday’s working group meeting, insert notes here*

* The reality of 2030 is …
	+ calculus and …
	+ calculus for whom… ?
* **Can we make a course that students will want to take because it is excellent?**
	+ **Topic: Critical thinking though in-depth problems**
	+ There is some marketing involved
		- *The name of the course is important*
	+ **Can the questions motivate this course and its content?**
		- *Learning objectives are key*
		- *If it is for everyone - it should be university-level objectives!*
	+ We have to think about a target audience
		- **Target 1**: students who need an **authentic mathematics course**
		- **Target 2**: students who want to see what mathematics is out there
			* Both of these have as broad an audience as possible
			* Downside of a broad audience?
				+ Various levels of incoming students (esp. nonscience) and we **do not** want to disadvantage students pursuing non-science majors
		- **Target 3**: students who want to be MD, Chief Science Officer, etc.
			* Well prepared students who we can push hard (they have high motivation)
	+ **Challenge**: we can’t teach all of the foundations, can we provide skills to interpret and understand mathematics in student’s contexts?
		- Build understanding and appreciation
	+ **Challenge**: concerns about scale and assessment are coming up…
* **Notable book**: “How to lie with statistics” - being critical while reading science
	+ Fake news versus “fake numbers”
* **Low-floor High-ceiling**
	+ Problems that have approachable entry and can be scaled easily
	+ Who can teach a course like this well?
* **Note**: we are not trying to create an “easy A” for science students

**Learning Objectives / Competencies for First-Year Mathematics**

* Students will learn to…
	+ ask a good (mathematical) question
	+ think critically about quantitative problems
	+ use abstraction
	+ develop problem solving strategies
* We want students to …
	+ Behave like mathematicians (thinking mathematically)
	+ Communicate mathematically

**Jumping off for next time**

* **Course 1:** For STEM Majors
* **Course 2**: For non-Majors

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**Wednesday August 10 - Our students in 2030**

We will try to envision who the potential students will be in 2030:

● How can we best respond to the constantly changing qualities of incoming university

students? What are these qualities?

● Students come to university for different reasons, but mostly to enhance their “market

value” ie, employability. However, instructors often believe that their students are

there because they like the discipline and are willing to learn. How to reconcile this

seeming contradiction/ tension?

● High school mathematics teachers face significant challenges in meeting learning

outcomes of the courses they teach. It is highly probable that, as a collateral damage

and for many years, teaching of mathematics at the post-secondary level will be

negatively affected by the fact that the incoming students will need even more time

and effort to bring their mathematical knowledge and skills to a required/desired

level.

*For Wednesday’s working group meeting, insert notes here*

**Our plan going forward for the next two days**

* What constraints do we want to consider? What will we ignore?
* Perhaps we build components that can be fit together to build many courses?
	+ Build the modules independent of what course it fits into
	+ *Focus on learning outcomes*
* One course or both courses?
	+ If one, then STEM (votes: i)
	+ If one, then non-STEM (votes: i)
	+ Let’s do both (votes: i)
* **What do we want to produce for Friday?**
	+ A course outline?
	+ A pitch to the department head?
	+ A marketing pitch for students?
	+ An experiment to evaluate if students are achieving these outcomes?
		- Compare to calculus?
	+ **A way to assess if students are achieving these outcomes**
		- **Meaningful measurement**

**What do we want our students to know & be able to do in 20 years?**

**Long list of goals / outcomes**

1. Employ logical reasoning
	* Formulate and evaluate logical arguments
	* Algorithmic thinking (computing/coding literacy)
2. Think mathematically
	* Pattern recognition / identify relationships
	* Abstraction (ability to see past specific situations) & generalization
	* Develop number sense (determine what is a reasonable result, big versus small, comparison in context, … )
3. Ask a good (mathematical) question
	* Develop mathematical curiosity (so what? what if?)
	* Conjecture
4. Think critically about quantitative problems
5. Develop problem solving strategies
	* Learn to be frustrated
	* Experiment / structured-play
	* Explore various approaches / representations
6. Explore proof versus evidence
7. Communicate mathematically (multiple representations of ideas)
8. Identify applications of mathematical ideas
9. Be inspired, have fun, find joy
10. Develop metacognitive awareness in mathematical context

**Teaching Content to achieve these goals (topics through which we achieve the goals)**

* Authentic applications
	+ Applications
* History of mathematics
* Logic puzzles, mathematical games, strategy
	+ Combinatorial games / single-player mathematical games
* Probability (e.g, Bayes theorem)
* Data visualization
	+ Math in the media
* Voting systems
* Finance
	+ Loan rates, investing, taxes, inflation, time value of money, tuition rates
* Discrete mathematics
	+ Graph theory
* Linear algebra
* Number theory
* Geometry
* Ethnomathematics
* Modeling
* Coding / programming
* Ethical use of technology/mathematics (e.g., “weapons of math destruction”)
* Anything else that you love that *isn’t* calculus (or calculus, if you must)

**Module One - Modular Arithmetic & Friends**

**Questions**

* How do you send a secret message? (Cryptography)
	+ E.g., here’s a code - decrypt it, how did we do it? is it good? can we make it better?

**Topics**

* Topic: cryptography
* Outcomes: asking questions, generalizing, patterns, exploring, logical thinking
* Topic: quadratic residues
* Outcomes: patterns, conjecture, asking questions
* Topic: modular arithmetic
* Outcomes: abstraction, exploration (structured play), logical thinking ((bi)conditional statement, examples, non-examples)

**Plan for Thursday**

* Calendar entry
* Marketing pitch
* Mechanism of course creation
	+ Online collaboration?
	+ Teaching release?
	+ Funding?
	+ Intensive workshop?
* Ways to assess effectiveness

**How does this connect to our future students in 2030?**

* Low-floor High-ceiling approach
	+ Welcoming to students with a variety of background and experience
	+ Rich experience and opportunity
	+ Lots of opportunity for success within this framework
* Prepare to welcome and support students from highschool
* Math for STEM students and for **future highschool teachers**
	+ These things could be taught in highschool

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**Thursday August 11 - Math curriculum in 2030**

Revamping mathematics curriculum: Re-think the organization of mathematics

curriculum; consider organizing it around big ideas in mathematics, and develop

important skills such as problem solving, logical reasoning and proof construction,

and communication of mathematical ideas. This is closely related to the emerging

pandemic and post-pandemic realities. There is no doubt that the knowledge of

mathematics (mathematical modelling, as well as applications of the outcomes of

emerging research in pure mathematics), will be even more dominant as a necessary

condition for a range of employment opportunities. How do we prepare our students

for that kind of world?

*For Thursday’s working group meeting, insert notes here*

**Plan for Today**

* Calendar entry
* Marketing pitch (to get into the calendar and count)
* Mechanism of course creation
	+ Online collaboration?
	+ Teaching release?
	+ Funding?
	+ Intensive workshop?
* Ways to assess effectiveness

**MATH 007 - Math for Survival**

**(What is it? “Introduction to Mathematics”)**

***First Try:*** *Through an active problem-based learning approach, students will develop mathematical habits of mind, including: how to communicate; how to explore, simulate and conjecture; in the mathematical world. Throughout this course students will develop inspiration and joy.*

***Second Take:*** *Students will develop knowledge, skills and attributes of mathematics as a human activity. This includes learning and doing mathematics through problem-solving, problem-posing, and exploration in a variety of contexts which will include components of applied and theoretical mathematics and statistics. Throughout this course, students will develop inspiration and joy. Students will flourish!*

**NB.** *Attributes means: attitudes, motivation, confidence, …*

**Friday plan**

* Marketing pitch (to get into the calendar and count for program credit in 2030)
* Plan for online collaboration to create the course modules tied to outcomes
	+ How to support instructors for teaching this
* **We need a presentation!**

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**Friday August 12 - So what will 2030 look like?**

Working groups present their work

*Before Friday’s working group presentation, let’s finalize the slides we previously created (e.g. adding some pictures, using concise wording, focusing on key takeaways, etc.). The slides can be found here:* [*https://docs.google.com/presentation/d/1pWaaYWaGT\_VoPbihiVCoOSfDSXb5OYAIL57s0-GLxdw/edit?usp=sharing*](https://docs.google.com/presentation/d/1pWaaYWaGT_VoPbihiVCoOSfDSXb5OYAIL57s0-GLxdw/edit?usp=sharing)