BIRS workshop 07w5104
The Many Strands of the Braid Groups
April 22–27, 2007

Final report

The meeting gathered mathematicians from nine countries: Brazil, Canada, France, Great Britain, Japan, Korea, New Zealand, Spain, United States. It was devoted to recent progress and new interactions involving Artin's braid groups (see below for a synopsis).

The very intense scientific program comprised thirty-two talks of twenty-five and forty-five minutes, plus a problem session, and a recollection of the work and life of X.S.Lin (1957–2007), to whose memory the meeting was dedicated. In order to take advantage of the connections between talks, the schedule was organized to provide homogeneous sessions, each devoted to one particular aspect.

According to the feedback from most participants, the meeting was a great success. In particular, the problem session opened promising perspectives for future developments (see below).

The more detailed report below has four parts.
1. A mathematical synopsis of the scientific theme of the meeting.
2. The schedule of the talks and their abstracts.
3. The list of questions raised in the problem session.
4. The complete list of participants to the meeting.

1 Braid groups: definitions and results

The braid groups B_n were introduced by E. Artin in 1926 [1] (see also [2]). They have been of importance in many fields – algebra, analysis, cryptography, dynamics, topology, representation theory, mathematical physics – and many of these aspects were represented in the BIRS workshop. This workshop involved not only leading experts in the field, but also, importantly, a number of young researchers, postdoctoral fellows and several graduate students. This made for an exciting and informative mix of ideas on the subject. Female mathematicians were well represented, and were among the leading contributors.

1.1 Many equivalent definitions

The importance of the braid groups is based, in part, on the many ways in which they can be defined. Below are six different definitions of the braid groups.

Definition 1: Braids as particle dances. Consider n particles located at distinct points in a plane. To be definite, suppose they begin at the integer points $\{1, \dots, n\}$ in the complex plane $\mathbb{C}$. Now let them move around in trajectories

$$\beta(t) = (\beta_1(t), \ldots, \beta_n(t)), \quad \beta_i(t) \in \mathbb{C}, \quad 0 \leq t \leq 1.$$

A *braid* is then such a time history with the proviso that the particles are noncolliding:

$$\beta_i(t) \neq \beta_j(t) \quad \text{if} \quad i \neq j$$

and end at the spots they began, but possibly permuted:

$$\beta_i(0) = i, \quad \beta_i(1) \in \{1, \ldots, n\}, \quad i = 1, \ldots, n.$$

If one braid can be deformed continuously into another (through the class of braids), the two are considered equivalent − we will say equal.

Braids α and β can be multiplied: one dance following the other, each at double speed. The product is associative but not in general commutative. The identity dance is to stand still, and each dance has an inverse; doing the dance in reverse time. These (deformation classes of) dances form the group B_n.

A braid β defines a permutation $i \to \beta_i(1)$ which is a well-defined element of the permutation group Σ_n. This is a homomorphism with kernel, by definition, the subgroup P_n of *pure* braids. P_n is sometimes called the *colored* braid group, as the particles can be regarded as having identities, or colors. P_n is of course normal in B_n, of index $n!$, and there is an exact sequence

$$1 \to P_n \to B_n \to \Sigma_n \to 1.$$

Definition 2: Braids as strings in 3-D. This is the usual and visually appealing picture. A braid can be viewed as the graph, or timeline, of a braid as in the first definition, drawn in real x, y, t-space, monotone in the t direction. The complex part is described as usual by $x + y\sqrt{-1}$. The product is then a concatenation of braided strings.

This viewpoint provides the connection with knots. A braid β defines a knot or link $\hat{\beta}$, its closure, by connecting the endpoints in a standard way so that no new crossings are introduced. J. W. Alexander showed that all knots arise as the closure of some braid and by a theorem of Markov (see [5] for a discussion and proof) two braids close to equivalent knots if and only if they are related by a finite sequence of moves and their inverses: conjugation in the braid group and a stabilization, which increases the number of strings.

Definition 3: B_n as a fundamental group of a configuration space. In complex n-space $\mathbb{C}^n$ consider the big diagonal

$$\Delta = \{(z_1, \ldots, z_n); \quad z_i = z_j, \quad \text{some} \quad i < j\} \subset \mathbb{C}^n.$$

Using the basepoint $(1, 2, \ldots, n)$, we see that

$$P_n = \pi_1(\mathbb{C}^n \setminus \Delta).$$

In other words, pure braid groups are fundamental groups of complements of a special sort of complex *hyperplane arrangement*, itself a deep and complicated subject.

To get the full braid group we need to take the fundamental group of the *configuration space*, of orbits of the obvious action of Σ_n upon $\mathbb{C}^n \setminus \Delta$. Thus

$$B_n = \pi_1((\mathbb{C}^n \setminus \Delta)/\Sigma_n).$$

Notice that since the singularities have been removed, the projection

$$\mathbb{C}^n \setminus \Delta \longrightarrow (\mathbb{C}^n \setminus \Delta)/\Sigma_n$$

is actually a covering map. As is well-known, covering maps induce injective homomorphisms at the π_1 level, so this is another way to think of the inclusion $P_n \subset B_n$.

Finally, we note that the space $(\mathbb{C}^n \setminus \Delta)/\Sigma_n$ can be identified with the space of all complex polynomials of degree n which are monic and have n distinct roots

$$p(z) = (z - r_1) \cdots (z - r_n).$$

This is one way in which the braid groups play a role in classical algebraic geometry, as fundamental group of the space of such polynomials.

Definition 4: The algebraic braid group. B_n can be regarded algebraically as the group presented with generators $\sigma_1, \ldots, \sigma_{n-1}$, where σ_i is the braid with one crossing, with the string at level i crossing over the one at level $i+1$ and the other strings going straight across.

These generators are subject to the relations

$$\sigma_i \sigma_j = \sigma_j \sigma_i, \quad |i - j| > 1,$$

$$\sigma_i \sigma_j \sigma_i = \sigma_j \sigma_i \sigma_j, \quad |i - j| = 1.$$

We can take a whole countable set of generators $\sigma_1, \sigma_2, \ldots$ subject to the above relations, to define the infinite braid group B_∞. If we consider the (non-normal) subgroup generated by $\sigma_1, \ldots, \sigma_{n-1}$, these algebraically define B_n. Notice that this convention gives "natural" inclusions $B_n \subset B_{n+1}$ and $P_n \subset P_{n+1}$.

Definition 5: B_n as a mapping class group. Going back to the first definition, imagine the particles are in a sort of planar jello and pull their surroundings with them as they dance about. Topologically speaking, the motion of the particles extends to a continuous family of homeomorphisms of the plane (or of a disk, fixed on the boundary). This describes an equivalence between B_n and the mapping class of D_n, the disk D with n punctures (marked points). That is, B_n can be considered as the group of homeomorphisms of D_n fixing ∂D and permuting the punctures, modulo isotopy fixing $\partial D \cup \{1, \ldots, n\}$.

Definition 6: B_n as a group of automorphisms. A mapping class $[h]$, where $h : D_n \to D_n$, gives rise to an automorphism $h_* : F_n \to F_n$ of free groups, because F_n is the fundamental group of the punctured disk. Using the interpretation of braids as mapping classes, this defines a homomorphism

$$B_n \to Aut(F_n),$$

which Artin showed to be faithful, i. e. injective.

The generator σ_i acts as

$$x_i \to x_i x_{i+1} x_i^{-1}; \quad x_{i+1} \to x_i; \quad x_j \to x_j, \quad j \neq i, i+1.$$

Thus B_n may be considered a group of automorphisms of $Aut(F_n)$ satisfying a condition made precise by Artin.

1.2 Representations of the braid groups

One of the most active aspects of braid theory is the study of linear representations. A major breakthrough has been the proof in 2000 by S. Bigelow [4] and D. Krammer [15] of the long-standing conjecture that Artin's braid groups B_n are linear groups. That is, there exists a faithful representation of B_n in a finite-dimensional linear group. The Lawrence–Krammer representation that provides a linear representation of B_n has dimension $n(n-1)/2$. After the result was established, considerable efforts have been made to better understand the algebraic underlying socle on which the representations arise. The general question is to identify the non-trivial finite-dimensional quotients of the group algebra $\mathbb{C}B_n$, on the shape of the Iwahori–Hecke algebra investigated in the past decades. The general philosophy is: the bigger the quotient algebra, the better the results. Until recently, the biggest known algebra was the Birman–Murakami–Wenzl algebra [7].

I. Marin discussed the image of representations of the braid groups and certain generalizations in $GL(N)$ and showed that their images are Zariski-dense. This has important algebraic consequences, discussed in his abstract listed below.

By way of representations which are not linear, F. Castel reviewed certain faithful representations of B_n in mapping class groups of surfaces. He showed that in some sense, these constitute all the possible embeddings, utilizing Nielsen-Thurston theory of surface automorphisms and rigidity of the embeddings involved.

1.3 Applications to knot theory

The most obvious applications of braid theory are to the study of knots. About two decades ago, work of V. Jones [14] established a new powerful knot invariant via representations of B_n. This work led to exciting and unsuspected connections with operator theory, statistical mechanics and other aspects of mathematical physics. It was also generalized to the so-called HOMFLYPT polynomial, the Kauffman polynomial and a plethora of other knot invariants.

An outstanding open question is whether the Jones polynomial detects the unknot. In other words, if the Jones polynomial $V_K(t)$ of a knot K is trivial, does it imply that K is unknotted? The corresponding question for links of two or more components was settled very recently by Eliahou, Kauffman and Thistlethwaite [10], who displayed infinite families of links with the same Jones polynomial as the unlink, but which are nontrivially linked.

It is also well-known that there are many examples of distinct knots with the same Jones (and HOMFLYPT) polynomial, using various techniques: Conway mutation, a construction of Kanenobu (producing an infinite family with common Jones polynomial), etc. H. Morton showed that, in the presence of extra symmetry, mutant knots have satellites which (unlike knots in general) also cannot be distinguished by their HOMFLYPT polynomials. Representation theory provided the tool for Morton's proof.

J. Birman spoke of the fascinating connection between a certain family of knots which arise in dynamical systems, called Lorenz knots, and number theory. These knots were originally studied as closed trajectories of a 3-dimensional dynamical system defined by the meteorologist E. Lorenz in 1963, contained in a celebrated "strange attractor." Work by Etienne Ghys, showing that they arise in a certain "modular flow" has inspired renewed interest in this family of knots. A wonderful exposition of this is in [13].

1.4 Knot homology theories

It was shown recently by Khovanov that the Jones polynomial can be considered as a sort of Euler characteristic of a homology theory related to a given knot. Several of the talks focussed on Khovanov theory, including a presentation by L. Watson of knots which cannot be distinguished by their Khovanov homology and a proof of a functoriality property by S. Morrison. Przytycki described a relationship between Khovanov homology and the more classical Hochschild homology theory. Another recent, and very fruitful, development in low-dimensional topology is Heegaard-Floer homology. Originally defined using methods of complex analysis, a new combinatorial version of this homology theory was presented by D. Thurston at the meeting.

1.5 Three-dimensional manifolds and TQFT's

Topological quantum field theory was codified by Atiyah [3] and Witten [16] in 1988. Witten showed that the Jones polynomial, originally defined using representations of the braid groups, could also be expressed as a certain configuration space integral. One of the most important tools in the study of 3-manifolds is the Casson invariant $\lambda(M)$, defined by A. Casson for any integral homology 3-sphere M. The original definition by Casson in 1984 involved counting $SU(2)$ representations of the fundamental group of M. G. Kuperberg and D. Thurston showed, in 1999, how to express $\lambda(M)$ as a configuration space integral. Other new invariants of manifolds have been devised using TQFT methods, and the connection between TQFT and braid theory remains an active area of research. Loop spaces of configuration spaces were shown at the meeting by T. Kohno to be instrumental in developing new invariants of knots and links.

Finite-type invariants, following Vassiliev, have been extremely important in the study of knots and 3-manifolds. C. Lescop presented surgery formulas for finite-type invariants associated with rational homology 3-spheres, that is, orientable 3-manifolds with trivial rational homology in dimension one.

1.6 Braids, combinatorics and algorithms

A very active area which was well-represented at the conference concerns ideas surrounding Garside's 1969 solution to the word and conjugacy problems in the braid groups [12]. An equivalent way of describing the framework is to introduce the notion of a Garside groupoid (small catgeory where all arrows are invertible). Technically, an extended Garside structure is specified by axiomatizing the intervals $[a, a\Delta]$ of a Garside monoid, where Δ is a Garside element. The main interest of this extended framework is to make it possible to define completely new Garside structures on braid groups — and, possibly, on more general mapping class groups, but this remains a conjecture. The construction starts with considering the braid group B_n as acting on a disk with n punctures, as in Definition 5 above.

Now, the new ingredient is to add q marked points on the boundary circle. By considering certain cell decompositions of such "bi-punctured" disks (punctures in the interior and on the boundary) up to isotopy, one obtains a lattice and, under a convenient version of Dehn's half-twist in which the boundary punctures are shifted, one obtains an action of the braid group B_n on that lattice. In the case $q = 2$ (only the North and the South poles of the disk are marked), the action is simply transitive, and one obtains the standard Garside structure of B_n. For $q \geq 3$, the action is not transitive, and one obtains a completely new structure. In particular, for $q = 3$ (3 punctures on the boundary disk), the lattice can be described explicitly, and, surprisingly enough, the famous MacLane pentagon shows up, and, more generally, the intervals $[a, a\Delta]$ are closely related with the Stasheff associahedra. This opens a new, fascinating connection between Artin's braid group and Richard Thompson's groups, and certainly much more is still to come.

The word and conjugacy problems in the braid groups have importance for their role in public key cryptography. It is well known that the complexity of the word problem in the braid group B_n is $(|W|^2 n)$, where $|W|$ is word length and n is braid index, whereas all solutions to the conjugacy problem known at this time are exponential. Codes have been designed which are based on the assumption that the conjugacy problem is fundamentally exponential, so a polynomial solution to the conjugacy problem would be of major importance. J. Gonzalez-Meneses outlined an ambitious program, with J. Birman and V. Gebhardt, to develop a polynomial-time algorithm to solve the conjugacy problem in braid groups, as well as the closely-related conjugacy search problem.

1.7 Generalizations of the braid groups

Because of the many definitions of the braid groups, there are various natural ways to generalize them, some of which have far-reaching applications. Several such generalizations were considered in the BIRS workshop, namely Artin groups (an algebraic generalization), mapping class groups (also known as modular groups), configuration spaces and their algebraic properties.

1.8 Artin groups and reflection groups

Deligne [8] and Brieskorn-Saito [6], introduced a family now referred to as Artin groups, which generalizes the braid groups and is also closely related to the so-called Coxeter groups which arise in the study of Lie groups and symmetries of Euclidean space. For a fixed positive integer n, consider an n by n matrix $M = \{m_{ij}\}$, where m_{ij} is a positive integer or ∞, with the assumption that $m_{ij} = m_{ji} \geq 2$ and $m_{ii} = 1$. The corresponding Artin group has a presentation with generators $x_1, \ldots, x_n$ and, for each pair i, j there is a relation:

$$x_i x_j x_i \cdots = x_j x_i x_j \cdots$$

where the product on each side has length m_{ij} ($m_{ij} = \infty$ indicates no relation is present). If one adjoins relations $x_i^2 = 1$, the result is the so-called Coxeter group corresponding to the given matrix.

In this context, the $n+1$ by $n+1$ matrix with entries equal to 3 just above and below the diagonal, and 2 in entries farther from the diagonal, corresponds exactly to the braid group B_n; in this case the Coxeter group is the symmetric group Σ_n. The Artin groups for which the corresponding Coxeter group is finite are an important subclass, referred to as "spherical." As with the braid groups, Artin groups of spherical type correspond to fundamental groups of configuration spaces associated to hyperplane arrangements.

The finite Coxeter groups can be considered as groups of reflections of $\mathbb{R}^n$, acting on configuration spaces, as described in Definition 3 for the case of the braid groups. Periodic elements in the sphrical Artin groups were described at the meeting by D. Bessis. B. Wiest outlined algorithmic solutions of the conjugacy problem in an important class of Artin groups, called right-angled as their corresponding reflection groups involve right angles.

1.9 Mapping class groups

The mapping class group $Mod(S)$ of an orientable surface S is well-known to be generated by Dehn twists about simple closed curves in S. An important subgroup of this is the Torelli subgroup, consisting of (classes of) homeomorphisms which induce the identity on the homology of S. In particular, the subgroup K of $Mod(S)$ generated by twists along separating curves of S, called the Johnson kernel, lies in the Torelli subgroup. An important advance in our understanding of this family of groups was described by D. Margalit, who gave an explicit calculation of the cohomological dimension of the Torelli group. Another aspect which promises to be quite fruitful was discussed by D. Kraamer, who showed that the Torelli groups can be analyzed using a structure similar to that used by Garside to solve the conjugacy problem in braid groups.

J. Marché's talk dealt with so-called quantum representations of $Mod(S)$, showing that asymptotically they are faithful, converging in a certain sense to the space of regular functions on a certain character variety.

1.10 Surface braid groups, string links and orderings

If S is a Riemann surface, one can consider braids in the product of S with an interval, just as classical braids are defined over the disk. This defines surface braid groups $B_n(S)$, codified by Fox and Neuwirth [11] in 1962. Many aspects of these surface braid groups are still not well understood. It is known that the only surfaces whose braid groups contain elements of finite order are the sphere and projective plane. At the conference J. Guaschi and D. Goncalves described their recent compilation of exactly which finite groups which can occur as subgroups of the braid group of the sphere.

String links are another generalization of braid groups, in which the strands are no longer required to be monotone in the "time" direction. Just as braids, they may be multiplied by concatenation, but they no longer form a group, as inverses do not always exist. N. Habegger and X.-S. Lin (in whose memory the conference was dedecated) showed that under J. Milnor's notion of link homotopy, in which one allows strands to pass through themselves but not each other, string links do form a group. They also derived an algorithm for comparing string links. The student K. Yurasovskaya discussed her recent work, showing that the groups of homotopy string links can be endowed with a strict total ordering which is invariant under left-multiplication. This was inspired by the celebrated result of P. Dehornoy that the classical braid groups are left-orderable. Another student, A. Clay, presented his work showing the seemingly paradoxical result that, although the Dehornoy ordering is discrete (in the sense of orderings), certain important subgroups – for example the commutator subgroup of B_n are order-dense, under the same ordering. An important open question is whether $B_n(S)$ is left-orderable for surfaces of positive genus.

The above discussion is just a sample of the progress in braid theory discussed at the BIRS conference. Further details may be found in the abstracts which are given below.

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2 The talks

2.1 General organization

As mentioned above, we managed to organize, as much as possible, homogeneous sessions. Roughly speaking, the themes were as follows:

- Monday morning: applications of braids to knot theory (3 talks),
- Monday afternoon: algebraic properties of braids (4 talks),
- Tuesday morning: connections with Heegard and Floer homology (4 talks),
- Tuesday afternoon: geometric aspects of braids (5 talks),
- Wednesday morning: Garside's theory of braids (4 talks),
- Thursday morning: braids and mapping class groups (4 talks),
- Thursday afternoon: more on Garside's theory (4 talks),
- Friday morning: geometric and ordered aspects of braids (4 talks).

2.2 Schedule

Here is the complete schedule and the abstracts of the talks.

Monday

9:15–9:45	Recollection of X.Lin and his work, by Joan Birman
10:00–10:25	H.Morton, Mutants with symmetry
11:00–11:45	H.Murakami, On a generalization of the volume conjecture
12:00–12:25	G.Zhang, Concordance crosscap number of a knot
14:45–15:30	P.Dehornoy, Alternating normal forms of braids
16:00–16:25	A.Clay, Normal subgroups of the braid groups and the Dehornoy ordering
16:45–17:30	J.Guaschi & D.Goncalves, Finite subgroups of the sphere braid groups
17:45–18:10	S.Humphries, Subgroups of braid groups generated by powers of Dehn twists

Tuesday

9:00–9:45	J.Przytycki, Two–braid intersection of Hochschild and Khovanov homologies
10:00–10:25	L.Watson, Knots with identical Khovanov homology
10:50–11:35	D.Thurston, Combinatorial Heegard–Floer homology for knots via grid diagrams
11:45–12:30	S.Morrison, Functoriality for Khovanov homology in S^3
14:45–15:30	C.Lescop, Surgery formulae for finite type invariants of rational homology 3–spheres
16:00–16:25	W.Menasco, A calculus for Legendrian and transversal knots
16:35–17:00	R.Fenn, Welded braids, links, their configuration spaces and other properties
17:10–17:35	H.Matsuda, A calculus on links via closed braids
17:45–18:10	J.Birman, Lorenz knots, templates and closed braids

Wednesday

9:00–9:45	D.Krammer, A Garside type structure on the Torelli group
10:00–10:25	D.Margalit, Dimension of the Torelli group
10:50–11:35	D.Bessis, Periodic elements in spherical type Artin groups
11:45–12:30	J.Gonzalez–Meneses, A project to find a polynomial solution to the conjugacy problem in braid groups

Thursday

9:00–9:45	J.Marché, On asymptotics of quantum representations of mapping class groups via skein theory
10:00–10:25	K.Kawamuro, Braid index and algebraic crossing number
11:00–11:45	F.Castel, Rigidity of the representations of the braid group in the mapping class group
12:00–12:25	S.Kamada, On braid presentation of knotted surfaces and the enveloping monoidal quandle

14:45–15:30	S.Lee, Translation numbers in Garside groups
16:00–16:25	E.Lee, Super summit property of abelian subgroups of Garside groups
16:45–17:30	I.Marin, Generalized braid groups as Zariski–dense subgroups of GL_N
17:40–18:05	B.Wiest, The conjugacy problem in right–angled Artin groups and their subgroups
20:00–21:30	Problem session

Friday

9:00–9:25	E.Kin, The ratio of the topological entropy to the volume for pseudo–Anosov braids
9:40–10:05	T.Kohno, Loop spaces of configuration spaces and link invariants
10:30–10:55	E.Yurasovskaya, String links and orderability
11:00–11:25	D.Rolfsen, Ordered groups and pseudo-Anosov maps

2.3 Abstracts

Here are (in alphabetic order by speaker surname) the abstracts of these talks.

Speaker: **David Bessis** (Ecole Normale Supérieure, Paris, France)
Title: *Periodic elements in spherical type Artin groups*
Abstract: In the braid group on n strings, the classification of periodic elements (elements with a central power) follows from a classical theorem of Kerekjarto. We generalize this to the other spherical type Artin groups and obtain a complete description of periodic elements, their conjugacy classes and their centralizers. A key ingredient is a categorical reformulation of a theorem by Bestvina.

Speaker: **Joan Birman** (Columbia University, New York, USA)
Title: *Lorenz knots, templates and closed braids*
Abstract: Lorenz knots were first defined in a 1983 paper that Bob Williams and I wrote. They arise as the periodic orbits in the flow associated to solutions to a particular ODE in 3-space which has since become a paradigm for chaos. They are of renewed interest right now because of work by Etienne Ghys, who proved that the identical family of knots (and their defining 'template') are as the closed orbits in the classical modular flow on the complement of the trefoil knot.

Speaker: **Fabrice Castel** (Université de Dijon, France)
Title: *Rigidity of the representations of the braid group in the mapping class group*
Abstract: In 1995, Perron and Vannier proved that the morphism from the braid group into the mapping class group of an orientable surface, that sends the generators of the braid group on Dehn twists, is injective. We show that under some restrictions on the genus, the embeddings between these two groups

arise all from the embedding defined by Perron and Vannier. Using the rigidity of such embeddings, one can for instance compute the group of automorphisms of the braid group as well as the group of automorphisms of the mapping class group. The proof of the theorem is based on Nielsen-Thurston theory and of a simultaneous action of the mapping class group on itself, on the complex of curves and on the complex of subsurfaces.

Speaker: **Adam Clay** (University of British Columbia, Vancouver, Canada)
Title: *Normal subgroups of the braid groups and the Dehornoy ordering*
Abstract: The braid groups admit a left-ordering, discovered by Dehornoy, which is discrete as an ordering. I will show that normal subgroups interact with the Dehornoy ordering in such a way that "nearly all" normal subgroups of the braid groups are densely ordered with respect to this ordering. In particular, some popular normal subgroups–such as the commutator subgroup and kernels of the Burau representations–can be easily analyzed. This is joint work with Dale Rolfsen.

Speaker: **Alissa Crans** (Loyola Marymount University, Los Angeles, USA)—talk cancelled due to illness
Title: *Analogues of self-distributivity*
Abstract: This is joint work with Scott Carter, Mohamed Elhamdadi, and Masahico Saito. Self-distributive binary operations have appeared extensively in knot theory in recent years, specifically in algebraic structures called 'quandles.' A quandle is a set equipped with two binary operations satisfying axioms that capture the essential properties of the operations of conjugation in a group. The self-distributive axioms of a quandle correspond to the third Reidemeister move in knot theory. Thus, quandles give a solution to the Yang-Baxter equation, which is an algebraic distillation of the third Reidemeister move. We formulate analogues of self-distributivity in the categories of coalgebras and Hopf algebras and use these to construct additional solutions to the Yang-Baxter equation.

Speaker: **Patrick Dehornoy** (Université de Caen, France)
Title: *Alternating normal forms of braids*
Abstract: We describe new types of normal forms for braid monoids, Artin-Tits monoids, and, more generally, all monoids in which divisibility has some convenient lattice properties ("locally Garside monoids"). We show that, in the case of braids, one of these normal forms turns out to coincide with the normal form introduced by Burckel and deduce that the latter can be computed easily. This approach leads to a new, simple description for the canonical well-order of B_n^+ in terms of that of B_{n-1}^+ which, in turn, leads to unprovability statements for certain games involving braids.

Speaker: **Roger Fenn** (University of Sussex, Brighton, GB)
Title: *Welded braids, links, their configuration spaces and other properties*
Abstract: Configuration spaces of the classical braids are well known. A configuration space for welded braids is given with a suggestion for possible invariants.

Speaker: **Daciberg Lima Goncalves** (Universidade de Sao Paulo, Brazil) & **John Guaschi** (Université de Toulouse, France)
Title: *Finite subgroups of the sphere braid groups*
Abstract: It is well known that the sphere braid groups $B_n(S^2)$ have torsion elements. Such elements were characterised by Murasugi. In this talk, we classify the finite subgroups of $B_n(S^2)$. Our work is partly motivated by the study of the generalisation of the Fadell-Neuwirth short exact sequence for pure braid groups to the 'mixed' subgroups of the full braid groups. By giving explicit constructions, we prove that for all $n \geq 3$, $B_n(S^2)$ contains subgroups isomorphic to the dicyclic groups of order $4n$ and $4(n-2)$. It follows that $B_n(S^2)$ contains two non-conjugate copies of the quaternion group of order 8 for all $n \geq 4$ even, one of which lies in the commutator subgroup of $B_n(S^2)$, the other not. Finally we classify the finite subgroups of $B_n(S^2)$: the maximal finite subgroups of $B_n(S^2)$ are either cyclic, dicyclic or binary polyhedral groups (their realisation depending on n). Two corollaries of this classification are: a) the binary tetrahedral group is a subgroup of $B_n(S^2)$ for all $n \geq 4$ even; b) if n is odd then the finite subgroups of $B_n(S^2)$ are cyclic or dicyclic.

Speaker: **Juan Gonzalez-Meneses** (University of Seville, Spain)
Title: *A project to find a polynomial solution to the conjugacy problem in braid groups*
Abstract: This is a joint work with Joan S. Birman and Volker Gebhardt. We present a project to find a polynomial solution to the conjugacy decision problem and the conjugacy search problem in braid groups, whose outline is the following. First we need to determine the geometric type of the braids involved, that is, to classify a given braid as periodic, reducible or pseudo-Anosov. In the periodic case, we give a polynomial solution by using some Garside structures of the braid groups and of Artin-Tits groups of type B. In the reducible case, one needs to find the reducing curves, and also to solve the question of finding the generators of the centralizer of a braid. In the pseudo-Anosov case, we show how one can simplify the situation by taking powers of the original braids, and reducing the problem to the conjugacy search problem for "rigid" braids. We will present our achievements, together with the open problems that remain.

Speaker: **Stephen Humphries** (Brigham Young University, Utah, USA)
Title: *Subgroups of braid groups generated by powers of Dehn twists*
Abstract: Let $F =< x_1, \ldots, x_n >$ be the free group on n generators and let $P_n =< A_{12}, \ldots, A_{n-1,n} >$ be the pure braid group with its standard (Dehn twist) generators. We identify F_n with the subgroup $< A_{1,n+1}, \ldots, A_{n,n+1} >$ of P_{n+1}. We are interested in the related questions: (1) when is a subgroup of F_n which is generated by a set of powers of conjugates of $x_1, \ldots, x_n$, of finite index in F_n; and (2) when is a subgroup of P_n which is generated by a set of powers of conjugates of $A_{12}, \ldots, A_{n-1,n}$, of finite index in P_n? For example, we give necessary and sufficient conditions for a subgroup of P_n of the form $< A_{12}^{e_{12}}, \ldots, A_{n-1,n}^{e_{n-1,n}} >$ to have finite index in P_n. The answer to question (1) involves Schur's theory of S-rings.

Speaker: **Seiichi Kamada** (Hiroshima University, Japan)
Title: *On braid presentation of knotted surfaces and the enveloping monoidal quandle*
Abstract: We introduce a method to describe a knotted surface in 4-space by a sequence of braids, Alexander's and Markov's theorem in dimension 4. It is natural to regard such a sequence as an element of the enveloping monoidal quandle in the sense of Kamada and Matsumoto.

Speaker: **Keiko Kawamuro** (Rice University, Houston, USA)
Title: *Braid index and algebraic crossing number*
Abstract: I will discuss a conjecture that the maximal Bennequin number of a knot is realized at its minimal braid representatives.

Speaker: **Eiko Kin** (Tokyo Institute of Technology, Japan)
Title: *The ratio of the topological entropy to the volume for pseudo–Anosov braids*
Abstract: We consider two invariants of pseudo-Anosov mapping classes. One is the dilatation of pseudo–Anosov homeomorphisms and the other comes from the volume of mapping tori. Both invariants measure a kind of complexity of pseudo–Anosov mapping classes. The mapping class group on the n–punctured disk is identified with the n–braid group up to full twist braids, and it makes to sense to speak of the dilatation and the volume for pseudo–Anosov braids. We are interested in a relation of these two invariants, the dilatation and the volume. In this talk we focus on the ratio of the logarithm of the dilatation namely the (topological) entropy to the volume. We show that there is a constant $c > 0$ such that the ratio of the entropy to the volume for the pseudo–Anosov 3–braids is greater than c. We also extend this result for a family of pseudo–Anosov braids with many strands. This is a joint work with Mitsuhiko Takasawa (Tokyo Institute of Technoloty).

Speaker: **Toshitake Kohno** (University of Tokyo, Japan)
Title: *Loop spaces of configuration spaces and link invariants*
Abstract: It is know by F. Cohen and S. Gitler that the homology of the loop spaces of configuration spaces of ordered points in the Euclidean space is a graded algebra defined by infinitesimal pure braid relations. Based on this result we give a description of a link homotopy invariant as an integral of de Rham cohomology class of the loop space of a configuration space.

Speaker: **Daan Krammer** (University of Warwick, GB)
Title: *A Garside type structure on the Torelli group*
Abstract: In 1969, Garside solved the word and conjugacy problems for braid groups. We now say that he proved braid groups to be Garside groups. In 1998 another Garside structure on the braid group was discovered by Birman-Ko-Lee (BKL).

A well-known class of groups generalising braid groups are the surface mapping class groups. The Torelli group of a surface is the subgroup of the mapping class group of those elements which act trivially on the first homology $H_1(S, Z)$ of the surface.

I will present a Garside type structure on the Torelli group. It depends on the choice of a lexicographic total ordering on $H_1(S, Z)$. It is a close relative of the BKL Garside structure on the braid group.

It is not precisely a Garside structure for a number of reasons:

(1) Rather than as a group, it should be regarded as a groupoid whose object set looks a lot like a topological space;

(2) The distinguished path between two points in general has an infinite number of intermediate stops in a mild way.

Still, the most important properties of Garside groups, such as the grid property, still hold.

Speaker: **Eon-Kyung Lee** (Sejong University, Seoul, Korea)
Title: *Super summit property of abelian subgroups of Garside groups*
Abstract: Garside groups provide a lattice-theoretic generalization of braid groups and finite type Artin groups. In the talk, we show that for every abelian subgroup H of a Garside group, some conjugate $x^{-1}Hx$ consists of super summit elements. Using this property, we show that the centralizer of H is a finite index subgroup of the normalizer of H. Combining with the results on translation numbers in Garside groups, we obtain an easy proof of the algebraic flat torus theorem for Garside groups.

Speaker: **Sangjin Lee** (Konkuk University, Korea)
Title: *Garside groups and translation numbers*
Abstract: The translation number of an element in a combinatorial group is defined as the asymptotic word length of the element. The discreteness properties of translation numbers have been studied for geometric groups such as biautomatic groups and hyperbolic groups. The Garside group is a lattice-theoretic generalization of braid groups and Artin groups of finite type. In this talk, we discuss recent results on the discreteness properties of translation numbers in Garside groups, and their applications to the conjugacy problem.

Speaker: **Christine Lescop** (Université of Grenoble, France)
Title: *Surgery formulae for finite type invariants of rational homology* 3*-spheres*
Abstract: I wish to present four graphic surgery formulae for the degree n part Z_n of the Kontsevich-Kuperberg-Thurston universal finite type invariant of rational homology spheres. Each of these four formulae determines an alternate sum of the form $\sum_{I \subset N}(-1)^{\sharp I} Z_n(M_I)$ where N is a set of disjoint operations to be performed on a rational homology sphere M, and M_I denotes the manifold resulting from the operations in I. The first formula treats the case when N is a set of $2n$ Lagrangian-preserving surgeries (a *Lagrangian-preserving surgery* replaces a rational homology handlebody by another such without changing the linking numbers of curves in its exterior). In the second formula, N is a set of n rational surgeries on the components of a boundary link. The third formula deals with the case of $3n$ surgeries on the components of an algebraically

split link. The fourth formula is for $2n$ surgeries on the components of an algebraically split link in which all Milnor triple linking numbers vanish. In the case of homology spheres, these formulae can be seen as a refinement of the Garoufalidis-Goussarov-Polyak comparison of different filtrations of the rational vector space freely generated by oriented homology spheres (up to orientation-preserving homeomorphisms).

Speaker: **Julien Marché** (Université Paris 7, France)
Title: *On asymptotics of quantum representations of mapping class groups via skein theory*
Abstract: We explain a simple proof of the asymptotic faithfulness of quantum representations of the mapping class group of a surface S. The idea is to show that in some sense, the quantum representations converge to the representation $H(S)$, where $H(S)$ is the space of regular functions on the character variety of S in $SL(2, C)$.

Speaker: **Dan Margalit** (University of Utah, USA)
Title: *Dimension of the Torelli group*
Abstract: In joint work with Mladen Bestvina and Kai-Uwe Bux, we prove that the cohomological dimension of the Torelli group for a closed surface of genus g at least 2 is equal to $3g - 5$.

Speaker: **Ivan Marin** (Université Paris 7, France)
Title: *Generalized braid groups as Zariski-dense subgroups of GL_N*
Abstract: Embeddings of every (irreducible) spherical-type Artin group in some GL_N have been described in recent years. We show that these embedings have Zariski-dense image, and use this to prove group-theoretical results on Artin groups. In particular we show that these groups are residually torsion-free nilpotent, and compute their Frattini and Fitting subgroups. We also generalize a classical result of D. Long which says that normal subgroups of braid groups which are not included in the center intersect non-trivially. The density result is based on a simple interpretation of these embeddings as monodromy representations, that we shall describe if time permits.

Speaker: **Hiroshi Matsuda** (Columbia University, New York, USA)
Title: *A calculus on links via closed braids*
Abstract: We improve "Markov Theorem Without Stabilization" of Birman and Menasco.

Speaker: **William Menasco** (University at Buffalo, USA)
Title: *A calculus for Legendrian and transversal knots*
Abstract: Using an extended example of the Etnyre-Honda (2,3) cabling of the (2,3) torus knot we discuss a calculus of isotopies associated with Legendrian and transversal knots in the standard contact structure of S^3 (Joint work with Douglas Lafountain, University at Buffalo).

Speaker: **Scott Morrison** (University of California, Berkeley, USA)
Title: *Functoriality for Khovanov homology in* S^3
Abstract: (Joint work with Kevin Walker.) I'll tell you what I mean by the Khovanov homology of a knot in S^3 (as opposed to the usual B^3). We can show that Khovanov homology is still functorial in this case, but it takes a bit more work beyond checking the 15 movie moves needed for functoriality in B^3.

Speaker: **Hugh Morton** (University of Liverpool, GB)
Title: *Mutants with symmetry*
Abstract: Mutants with certain extra symmetry, for example the pretzel knots $K(a_1, ..., a_k)$ with k and all a_i odd, can be shown to share many more of their Homfly satellite invariants than is the case for a general mutant. The proofs make use of representation theory of quantum $sl(N)$ modules.

Speaker: **Hitoshi Murakami** (Tokyo Institute of Technology, Japan)
Title: *On a generalization of the volume conjecture*
Abstract: The volume conjecture says that the large N limit of the N-colored Jones polynomial of a knot evaluated at the N-th root of unity would determine the volume of the knot complement. In this talk we will consider what happens if we change the evaluation.

Speaker: **Jozef Przytycki** (George Washington University, Washington DC, USA)
Title: *Two-braid intersection of Hochschild and Khovanov homologies*
Abstract: We show that Khovanov homology and Hochschild homology theories share common structure. In fact they overlap: Khovanov homology of a $(2, n)$-torus link can be interpreted as a Hochschild homology of the algebra underlining the Khovanov homology. In the classical case of Khovanov homology we prove the concrete connection. In the general case of Khovanov-Rozansky, $sl(n)$, homology and their deformations we conjecture the connection. The best framework to explore our ideas is to use a comultiplication-free version of Khovanov homology for graphs developed by L. Helme-Guizon and Y. Rong and extended here to M-reduced case, and in the case of a polygon to noncommutative algebras. In this framework we prove that for any unital algebra A the Hochschild homology of A is isomorphic to graph cohomology over A of a polygon.

Speaker: **Dale Rolfsen** (University of British Columbia, Vancouver, Canada)
Title: *Ordered groups and pseudo-Anosov maps*
Abstract: This is a report on work in progress regarding finding orderings of groups invariant under a given automorphism. One goal is to show that for a pseudo-Anosov homeomorphism of a surface, there is an ordering of the surface group invariant under the action of the induced mapping. This would imply the bi-orderability of fundamental groups of hyperbolic 3-manifolds which fibre over the circle.

Speaker: **Dylan Thurston** (Barnard College, Columbia University, New York, USA)
Title: *Combinatorial Heegaard-Floer homology for knots via grid diagrams*
Abstract: We give a combinatorial definition of Heegaard-Floer homology. In particular, this yields a very simple algorithm for computing the knot genus. Our method is based on grid diagrams, a representation for knots that, with restrictions on the allowed moves, also yields transverse or Legendrian knots or closed braids up to isotopy.

Speaker: **Liam Watson** (Université du Québec Montréal, Canada)
Title: *Knots with identical Khovanov homology*
Abstract: While it is well know that mutation is not detected by the Jones polynomial, it is presently unknown if mutation of knots preserves Khovanov homology. In this talk we will present a technique for producing pairs of distinct knots that cannot be distinguished by Khovanov homology. As an application, this construction may be applied to produce families of examples of mutant pairs that have identical Khovanov homology.

Speaker: **Bert Wiest** (Université de Rennes, France)
Title: *The conjugacy problem in right-angled Artin groups and their subgroups*
Abstract: We prove that the conjugacy problem in right-angled Artin groups and a large class of their subgroups can be solved in linear time. This concerns in particular all graph braid groups. Some of this talk is joint work with J.Crisp, some with J.Crisp and E.Godelle.

Speaker: **Ekaterina Yurasovskaya** (University of British Columbia, Vancouver, Canada)
Title: *String links and orderability*
Abstract: The group of homotopy classes of string links $H(k)$ has first been described by Nathan Habegger and Xiao-Song Lin in 1990 and provided the main tool to classify links up to link-homotopy. Since then $H(k)$ became an object of interest in itself. I shall discuss $H(k)$ as an example of orderable groups appearing in topology.

Speaker: **Gengyu Zhang** (Tokyo Institute of Technology, Japan)
Title: *Concordance crosscap number of a knot*
Abstract: We define the concordance crosscap number of a knot as the minimum crosscap number among all the knots concordant to the knot. The four-dimensional crosscap number is the minimum first Betti number of non-orientable surfaces smoothly embedded in 4-dimensional ball, bounding the knot. Clearly the 4-dimensional crosscap number is smaller than or equal to the concordance crosscap number. We construct two infinite sequences of knots for which the 4-dimensional one is strictly smaller than the concordance one. In particular, the knot 7_4 is one of the examples.

3 The problem session

The problem session on Thursday night was a great moment. We think that the following list, which grew out of the discussion and was subsequently elaborated, contains very interesting and deep problems.

Question 1 (Famous open question). *Is it true that if two elements of PB_n do not commute, then they generate a free group?*

Question 2 (Joan S. Birman). *What are the (interesting) finite quotients of B_n? Find a constructive proof of the known fact that the braid groups B_n are residually finite. That is, a proof that produces (interesting) finite quotients of B_n.*

Question 3 (Józef H. Przytycki). *What if we adjoin the relation $\sigma_i^p = 1$ and $\Delta^4 = 1$ to B_n? (For which n, p is the quotient finite?)*

Question 4 (Dan Margalit). *Let B_n^k be the subgroup of B_n fixing the first k punctures. Note that $B_n^0 = B_n$, $B_n^1 = A(B_n)$, $B_n^{n-1} = B_n^n = PB_n$. What is $Aut(B_n^k)$? (The answer is known in the aforementioned cases [Dyer-Grossman, Ivanov, Charney-Crisp, Bell-Margalit]. Bell-Margalit proved that $Aut(B_n^k)$ surjects onto $Aut(B_n^k/Z(B_n^k))$, i.e. the automorphism group can exchange any of the fixed punctures with the boundary of the disk.*

Question 5 (Dan Margalit). *Define the kth term of term of the Johnson filtration to be the kernel of the map $B_n \to Aut(F_n/F_n^k)$, where F_n^k is the kth term of the lower central series of F_n. What is this filtration? Note that the first two terms are B_n and PB_n. (Proposed generating set: push punctures about elements of F_n^k.)*

Question 6 (Ivan Marin). *What is the topological closure of the Lawrence-Krammer representation of B_n, depending on the two complex parameters?*

Question 7 (Joan S. Birman). *An open problem is whether there is a solution to the conjugacy search problem in the braid groups which is polynomial both in the braid index and word length of a braid. If one wishes to use the Nielsen-Thurston classification, then a subquestion concerns its complexity. We suggest as a starting point to compute the complexity of the existing algorithms to decide the Nielsen-Thurston type of a braid. If it turns out that none of the existing algorithms have the desired polynomial properties, we suggest that this problem be studied.*

Question 8 (Patrick Dehornoy). *Let M_n be the incidence matrix for the standard greedy normal form of braids, i.e., the matrix with rows and columns indexed by permutation braids such that the (x, y)-entry is 1 if (x, y) is left weighted, and 0 otherwise. Is the spectrum of M_n included in the spectrum of M_{n+1}? (true for $n \leq 13$; ref: J. Combinatorial Th. Series A; 114 (2007) 389-409.)*

Question 9 (Juan González-Meneses). *What is a pseudo-Anosov element in an Artin-Tits group? (There are natural definitions of reducible and periodic elements in these groups, but pseudo-Anosov ones are just defined as "none of the above". Some Artin-Tits groups embed into the braid group, so this can give a partial answer. The others embed in E_8. So the question could be: What is a pseudo-Anosov element in E_8? But a general answer would be much better.)*

Question 10 (Stephen Humphries). *Does the Artin-Tits group E_8 embed in $Aut(F_n)$ for some n? (A positive answer would solve the previous question, since an element of E_8 would be pseudo-Anosov just as maps to an infinite order, irreducible automorphism of F_n)*

Question 11 (Dale Rolfsen). *Are spherical type Artin-Tits groups left-orderable? (true provided it is true for E_8)*

Question 12 (Ivan Marin). *Do the exceptional type pure spherical Artin-Tits groups admit non-abelian free normal subgroups?*

Question 13 (Ivan Marin). *Can they be decomposed as an iterated semi-direct product of free groups?*

Question 14 (Fabrice Castel). *What is the kernel of the standard embeddings of Artin-Tits groups E_6, E_7 and E_8 in a mapping class group? What are the outer automorphisms of these groups?*

Question 15 (Dan Margalit). *Let B_n^k be the subgroup of B_n fixing the first k punctures. Note that $B_n^0 = B_n$, $B_n^1 = A(B_n)$, $B_n^{n-1} = B_n^n = PB_n$. What is $Aut(B_n^k)$? (The answer is known in the aforementioned cases [Dyer-Grossman, Ivanov, Charney-Crisp, Bell-Margalit]. Bell-Margalit proved that $Aut(B_n^k)$ surjects onto $Aut(B_n^k/Z(B_n^k))$.*

Question 16 (Seiichi Kamada). *Is there an algorithm to decide if two given n-tuples of elements of a group G are in the same orbit under the Hurwitz action of B_n on G^n? (specially for $G = B_n$ and $G = MCG(\Sigma)$)*

Question 17 (Daciberg Lima Goncalves). *For which integers $m < n$ is $B_m(S^2)$ a subgroup of $B_n(S^2)$? For $m = 3$ it is known to be true if and only if $n \equiv 0, 2$ $(mod\ 3)$.*

Question 18 (Daan Krammer). *Luis Paris proved that Artin monoids embed into groups, but his proof is rather indirect. Distill a combinatorial proof from his methods.*

Question 19 (Daan Krammer). *Is every Garside group linear? (Guess: no). Find combinatorial necessary conditions for a Garside group to be linear (more precisely, for it to have a faithful representation over a totally ordered field which realises the Garside structure).*

Question 20 (Daan Krammer). *One of the equivalent definitions of Garside groups (namely, the grid property) is a combinatorial analog of convex sets in real vector spaces, or equivalently, in real hyperbolic space. Weaken Garside groups by modelling them on convex sets in complex hyperbolic space.*

Question 21 (Ivan Marin). *Is the Frattini subgroup of a Garside group always trivial/central?*

Question 22 (Dale Rolfsen). *Assume Σ is a surface of positive genus. Is $B_n(\Sigma)$ left orderable? [$P_n(\Sigma)$ is bi-orderable]*

Question 23 (Joan Birman). *Can one find a bound on the volume of the complement of a Lorentz knot?*

Question 24 (Hugh Morton). *Let us regard a Lorentz knot as a framed knot (by the template); describe a Lorentz pattern as a framed pattern in the standard annulus by including the Lorentz pattern in the annulus. If a Lorentz knot is a satellite, is it constructed as a satellite of a Lorentz knot using a Lorentz pattern? (the satellite of any Lorentz knot using a Lorentz pattern is always a Lorentz knot).*

Question 25 (Roger Fenn). *Are there interesting polynomials that are invariants of welded links (other than the Alexander...)?*

Question 26 (Dale Rolfsen). *Is there a practical algorithm to decide for a braid β whether $\widehat{\beta}$ is fibred?*

Question 27 (Michel Boileau). *What are the positive braid presentations of a torus knot? (think of the Lorentz presentations)*

Question 28 (Michel Boileau). *Let Σ be a closed surface, φ a pseudo-Anosov homeomorphism of Σ. Then $\Sigma \rtimes S^1$ has a representation in $PSL(2,\mathbb{C})$ which induces a representation ρ of $\pi_1(\Sigma)$ in $PSL(2,\mathbb{C})$. Consider $H_1(\Sigma)$ with coefficients twisted by ρ^*. Look at the action of φ on $H_1(\Sigma)$. It is described by a matrix $M(\varphi)$ which is nontrivial. What does $M(\varphi)$ say about the dynamics of φ?*

Question 29 (Patrick Dehornoy). *Can one (fruitfully) use self-distributive systems that are not racks in knot theory? (In other words: algebraic systems that encode invariance under Reidemeister move III, but not necessarily move II; comment: highly non-trivial examples of such systems are known.)*

4 List of participants

Here is the complete list of participants to the meeting.

Bachman, David, Pitzer College (US)
Bessis, David, DMA - Ecole normale suprieure (FR)
Birman, Joan, Barnard College, Columbia University (US)
Boileau, Michel, Universite Paul Sabatier (FR)
Budden, Stephen, University of Auckland (NZ)
Castel, Fabrice, University of Bourgogne (FR)
Clay, Adam, University of British Columbia (CA)
Dehornoy, Patrick, University of Caen (FR)

Digne, Franois, Universit de Picardie Jules-Verne (FR)
Fenn, Roger, University of Sussex (GB)
Gebhardt, Volker, University of Western Sydney (AU)
Gonzalez-Meneses, Juan University of Seville (SP)
Guaschi, John, Universit Paul Sabatier (FR)
Humphries, Stephen, Brigham Young University (US)
Kamada, Naoko , Osaka City University (JP)
Kamada, Seiichi, Hiroshima University (JP)
Kania-Bartoszynska, Joanna, National Science Foundation (US)
Kawamuro, Keiko, Rice University (US)
Kin, Eiko, Tokyo Institute of Technology (JP)
Kohno, Toshitake, University of Tokyo (JP)
Krammer, Daan, University of Warwick (GB)
Lee, Sang-Jin, Konkuk Univ (KR)
Lee, Eon-Kyung , Sejong University (KR)
Lescop, Christine, University of Grenoble (FR)
Lima Goncalves, Daciberg, Universidade de Sao Paulo (BR)
Marche, Julien, University Paris 6 (FR)
Margalit, Dan, University of Utah (US)
Marin, Ivan, University Paris 7 (US)
Matsuda, Hiroshi, Columbia University (JP)
Menasco, William, University at Buffalo (US)
Michel, Jean, University Paris 7 (FR)
Morrison, Scott, University of California, Berkeley (US)
Morton, Hugh, University of Liverpool (GB)
Murakami, Hitoshi, Tokyo Institute of Technology (JP)
Przytycki, Jozef, George Washington University (US)
Rolfsen, Dale, University of British Columbia (CA)
Thurston, Dylan , Barnard College, Columbia University (US)
Watson, Liam, L'Universit du Qubec Montral (CA)
Wiest, Bert, Universite de Rennes (FR)
Yurasovskaya, Ekaterina, University of British Columbia CA)
Zhang, Gengyu, Tokyo Institute of Technology (JP)