

# **Report on BIRS 08w5065**

## **Inverse Problems: Recent Progress and New Challenges**

Adrian Nachman and Fasil Santosa

### **Scientific description of workshop**

Inverse problems lie at the heart of scientific inquiry and technological development. The topics covered in this workshop included imaging problems, inverse scattering, optimal design, and parameter estimation. Different imaging modalities were represented. There were talks on MRI, photo-acoustic tomography, optical imaging, elastography, electrical impedance tomography, as well as, talks on more specialized applications of these technologies. Data assimilation, an important problem in climate modeling and weather prediction, and system biology, were examples of parameter estimation problems discussed at the meeting. While many of the presentations delved into the practical and computational aspects of these applications, we had an equally rich array of mathematical talks on recent theoretical developments in inverse problems. We had talks on sharp conditions on medium parameters for which there is visibility and beyond which cloaking becomes possible, results on approximate cloaking with isotropic materials, new approaches to uniqueness in anisotropic problems using limiting Carleman weights, a breakthrough result on recovering metrics from lengths of broken geodesics, new analytic treatment of inverse problems with imperfectly known boundaries, a novel inverse conductivity problem and its solution using minimal surfaces, major progress on transmission eigenvalues in scattering theory, and the information which can be extracted from them about both isotropic and anisotropic media, nonlinear obstacle problems, some inverse problems in systems biology, use of Wasserstein distance from optimal transport theory to analyze stability of Radon transforms, and genuinely nonlinear novel approaches to regularization.

### **Goals and objectives**

The workshop was designed to bring together scientists working in the forefront of the subject of inverse problems to address new challenges arising in this field. A goal in this workshop is to create interdisciplinary teams to work on inverse problems which potentially have great impact in development of new technologies.

### **Outcome**

We were successful in bringing together leading researchers from both the pure and applied side of inverse problems. The format of the workshop, consisting of one-hour tutorials and half-hour research talks, was conducive to creating open-ended discussions. The tutorials managed to give the big picture problems and remaining challenges, whereas the research talks highlight recent developments.

The organizers of the workshop were particularly pleased by the mix of mathematicians and scientists from other disciplines. It made for a truly interdisciplinary, exciting workshop. The workshop also featured technologists who work on ground breaking

imaging methodologies. It was gratifying to see that they were able to convey the mathematical problems behind these new technologies, while being surprised at the depth and breath of advances made on the theoretical side. We feel optimistic that some new collaborations will result from this workshop. It is clear that new mathematical research is needed in order to bring new imaging technologies into practice.

### **List of tutorial talks**

Simon Arridge, Multimodal and hybrid optical imaging  
Kari Astala, Constructing solutions to the conductivity equation with bounded coefficients, and beyond  
Paul Barbone, Some math and mechanics of biomechanical imaging: current status and open questions  
Jacques Blum, Data assimilation for geophysical problems: variational and sequential techniques  
David Dos Santos Ferreira, Carleman estimates and anisotropic inverse problems  
Andreas Kirsch, Inverse scattering problems for the time-harmonic Maxwell system and the interior transmission eigenvalue problem  
Ricardo Otazo, Parallel magnetic resonance imaging  
Xiaochuan Pan, Thoughts on some mathematical and practical issues in tomographic imaging  
John C. Schotland, Optical tomography in one dimension  
Lihong V. Wang, Photoacoustic tomography: High-resolution imaging of optical contrast in vivo at new depths

### **List of research talks**

Leon Axel, Studying tissue perfusion with first-pass contrast enhancement imaging  
Guillaume Bal, Physics-based modeling of measurement correlations  
Eric Bonnetier, Asymptotics of the potential in a periodic composite perturbed by small localized defects  
Fioralba Cakoni, Eigenvalue problems in inverse scattering theory  
David Dobson, Optimization of periodic composite structures for sub-wavelength focusing  
Heinz W. Engl, Inverse problems in systems biology  
Alan Greenleaf, Approximate acoustic and quantum cloaking  
Bojan Guzina, Bits and pieces put together to present a semblance of an elastic whole  
Michael Joy, Imaging conductivity using a magnetic resonance imager  
Matti Lassas, Broken geodesics and inverse problems for radiative transfer equation  
Alison Malcolm, Ultrasound vibro-acoustography  
Stephen McDowall, Optical tomography in media with varying index of refraction.  
Petri Ola, Electric impedance tomography with an imperfectly known boundary  
Lassi Paivarinta, Transmission eigenvalues in scattering theory  
Luca Rondi, Inverse problems and free-discontinuity problems  
William Rundell, Inverse obstacle problems: recovering shape, impedance and surrounding conductivities  
Samuli Siltanen, Regularization of the D-bar method for electrical impedance tomography  
Plamen Stefanov, The geodesic X-ray transform with caustics  
Ben Stephens, Measuring the geodesic Radon transform with mass transport  
Alexandru Tamasan, Current density based conductivity reconstruction: A new hybrid imaging method

## List of participants and their home department

[Arridge, Simon](#), University College London, Computing Sciences  
Astala, Kari, University of Helsinki, Mathematics  
Axel, Leon, New York University Langone Medical Center, Radiology  
Bal, Guillaume, Columbia University, Applied Physics and Applied Mathematics  
Barbone, Paul, Boston University, Mechanical Engineering  
[Blum, Jacques](#), University of Nice-Sophia-Antipolis, Mathematics  
[Bonnetier, Eric](#), Université Joseph Fourier, Mathematics  
[Cakoni, Fioralba](#), University of Delaware, Mathematics  
[Colton, David](#), University of Delaware, Mathematics  
[Dobson, David](#), University of Utah, Mathematics  
[Dos Santos Ferreira, David](#), Université Paris 13, Mathematics  
[Engl, Heinz](#), Austrian Academy of Sciences, Applied Mathematics  
[Greenleaf, Allan](#), University of Rochester, Mathematics  
[Guzina, Bojan](#), University of Minnesota, Civil Engineering  
Joy, Michael, University of Toronto, Biomedical Engineering  
[Kirsch, Andreas](#), University of Karlsruhe, Mathematics  
[Lamoureux, Michael](#), University of Calgary, Mathematics  
Lassas, Matti, Helsinki, University of Technology, Mathematics  
[Malcolm, Alison](#), Massachusetts Institute of Technology, Geophysics  
[Mazzucato, Anna L](#), Penn State University, Mathematics  
[McDowall, Stephen](#), Western Washington University, Mathematics  
Nachman, Adrian, University of Toronto, Mathematics  
Ola, Petri, University of Helsinki, Mathematics  
Otazo, Ricardo, New York University Langone Medical Center, Radiology  
Pan, Xiaochuan, University of Chicago, Radiology  
Päivärinta, Lassi, University of Helsinki, Mathematics  
Rondi, Luca, Università degli Studi di Trieste, Mathematics  
Rundell, William, Texas A&M University, Mathematics  
[Santosa, Fadil](#), University of Minnesota, Mathematics  
Schotland, John, University of Pennsylvania, Applied Sciences  
[Siltanen, Samuli](#), Tampere University of Technology, Mathematics  
Stefanov, Plamen, Purdue University, Mathematics  
[Stephens, Ben](#), University of Toronto, Mathematics  
Street, Brian, University of Toronto, Mathematics  
Tamasan, Alexandru, University of Central Florida, Mathematics  
[Wang, Lihong](#), Washington University - St Louis, Biomedical Engineering  
Zarate Saiz, Ramon, University of British Columbia, Mathematics