Advances in hyperkähler and holomorphic symplectic geometry

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1 Overview of the Field

Hyperkähler geometry, defined by Calabi in 1978, is endowed with such a high degree of structure that it has occupied a special status within many fields of geometry simultaneously. As a result, breakthroughs in hyperkähler geometry have come from Riemannian geometry, algebraic geometry, symplectic geometry, integrable systems, and quantum field theory in physics.

From a Riemannian point of view, hyperkähler metrics are Ricci-flat Kähler metrics with a surplus of constant spinors. They are also endowed with a natural family of real symplectic forms, making hyperkähler geometry amenable to methods of symplectic geometry, such as the symplectic quotient construction. Within algebraic geometry, they may be viewed as holomorphic symplectic manifolds, a fact which has led to the construction of many examples of hyperkähler manifolds, and which accounts for the appearance of hyperkähler geometry in the study of algebraically completely integrable systems. They have been of interest to physicists because they are required in supersymmetric quantum field theories, but also for the reason that hyperkähler geometry describes the behaviour of a topological field theory related to 4-dimensional Yang-Mills theory. Perhaps the most interesting hyperkähler manifold is Hitchin's moduli space of Higgs bundles, a central object of study in the geometric Langlands programme.

In the last five years, there have been several explosive developments in the study of hyperkähler manifolds, coming from different lines of inquiry by researchers working in very different fields of mathematics.

- **The classification or Torelli problem.** Holomorphic symplectic geometry is the algebro-geometric manifestation of hyperkähler geometry. By Yau's Theorem, every compact Kähler holomorphic symplectic manifold admits a hyperkähler metric. In two dimensions, K3 surfaces and complex tori are the only examples. One of the main focuses of algebraic geometers working in this area has been to extend to higher dimensions the well-understood classification theory of K3 surfaces. For instance, moduli of deformations of irreducible symplectic manifolds are described by a period map, which is known to be a local isomorphism (Beauville 1983). Huybrechts' (1999) proof of the surjectivity of the period map uses twistor spaces, and is a fine example of the interplay of the differential geometric and algebraic approaches. A complete understanding of the global behaviour of the period map has so far been out of reach, but there has been recent significant progress on this "Global Torelli Theorem" by Verbitsky (2009).

- **Mirror symmetry for hyperkähler manifolds.** An elliptic K3 surface is a K3 surface fibred over $\mathbb{P}^1$ with generic fibre an elliptic curve. The higher-dimensional analogue of this is a holomorphic Lagrangian fibration, which is a holomorphic symplectic manifold fibred by n-dimensional complex tori over $\mathbb{P}^n$. Hyperkähler rotation turns this into a special Lagrangian fibration, a structure of central importance in the Strominger-Yau-Zaslow formulation of Mirror Symmetry; moreover, this is currently the only way known of producing special Lagrangian fibrations on compact manifolds. Matsushita (1999) proved that Lagrangian fibrations are the only possible fibrations on holomorphic symplectic manifolds. A major goal is to prove the "hyperkähler SYZ conjecture", which states that these fibrations always exist, at least after a small deformation of the manifold (Matsushita, Verbitsky 2008). Singular fibres of Lagrangian fibrations were recently classified by Hwang and Oguiso (2009), who are now working on understanding the local structure of the fibration near the singular fibres.

- **The study of compact hyperkähler manifolds.** Compact hyperkähler manifolds are exceedingly rare, and their complete classification is one of the main outstanding problems in the field. Examples are found using algebraic geometry, by constructing holomorphic symplectic varieties. Almost all examples of higher-dimensional holomorphic symplectic manifolds can be described in terms of moduli spaces of stable sheaves on K3 or abelian surfaces. Such moduli spaces can sometimes be singular, and it came as a surprise in 1998 when O'Grady constructed explicit symplectic desingularizations of some of these spaces. Subsequent investigations by Lehn and Kaledin (2004) revealed that this behaviour is rare: the moduli spaces considered by O'Grady are essentially the only ones that admit symplectic desingularizations. The topology and birational geometry of O'Grady's spaces remains a topic of interest (Rapagnetta, Nagai 2010).

- **Rozansky-Witten theory.** Hyperkähler and holomorphic symplectic geometry figures prominently in many physical theories. For example, Rozansky-Witten theory (1997) used a hyperkähler manifold X as the target space for a 3-dimensional sigma model. The theory was originally used to study knot and 3-manifold invariants, but recently, the theory has been revisited by Kapustin, Rozansky, and Saulina (2009), who discovered that by studying the boundary conditions in the original Rozansky-Witten theory, one uncovers the rich structure of an extended topological field theory, which lends an unexpected (and poorly understood) structure on the category of holomorphic Lagrangian submanifolds of X, connecting them with the theory of matrix factorizations in the study of noncommutative algebra.

- **Wall-crossing and explicit hyperkähler metrics.** One of the long-standing mysteries of modern geometric analysis is that while the theorem of Yau implies that Calabi-Yau and hyperkähler metrics exist, the metric is not explicitly known even in the simplest interesting cases on compact manifolds. Even for non-compact manifolds, few explicit hyperkähler metrics are known, and those that are are celebrated examples such as the Gibbons-Hawking metric, frequently used in various branches of geometry and physics. Recently, using results of Kontsevich-Soibelman (2008) on wall-crossing formulas describing enumerative invariants on Calabi-Yau manifolds, physicists Gaiotto, Moore, and Neitzke (2009) were able to explicitly compute hyperkähler metrics on certain noncompact varieties in terms of these enumerative invariants. This work provides a completely new viewpoint on the relationship between the Riemannian aspect of hyperkähler geometry and the algebro-geometric aspect.

- **The geometry/topology of hyperkähler quotients.** The hyperkähler quotient construction of Hitchin-Karlhede-Lindström-Roček (1987) generalizes symplectic reduction, and has been the main method of constructing non-compact hyperkähler manifolds. Surprisingly, many equations arising in gauge theory (certain anti-self-dual equations, the Bogomolny equations, Nahm's equations) can be naturally interpreted as hyperkähler moment maps. The corresponding moduli spaces (of instantons, magnetic monopoles, Higgs bundles, etc.) thereby inherit hyperkähler metrics. Understanding the geometry and topology of hyperkähler quotients will therefore shed some light on the geometry and topology of many interesting physical moduli spaces; contributors in this area include Hitchin, Roček, Swann, and Nakajima-Yoshioka. In addition to the many results of Hausel and his collaborators, a recent development has been a criteria for desingularization of a quotient by Jeffrey-Kiem-Kirwan (2009), who also investigate the surjectivity of an analogue of the Kirwan map to the cohomology of a hyperkähler quotient.

A great deal is known about hyperkähler quotients constructed from torus actions on flat quaternionic vector spaces, which are known as ”toric hyperkahler manifolds” or ”hypertoric varieties”. Bielawski-Dancer (2000) classified such hyperkähler quotients in terms of hyperplane arrangements; they also determined a Kähler potential and gave an explicit local form of the metric. Konno (1999, 2000) studied the cohomology rings. Harada-Proudfoot (2004) computed the equivariant cohomology. Hausel-Proudfoot (2005) used hypertoric varieties to help understand the cohomology rings of general hyperkähler quotients. The topology of hypertoric varieties continues to be a topic of great interest, with recent results of Braden-Proudfoot (2009) on the intersection cohomology ring, and of Stapledon (2009) on orbifold cohomology and Ehrhart polynomials of Lawrence polytopes.

2 Objectives of the workshop

The meeting was intended as a 5-day workshop involving the main researchers in the fields above, both faculty and postdoctoral, together with graduate students who have taken up the subject in their doctoral work. For students and postdocs, the meeting was to provide two parallel opportunities: first, to learn well-established aspects of the theory of holomorphic symplectic and hyperkähler manifolds, and second, to be exposed to the plethora of open questions ripe for investigation, deriving from the recent advances listed above.

There were two main scientific objectives. The first was to summarize and understand the recent developments and main questions within the different groups of researchers focusing on hyperkähler geometry, including the recent work on the Torelli theorem, the algebro-geometric study of holomorphic Lagrangian fibrations, and the study of hypertoric varieties. The second aim was to introduce new ideas coming from physics in the recent papers of Kapustin-Rozansky-Saulina and Gaiotto-Moore-Neitzke and to work on some of the many open questions resulting from this seminal work. Specifically, the three main questions which must be resolved are whether the categorical structure they obtain on holomorphic Lagrangians can be made mathematically precise, whether the structure can be computed in known special cases, and what impact this has on the study of holomorphic symplectic manifolds, and possibly on holomorphic Poisson manifolds.

3 Overview of the meeting

Here are the abstracts of the talks, in alphabetical order by speaker surname:

Speaker: **Philip Boalch** (Centre National de la Recherche Scientifique)
Title: “Irregular connections, Dynkin diagrams, and fission”

Abstract: I’ll survey some results (both old and new) related to the geometry of hyperkahler moduli spaces of irregular connections on curves. If time permits this will include: 1) new nonlinear group actions generalising the well known actions of the mapping class/braid groups on character varieties, 2) new ways to glue Riemann surfaces together to obtain (symplectic) generalisations of the complex character varieties of surfaces, and 3) a precise conjecture that the Hilbert scheme of points on any 2d meromorphic Hitchin system is again a Hitchin system.

Speaker: **Sabin Cautis** (Columbia University)
Title: “Flops and about”

Abstract: Stratified flops of type A, D and E show up in the birational geometry of holomorphic symplectic varieties. For example, by a result of Namikawa all Springer resolutions of the closure of nilpotent orbits are related by a sequence of such flops.

Two varieties related by such a flop are expected to have equivalent derived categories. Concentrating on the A and D types, I will discuss the geometry of such flops, explain how they induce derived equivalences and speculate on various open questions.

Speaker: **François Charles** (IRMAR – Université de Rennes 1)
Title: "Some arithmetic aspects of specialization of Néron-Severi groups for holomorphic symplectic varieties"

Abstract: For a given family of smooth projective complex varieties, the theory of variations of Hodge structures gives a precise description of the variation of the Picard number of the members of the family. In the case of families of holomorphic symplectic varieties, the two following properties are well-known. On the one hand, the Picard number of a very general member of the family is equal to the Picard number of the generic fiber of the family. This is an easy consequence of Baire's theorem. On the other hand, if the family is not isotrivial, the locus in the parameter space of varieties with Picard number strictly bigger than that of the generic fiber is topologically dense. This is a consequence of the local Torelli theorem and of Lefschetz' theorem on $(1,1)$-classes which was pointed out by M. Green.

The goal of this talk is to investigate the extent to which the behaviour described above still appears in the arithmetic situation where the parameter space is replaced with the ring of integers of a number fields. This amounts to investigating specialization of Néron-Severi groups for holomorphic symplectic varieties defined over number fields after reduction to a finite field.

In this situation, the first result above does not hold. However, we will describe precisely the extent to which the Picard number can be forced to jump after specialization to a finite field. If time allows, we will describe a proof of the arithmetic analog of the theorem of Green in the special case of products of elliptic curves and discuss its arithmetic significance.

These problems have implications outside of arithmetic geometry, as was pointed out by recent results on the existence of rational curves on K3 surfaces by Bogomolov-Hassett-Tschinkel and Li-Liedtke. We will describe how they can be used to get an algorithm that allows one to compute the Picard number of any holomorphic symplectic variety.

Speaker: **Sergey Cherkis** (University of Arizona)
Title: "Doubly-periodic monopoles and their moduli spaces"

Abstract: A monopole wall is a solution of the Bogomolny equation on $\mathbb{R} \times T^2$; in other words it is a doubly periodic monopole. Moduli spaces of monopole walls are hyperkähler and, when the dimension is minimal, deliver examples of gravitational instantons. We formulate spectral description of a monopole wall of any given charges and use it to compute the dimension of its moduli space.

The Nahm transform maps a monopole wall to a monopole wall establishing the isometry between their respective moduli spaces. We find $SL(2,\mathbb{Z})$ group action on monopole walls, such that the Nahm transform is its S element. We conclude by identifying all monopole walls with four real moduli, up to this $SL(2,\mathbb{Z})$ equivalence.

These results are obtained in collaboration with Richard Ward.

Speaker: **Andrew Dancer** (Oxford University)
Title: "Implosion for hyperkähler manifolds"

Abstract: Implosion is an abelianisation construction in symplectic geometry, due to Guillemin, Jeffrey and Sjamaar. In this talk we describe joint work with Frances Kirwan and Andrew Swann on developing an analogous construction for hyperkähler spaces.

Speaker: **Tamás Hausel** (Oxford University)
Title: "Symmetries of SL(n) Hitchin fibres"

Abstract: In this talk we show how the computation of the group of components of Prym varieties of spectral covers leads to cohomological results motivated by mirror symmetry on the cohomology of moduli space of

Higgs bundles and in turn to cohomological results on the moduli space of stable bundles on curves originally due to Harder-Narasimhan. This is joint work with Christian Pauly.

Speaker: **Daniel Huybrechts** (University of Bonn)
Title: "Chow groups and stable maps"

Speaker: **Jun-Muk Hwang** (Korea Institute for Advanced Study)
Title: "Webs of Lagrangian tori in projective symplectic manifolds"

Abstract: For a Lagrangian torus A in a simply-connected projective symplectic manifold M, we prove that M has a hypersurface disjoint from a deformation of A. This implies that a Lagrangian torus in a compact hyperkaehler manifold is a fiber of an almost holomorphic Lagrangian fibration, giving an affirmative answer to a question of Beauville's. Our proof employs two different tools: the theory of action-angle variables for algebraically completely integrable Hamiltonian systems and Wielandt's theory of subnormal subgroups. This is a joint-work with Richard Weiss.

Speaker: **Emanuele Macrì** (The Ohio State University)
Title: "Projectivity and birational geometry of Bridgeland moduli spaces"

Abstract: In this talk we will present a construction of a family of nef divisor classes on every moduli space of stable complexes in the sense of Bridgeland. For a generic stability condition on a K3 surface, we will prove that these classes are ample, thereby generalizing a recent result of Minamide, Yanagida, and Yoshioka.

We will apply this construction to describe a region in the ample cone of a moduli space of Gieseker-stable sheaves on a K3 surface and to study its birational geometry.

This is joint work in progress with Arend Bayer.

Speaker: **Dimitri Markushevich** (Universite Lille 1)
Title: "Some examples of Prym Lagrangian fibrations"

Abstract: The objective of the talk is to describe several constructions of holomorphically symplectic varieties equipped with Lagrangian fibrations. The constructions are related to the variations of mixed Hodge structures, and the fibers of the obtained Lagrangian fibrations are intermediate Jacobians of algebraic varieties. Several examples are produced when the latter algebraic varieties are conic bundles and the intermediate Jacobians are Prym varieties of double covers. A work is in progress on their (partial) compactification.

Speaker: **Yoshinori Namikawa** (Kyoto University)
Title: "On the structure of homogeneous symplectic varieties of complete intersection"

Abstract: If X is a symplectic variety embedded in an affine space as a complete intersection of homogeneous polynomials, then X coincides with a nilpotent orbit closure of a semisimple Lie algebra. Moreover, if X is a homogeneous symplectic hypersurface, then $\dim X = 2$ and X is an A_1-surface singularity.

Speaker: **Andrew Neitzke** (University of Texas at Austin)
Title: "Spectral networks"

Abstract: I will describe some objects called "spectral networks." A spectral network is a set of paths drawn on a punctured Riemann surface, obeying some local conditions. Spectral networks arise naturally in a new construction of the hyperkähler structure on moduli spaces of Higgs bundles with gauge group SU(N). This is joint work with Davide Gaiotto and Greg Moore.

Speaker: **Kieran G. O'Grady** (Rome (Sapienza))
Title: "Vector-bundles and zero-cycles on $K3$ surfaces"

Abstract: Let X be a projective complex $K3$ surface. Let $A^q(X)$ be the Chow group of codimension-q cycles on X modulo rational equivalence. Beauville and Voisin singled out a class $c_X \in A^2(X)$ of degree 1: it is represented by any point lying on an arbitrary rational curve (an irreducible curve whose normalization is rational). The class c_X has the following remarkable property:

$$\text{Let } D_1, D_2 \in A^1(X)\text{: then } D_1 \cdot D_2 \in \mathbb{Z}c_X.$$

Moreover $c_2(X) = 24c_X$. (Conjecturally the Chow ring of Hyperkähler varieties has similar properties.) In particular one has the *Beauville-Voisin ring* $A^0(X) \oplus A^1(X) \oplus \mathbb{Z}c_X$. Huybrechts proved that if E is a spherical object in the bounded derived category of X then the Chern character of E belongs to the Beauville-Voisin ring provided $\mathrm{Pic}(X)$ has rank greater than 2 or $c_1(E) \equiv \pm 1$ (mod rk(E)) if $\mathrm{Pic}(X) = \mathbb{Z}$. A rigid simple vector-bundle on X is a particular case of spherical object - in fact the key case in the proof of Huybrecths' result. One may summarize Huybrechts' result as follows. Let F_1, F_2 be rigid vector-bundles on X (the additional hypotheses mentioned above are in force): then $c_2(F_1) = c_2(F_2) + ac_X$ where $a := (\deg c_2(F_1) - \deg c_2(F_2))$. We believe that the following more general statement (with no additional hypotheses) holds. Let $\mathfrak{M}_1^{st}$ and $\mathfrak{M}_2^{st}$ be moduli spaces of stable pure sheaves on X with Mukai vectors v_1 and v_2 respectively. Suppose that $\dim \mathfrak{M}_1^{st} = \dim \mathfrak{M}_2^{st}$ i.e. v_1 and v_2 have equal norm with respect to Mukai's pairing: then the subset of $A^2(X)$ whose elements are $c_2(F_1)$ where $[F_1] \in \overline{\mathcal{M}}_1^{st}$ (the closure of $\mathfrak{M}_1^{st}$ in the moduli space of semistable sheaves) is equal to the subset of $A^2(X)$ whose elements are $c_2(F_2) + ac_X$ where $[F_2] \in \overline{\mathcal{M}}_2^{st}$ and $a := (\deg c_2(F_1) - \deg c_2(F_2))$ (notice that $(\deg c_2(F_1) - \deg c_2(F_2))$ is independent of F_1 and F_2). We will prove that the above statement holds under some additional assumptions.

Speaker: **Keiji Oguiso** (Keio University)
Title: "K3 surface automorphisms and hyperkähler automorphisms inspired by complex dynamics"

Abstract: I would like to discuss some nature of automorphisms of K3 surfaces and compact hyperkähler manifolds from the following basic and natural aspects in complex dynamics with concrete examples:
(1) topological entropy;
(2) Tits' alternatives;
(3) fixed point set;
(4) relations with ambient spaces.

Speaker: **Nicholas Proudfoot** (University of Oregon)
Title: "Quantizations of conical symplectic resolutions"

Abstract: The most studied example of a conical symplectic resolution is the cotangent bundle M of the flag manifold G/B, which resolves the nilpotent cone in Lie(G). Much of what goes under the name "geometric representation theory" is the study of this resolution, called the Springer resolution. Here are two cool features of this subject:

- If you construct a deformation quantization of M and take global sections, you get the ring of global (twisted) differential operators on the flag variety, which is isomorphic to a central quotient of the universal enveloping algebra of Lie(G). This allows you to study representations of Lie(G) in terms of sheaves on M.
- There is a natural action of "convolution operators" on the cohomology of M which provides a geometric construction of the regular representation of the Weyl group of G. This action can be promoted to a braid group action on a category by replacing cohomology classes with sheaves.

I will make the case that these two phenomena fit neatly into a theory that applies to arbitrary conical symplectic resolutions, including (for example) quiver varieties, hypertoric varieties, and Hilbert schemes of points on ALE spaces.

This is joint work with Braden, Licata, and Webster.

Speaker: **Brent Pym** (University of Toronto)
Title: "Residues of Poisson structures and applications"

Abstract: A holomorphic Poisson manifold is foliated by symplectic leaves, and the locus consisting of all leaves of dimension $2k$ or less is called the $2k^{th}$ degeneracy locus. In recent work with Marco Gualtieri, we explain that a Poisson structure has natural residues along its degeneracy loci, which are direct analogues of the Poincaré residue of a meromorphic volume form. As applications, we prove that the anti-canonical divisor along which a generically symplectic Poisson structure degenerates is singular in codimension two, and provide new evidence in favour of Bondal's conjecture that the $2k^{th}$ degeneracy locus of a Poisson Fano variety has dimension $\geq 2k+1$.

Speaker: **Giulia Saccà** (Princeton University)
Title: "Fibrations in abelian varieties and Enriques Surfaces"
Abstract: I will discuss two classes of fibrations in abelian varieties that can be associated to a linear system on an Enriques surface. The first class corresponds to Lagrangian subvarieties of certain HK manifolds, whereas the second class corresponds to singular symplectic subvarieties of certain singular moduli spaces of sheaves. The second class is a joint work in progress with E. Arbarello and A. Ferretti.

Speaker: **Misha Verbitsky** (SU-HSE, Faculty of Maths)
Title: "Trisymplectic manifolds"

Abstract: A trisymplectic structure on a complex 2n-manifold is a triple of holomorphic symplectic forms such that any linear combination of these forms has rank 2n, n or 0. We show that a trisymplectic manifold is equipped with a holomorphic 3-web and the Chern connection of this 3-web is holomorphic, torsion-free, and preserves the three symplectic forms. We construct a trisymplectic structure on the moduli of regular rational curves in the twistor space of a hyperkähler manifold.

4 Presentation Highlights/Scientific Progress Made

Examples of recent breakthroughs:

1. In the last five years, a discovery in quantum field theory about the counting of stable configurations of particles and black holes has shown itself to be an extremely powerful mathematical tool, with ramifications far beyond its initial applications in physics.

   The discovery concerns a "wall-crossing formula" which explains how certain numerical invariants, such as the number of stable configurations in a system, change when the parameters describing the system pass through a "wall", which is a real hypersurface in the parameter space.

   In recent work of Gaiotto, Neitzke, and Moore, the physical meaning of the formula was explained in a remarkable way using the theory of supersymmetric sigma models: they interpret the jumping of invariants as a change in the behaviour of instanton contributions to a hyperKähler metric; the wall-crossing formula is then viewed as a sum of multi-instanton contributions to this metric. In this way, enumerative invariants in algebraic geometry become involved in the definition of a smooth, non-algebraic object, namely a Riemannian metric.

   At BIRS, Andrew Neitzke presented this new research to an audience of experts and it brought tremendous excitement to many of them. Perhaps the greatest promise that it holds is the possibility that, using the wall-crossing ideas, it might actually be possible to explicitly determine the famous hyperKähler metrics on the Hitchin moduli space of Higgs bundles — an object of central study in differential geometry, algebraic geometry, and number theory through the geometric Langlands programme.

2. Jun-Muk Hwang's talk was a spectacular unveiling of his recent solution to a conjecture of Beauville concerning the existence of Lagrangian foliations in holomorphic symplectic manifold. In a tour-de-force, Hwang combined a beautiful argument using the theory of integrable systems with intricate

estimates coming from the theory of discrete groups. The audience immediately appreciated that this was a landmark achievement and there were many discussions afterward about possible extensions of the argument.

3. Kieran O'Grady is well-known in the field of holomorphic symplectic geometry as one of the primary innovators that sets the direction for the field. One of the main sources of examples of holomorphic symplectic manifolds are moduli spaces of sheaves – in his talk, O'Grady explained his recent detailed study of certain homological properties of moduli spaces of sheaves on K3 surfaces. His discovery, which extends the work of several other researchers in attendance such as Huybrechts, is a precise formula which relates the possible cohomological supports for different moduli spaces over the same K3.

4. Tamàs Hausel presented his recent work with Pauly concerning the detailed behaviour of Prym varieties of spectral covers in the $SL(n)$ Hitchin system. The work he explained at BIRS was instrumental in his recent work with de Cataldo and Migliorini [3], which appeared in the Annals of Mathematics.

5. The presentation by François Charles was a very interesting one for the conference, as it dealt with holomorphic symplectic geometry over fields other than the complex numbers. His work was shocking to many in the audience, because of how differently the varieties of interest behave in this situation as compared to the usual complex case. Naturally, this opened the eyes of many attendees and suggested many other questions about whether known results for complex K3 surfaces extended to number fields.

5 Outcome of the Meeting

We had *thirty-eight* participants. The participants consisted of main researchers in the area, both faculty and postdoctoral, and of graduate students who have taken up the subject in their doctoral work. It was important to us that the workshop provide an opportunity for graduate students to interact with experts in the field. We were very successful in that respect, with the participation of *seven* graduate students from the Universities of Toronto, Princeton, Roma Tre, and Pierre-et-Marie-Curie (Paris VI), and from the Institut Fourier, *two* of whom gave talks on their research. There were also *four* postdoctoral researchers, from the Universities of Toronto, Columbia, and Bonn, and from ETH Zürich.

Hyperkähler and holomorphic symplectic geometry figures prominently in many physical theories, and the work of physicists has, on numerous occasions, uncovered new structures and relationships that have advanced our understanding of some of the objects appearing in the field. We therefore also had the participation of physicists. In fact, two talks were given by physicists, Prof. Neitzke and Prof. Cherkis. The talk of Prof. Neitzke, in particular, sparked the interest of many mathematicians in attendance.

The workshop stimulated a number of fruitful discussions and collaborations. Those we are aware of are the following: Dr. Charles and Prof. Huybrechts intensively discussed Maulik's proof of the Tate conjecture for K3 surfaces [4] during the workshop (they tried to simplify his proof and get rid of additional assumptions on the degree and the characteristic). This eventually led to a wonderful paper [1] by Dr. Charles, which was accepted for publication in *Inventiones Mathematicae* in November 2012. Dr. Charles mentions BIRS in his acknowledgements. The ongoing collaboration between Profs. Moraru and Verbitsky also benefited from the conference, and some of the work they discussed at BIRS will appear shorly.

The BIRS meeting also provided an ideal environment for the collaboration between M. Gualtieri and B. Pym, who completed their paper concerning holomorphic Poisson structures [2] during the conference; also B. Pym spoke on this work at the conference.

Another interesting and unexpected benefit of the BIRS research environment was an impromptu after-hours lecture series given by A. Odesskii concerning the famous Feigin–Odesskii holomorphic Poisson structures. This was a subject of intense interest for a group of seven participants including M. Gualtieri, B. Pym and J. Fisher.

The participants were very enthusiastic about the scientific content of the workshop, as well as the facilities and breathtaking natural setting of BIRS. Moreover, the warm hospitality and professionalism of the staff were very much appreciated.

References

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[4] D. Maulik, Supersingular K3 surfaces for large primes. Preprint: arXiv:1203.2889.