QUANTUM FIELD FRAMEWORK FOR STRUCTURED LIGHT INTERACTIONS

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1 Overview

It is widely recognised that structured light, whose theoretical foundations were first laid twenty-five years ago, serves as a near-perfect test case for studying the quantum-classical boundary in physics. The pace of advances in both theory and experiment, since the original concept was developed, has led to the research community identifying a wide range of issues demanding urgent attention, to resolve conflicts between representations, with the aim of securing a consistent, agreed framework to describe the photonic interactions of such light. Specific problems had already been identified in connection with canonical and non-canonical forms of operator, coherence and quantum uncertainty, and the quantized forms of both linear and angular optical momentum.

Such is the backdrop to the initial 2015 proposal for this workshop which, focused on the theory and mathematics of the subject, from the outset received enthusiastic international support from all quarters. Although unconnected, the importance of this topical area was substantially underscored by the publication, only three months before the workshop took place, of a major roadmap review of the significance and practical potential for the whole field of structured light. [1] Six of the participants at this workshop featured in the long list of authors.

It was rewarding to find that only a handful of the experts who had been informally approached were ultimately unable to accept formal invitation; shortly before the workshop took place, almost every one of the places available was filled. Most participants were theorists, but a handful of especially knowledgeable experimentalists were also included, to sharpen the focus on issues of key practical significance. A few individuals had to pull out just before the conference began, all of them for family or health-related reasons. In the event, thirty-five individuals were able to participate, representing ten nations: Australia, Canada, Germany, Japan, Kazakhstan, Netherlands, Poland, UK, USA and Singapore.

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[1] H. Rubinsztein-Dunlop, A. Forbes, M. V. Berry, M. R. Dennis, D. L. Andrews, M. Mansuripur, C. Denz, C. Alpmann, P. Banzer, T. Bauer, E. Karimi, L. Marrucci, M. Padgett, M. Ritsch-Marte, N. M. Litchinitser, N. P. Bigelow, C. Rosales-Guzmn, A. Belmonte, J. P. Torres, W. N. Tyler, M. Baker, R. Gordon, A. B. Stilgoe, J. Romero, A. G. White, R. Fickler, A. E. Willner, G. Xie, B. McMorran, and A. M. Weiner, J. Opt. **19**, 013001 (2017).

The workshop was structured to provide an opportunity, without obligation, for each member to deliver a short lecture (some participants needed this explicit opportunity in order to secure independent funding to travel to the meeting). In the event, twenty-four talks were given, as listed in Section 2 below, and several other papers were also circulated during the meeting. The aim was to allow some preliminary points of discussion to be recognized, prior to full discussion sessions later in the week.

By mid-week the workshop had identified eight different topics, as shown in Section 3, which a show of hands indicated were considered the most worth revisiting in more detailed discussions. Together, these addressed a sizeable subset of the issues that had been identified as possible topics, at the planning stage. Not surprisingly, many participants were interested in all of them. Nonetheless, for expeditious discussion in groups of a practicable size, three groups were formed to pursue, in parallel sessions, loosely cognate sets of topic. Each group was assigned two members to report back to the reassembled full meeting the essential content and upshot of their discussions, as detailed in Section 4. A summing up session for all attendees was held on the last day of the workshop.

Immediately following the meeting, the Contact Organizer solicited brief feedback from all participants. Along with numerous messages of congratulations and thanks for the meeting, some very significant and promising new lines of investigation were identified, many of them involving a prospective collaboration between workshop members – and in many cases interactions between individuals who had not worked or even met before. They represent a substantial legacy, one that can be expected to lead to tangible fruit in the form of new work and publications in due course.

2 Lecture schedule

1. Iwo Bialynicki-Birula. *Do electromagnetic waves with fixed orbital angular momentum exist?* Examining orbital angular momentum and spin angular momentum shows that a relativistic treatment introduces a coupling between them. Neither of them satisfies a conservation relation; only the total angular momentum is a conserved quantity.

2. Elliot Leader: *The elementary particle angular momentum controversy: lessons from laser optics.* Informing participants in the field of laser optics about the recent angular momentum controversy in the particle physics community.

3. Ivan Fernandez-Corbaton: *An algebraic approach to light-matter interactions.* Expounding an S-matrix based methodology for the study and engineering of light-matter interactions, exhibits the ease with which symmetries and conservation laws enter the formalism, with an algebraic character that is suitable for computer implementation.

4. Garth Jones: *Spherical descriptions of photon fields compared with plane wave descriptions.* Typically when describing photonic events, the photon fields are cast as plane wave vector fields. In an alternative approach the fields are described in terms of spherical waves, expressible in the mathematical language of vector spherical harmonics. This approach is complementary to that of plane waves, and is more amenable to modelling specific types of phenomena, such as isotropic processes and condensed phase energy transfer.

5. Wolfgang Löffler: *Perfect darkness in exact solutions of Maxwell's equations.* Possibilities exist to identify nontrivial lines of dark vortices in exact solutions, leading to an examination of the importance of relativistically invariant null solutions. One issue with such solutions is their very broad plane-wave frequency spectrum; there are a number of strategies to narrow this down, that could lead to experimental demonstration.

6. Alexander Lvovsky: *Beating the Rayleigh limit.*

7. Miguel Alonso: *Structured Gaussian beams: ray and wave pictures*. The analogy between structured Gaussian beams and other systems can be described by two-dimensional harmonic oscillators. A ray-based (semiclassical) treatment based on a Poincaré sphere reveals the underlying geometry including beam shape and geometric (Berry) and Gouy phases, and new families of structured Gaussian beams emerge.

8. Masud Mansuripur: *Electromagnetic energy, force, torque, linear momentum, and angular momentum*. Maxwell's equations, manipulated in different ways, allow both familiar and unfamiliar treatments of electric and magnetic dipole interactions with the electromagnetic field. Different definitions of the Poynting vector and the stress tensor lead to different expressions for energy, force, torque, linear momentum, and AM. Each approach to the classical theory is self-consistent, as well as consistent with conservation laws; each has advantages and disadvantages. The traditional definition of the Poynting vector in conjunction with the

Einstein–Laub stress tensor provides the most reliable and physically sensible approach to the interactions of light and matter.

9. Duncan O'Dell: *The Abraham-Minkowski controversy: an ultracold atom perspective*. Whilst the momentum of light in vacuum is well understood, the complexity of real media means that when light propagates inside a dielectric it is hard to separate the momentum of the electromagnetic field from that of the constituent atoms. A dilute gas of ultracold atoms constitutes a particularly simple dielectric where experiments and theory can be understood from first principles. In particular, the well-known experiment by Campbell et al. [Phys. Rev. Lett. 94, 170403 (2005)] is usually considered to verify the Minkowski result. But this is wrong: both Abraham and Minkowski give the same result for their particular set-up. Also, a geometric phase is associated with the Abraham-Minkowski problem – an optical analogue of the He-McKellar-Wilkens phase of an electric dipole moving in a magnetic field. This phase can be measured using an atom interferometer.

10. Halina Rubinsztein-Dunlop: *Quantum aspects of structured light interactions, and ultracold atomic ensembles*.

11. Mohamed Babiker: *The current status and future of structured matter vortex waves.* A critical review of current work focuses on the practical limitations, i.e. the feasibility of experimental work in various types of matter wave, questioning what new physics is expected in each category, and what new applications may be envisaged.

12. Konstantin Bliokh: *Edge modes, degeneracies, and topological numbers in non-Hermitian systems.* Fundamental problems arise at the interface between two hot areas of modern physics: (a) non-Hermitian quantum mechanics and (b) topological states in wave systems. Non-Hermitian systems can have nontrivial topological properties and new type of chiral edge modes, determined by so-called 'exceptional points' (i.e. degeneracies in the bulk spectrum) and two types of topological numbers.

13. Kobus Kuipers: *Using local field topology for controlling quantum transitions.* Two topics: firstly, how to measure various vectorial components of both the electric and magnetic fields of light guided in nanophotonic structures, where phase- and polarization singularities are ubiquitous near the nanostructures. Secondly, it emerges that C-points and the field topology around them could be used to deterministically direct the emission of quantum transitions involving a change in orbital angular momentum.

14. Robert Boyd: *The promise of orbital angular momentum for quantum communications.* Hanbury Brown and Twiss interferometry with twisted light.

15. Robert Fickler: *Experimental limitations for large orbital angular momentum quantum numbers.* Several approaches generate photons with large OAM quantum numbers. One technique involving spin-to-orbit coupling uses tailored q-plates to generate photons with up to 200 quanta of the reduced Plancks constant, $\hbar$. Off-the-shelf spatial light modulators are limited to generating photons with up to 300 quanta. A third, recently established spiral phase mirror technique makes outputs with quantum numbers of 10000 possible. The mirrors are cut surface into such that the depth modulation corresponds to the azimuthal phase modulation, on optical reflection.

16. Ebrahim Karimi: *High-dimensional intra-city quantum cryptography, and quantum hacking with structured photons.*

17. David Andrews: *Representations of structured light and their quanta.* Questions arise over the extent to which common figurative representations of structured light and photons are defensible, and the possibility of their conveying misinformation. A look at quantum uncertainty in the light of a reformulated expression for optical mode orthogonality suggests implications for the number-phase quantum uncertainty in vortex light.

18. Enrique Galvez: *High-dimensional spaces of polarization and spatial mode of photons.* There are modes of light that give rise to spatially-variable polarization; this is a topic that touches of the fundamentals of light, and especially on the information that can be encoded into it.

19. Peter Banzer: *Transverse angular momentum of light.*

20. Martin Lavery: *Atmospheric turbulence and the propagation of structured modes.*

21. Alexander Khanikaev: *Photonic topological insulators: from theory to experimental realization.*

22. Daniel Leykam: *Topological phases and topological photonics.* Connections are established between topological photonic systems and structured light. Firstly it can be shown how Bloch band degeneracies such as Dirac points display analogies with optical vortices. Secondly, examination of the microscopic structure of optical fields in topological photonic systems, such as the presence of polarization singularities or optical vortex rings, may be used to diagnose topological phases

23. Akbar Salam: *Quantisation of the electromagnetic field in magnetodielectric media*. Considering the formulation of electrodynamics in a magnetoelectric medium, from both the classical and quantum mechanical points of view, raises questions regarding the potential utility of employing a macroscopic theory of QED to study structured light-matter interactions.

24. Nader Engheta: *Zero-index materials: some wave physics and quantum optical features*.

3 Headline topics

The following listing of topics were decided upon as especially worth focusing upon in detailed discussion:

(i) the separability of quantum spin and orbital angular momentum, and the role laser optics might play in determining their limits of measurability;

(ii) degenerate down-conversion, and the implications of quantum uncertainty on the spatial origin of correlated photon pairs;

(iii) defensible forms of the quantum number - phase uncertainty relation, as regards structured modes of light;

(iv) the relativistic origin of spin-orbit coupling, and its implications for electron vortices;

(v) dispersive effects in the Abraham and Minkowski formulations of linear and angular momentum, for photons travelling through a dispersive medium;

(vi) potential limitations of a fundamental quantum nature on the information content of an individual photon;

(vii) the role of local field symmetry and topology in structured light and other chiroptical interactions with matter;

(viii) cold atoms, elementary particles and their connections with quantised orbital angular momentum.

4 Session discussions

Three discussion groups were subsequently convened to address these issues, with each participant free to attend and participate in whichever proved to interest them most. Two reporters were assigned from within each group:

(A). Spin and orbital angular momentum separability [Konstantin Bliokh (Japan) and Ivan Fernandez-Corbaton (Spain)] (Room 202);

(B). Structured matter waves and quantised angular momentum [Halina Rubinsztein-Dunlop (Australia) and Duncan O'Dell (Canada)] (Room 107);

(C). Quantum uncertainty in structured light and down-conversion [Martin Lavery (United Kingdom) and Jrg Gtte (China)] (Lecture Theatre).

The following summarizes the outcome of these discussions, as presented to the reconvened full workshop.

In *Group A*, where there were extensive discussions about the separability of orbital and spin angular momentum, it became clear that there are two distinct frameworks, whose individual usefulness depended to some extent on the realm of application.

In Framework 1, helicity is an independent physical quantity related to dual symmetry. The integral values of spin and orbital angular momentum (AM) can be separated in the most generic EM fields in free space. However, local spin and orbital AM densities can be separated only in monochromatic optical fields. To obtain these quantities one can use either (i) canonical operators – which do not satisfy the transversality constraint, but obey canonical SO(3) commutation relations – or (ii) modified operators compatible with transversality, but obeying modified commutation relations. Modified operators with unusual commutation relations are natural in the presence of gauge fields (e.g. the momentum operator of a charged particle in a magnetic field). In the case of light, this is the Berry-connection field in momentum space. Importantly, the densities of the spin AM and momentum in a monochromatic field are directly measurable via radiation torque and force on a point dipole particle. They also satisfy separate local conservation laws (continuity equations).

In Framework 2, only transverse operators are allowed. The operators that are used are connected with symmetry transformations, and this is exploited in applications to experiments. The typical explanations of experimental results that are made using spin and orbital AM have a symmetry explanation in this framework. Sometimes, there are two different symmetry explanations for what would correspond to spin to orbital AM transfer in Framework 1. Total angular momentum is the only operator considered to be connected with physical rotations. Helicity plays an important role in this framework.

In *Group B* there were three sessions focused on structured matter waves: 1) Cold atoms and optical angular momentum; 2) Electron, neutron, and other matter waves; 3) The Abraham-Minkowski controversy.

1) The term 'cold atoms' refers to atoms cooled to less than one millionth of one degree above absolute zero, sometimes even 1000 times colder. At these temperatures the de Broglie wavelength becomes long enough to extend between the atoms in a trapped gas so that the gas becomes quantum degenerate. If the atoms are bosons they will Bose condense into a single quantum state forming a macroscopic quantum matter wave with a high degree of coherence. Bose-Einstein condensates (BECs) with orbital angular momentum (OAM) were first made by stirring a condensate with a laser beam, forming quantized vortices in the superfluid. The meeting discussed a more sophisticated method developed in a series of experiments by Bill Phillip's group at NIST [M. F. Andersen et al, "Quantized Rotation of Atoms from Photons with Orbital Angular Momentum", Phys. Rev. Lett. 97, 170406 (2006); C. Ryu, "Observation of Persistent Flow of a Bose-Einstein Condensate in a Toroidal Trap", Phys. Rev. Lett. 99, 260401 (2007)] where a Laguerre–Gaussian laser beam was interfered with a counter-propagating Gaussian beam to form a corkscrew-like interference pattern. The atoms diffract off this intensity pattern (via a stimulated Raman transition) and in so doing worth of angular momentum is transferred from the light to the external motion of each atom. This method can be used to introduce a singly-quantized vortex into a BEC, and also to rotate a BEC in a ring trap. Related experiments have also been performed in Bigelows group [K. C. Wright, L. S. Leslie, and N. P. Bigelow, "Optical control of the internal and external angular momentum of a Bose-Einstein condensate", Phys. Rev. A 77, 041601(R) (2008)].

An interesting basic question that has been tackled by some of the participants is whether optical OAM influences transitions between internal states in atoms. The selection rules for a dipole transition are $\Delta L = \pm 1$, $\Delta m = 0, \pm 1$. In traditional experiments where the laser carries no OAM the change in angular momentum of the atom is supplied by the 'spin angular momentum' (SAM) of the light: circularly polarized light carries $\pm\hbar$ of SAM depending on its handedness. The question is then whether OAM can play a similar role in transitions as SAM. A theory paper in 2002 [M. Babiker, C. R. Bennett, D. L. Andrews, and L. C. Davila Romero, "Orbital Angular Momentum Exchange in the Interaction of Twisted Light with Molecules", Phys. Rev. Lett. 89, 143601] suggested that dipole transitions would be unaffected by OAM and the lowest multipole transition that could be affected by OAM is the electric quadrupole transition. A very recent experiment has confirmed the first part of this prediction [F. Giammanco et al., "Influence of the photon orbital angular momentum on electric dipole transitions: negative experimental evidence", Opt. Lett. 42, 219 (2017)].

Open questions in this area include whether there are efficient methods to generate OAM in beams of atoms (as opposed to the case where the atoms are held in a trap) and whether having cold atoms with OAM would facilitate precision measurements. In contrast to light, atoms are massive and thus sensitive to gravity, and typically they have magnetic moments and are also electrically polarizable, and thus sensitive to material properties. Moreover, in contrast to electrons they are neutral and 1800 times more massive.

2) Electron beams are a very active area of investigation, not least because of their technological applications such as in electron microscopes. Like light and atoms, beams of these particles can be structured, including Airy and vortex beams [J. Harris et al., "Structured quantum waves", Nature Phys. 11, 629 (2015); H. Larocque and E. Karim, "A New Twist on Relativistic Electron Vortices", Physics Viewpoint Physics 10, 2 (2017)]. A significant motivation for studying such structured beams is the possibility of developing probes for magnetic structure at the nanoscale, benefitting from the fact that a vortex is a sub-wavelength structure.

Airy beams are wave-packets with the remarkable property that they appear as though they are accelerating, even though they are propagating in free space [N. Voloch et al., "Generation of Electron Airy Beams", Nature 494, 331 (2013)]. These beams also 'self-heal' if an obstacle is placed in their path. Vortex beams of electrons were first created in the lab in 2010 [J. Verbeeck, H. Tian, and P. Schattschneider, "Production and application of electron vortex beams", Nature 467, 301 (2010)]. The theoretical analysis of electron beams is involved and depends upon whether one is in the non-relativistic regime (described by the Schrödinger-Pauli equation) or the relativistic regime (described by the Dirac equation). In non-relativistic electron beams OAM and SAM are independently conserved [K. Y. Bliokh, Y. P. Bliokh, S. Savel'ev, and F. Nori, "Semiclassical Dynamics of Electron Wave Packet States with Phase Vortices," Phys. Rev. Lett. 99, 190404 (2007)].

However, in the relativistic regime these two angular momenta become coupled [K.Y. Bliokh, M. R. Dennis, and F. Nori, “Relativistic electron vortex beams: Angular momentum and spin-orbit interaction” Phys. Rev. Lett. 107, 174802 (2011)]. There has recently been some controversy as to whether a true vortex can exist in an electron beam. In particular, in a theoretical paper by one of the participants [I. Bialynicki-Birula and Z. Bialynicka-Birula, “Relativistic Electron Wave Packets Carrying Angular Momentum,” Phys. Rev. Lett. 118, 114801 (2017)] it was shown that the SAM and OAM contributions to the vorticity exactly cancel each other out. A non-zero vorticity is seen as a crucial feature of a vortex. Nevertheless, the probability density does coil around the beam in a manner highly reminiscent of a vortex; practical applications are unlikely to be affected by this cancellation.

A fundamental issue with free electrons is that they are hard to polarize; as first realized by Pauli and also Bohr. The standard Stern-Gerlach method of sorting particles according to their magnetic moment by using an inhomogeneous magnetic field fails for electrons, as the Lorentz force on such light particles leads to transverse forces which completely overwhelm the force on the magnetic moment. However, recent work by some of the participants [E. Karimi, L. Marrucci, V. Grillo, E. Santamato, “Spin-to-orbital angular momentum conversion and spin-polarization filtering in electron beams”, Phys. Rev. Lett. 108, 044801 (2012); E. Karimi, V, Grillo, R. W. Boyd, and E. Santamato, “Generation of a spin-polarized electron beam by multipole magnetic fields”, Ultramicroscopy 138, 22 (2014)] has shown that so-called ‘q-filters’, i.e. inhomogeneous magnetic fields with a quadrupole distribution, can give rise to a spin-orbit interaction that couples the OAM and the SAM of the electron beam. If a beam with the correct OAM is input to the system then a spin-polarized beam is produced.

Questions for future investigation include the further development of efficient methods for producing spin-polarized electrons, and also extending the idea of matter-wave vortex beams to neutrons [C.W. Clarke et al, “Controlling neutron orbital angular momentum”, Nature 525, 504 (2015)]. Concerning this last point, it was pointed out during the meeting that the neutron interferometer used to detect the vortex beam was essentially a white light (Michelson) style interferometer, due to the very low degree of transverse coherence in the neutron beam, and that this meant that the results were not very clear cut. The important problem of vortex beams of neutrons therefore remains ripe for further work. It was also noted that there are intriguing suggestions that self-accelerating beams can be used to prolong the lives of unstable particles [I. Kaminer et al., “Self-accelerating Dirac particles and prolonging the lifetime of relativistic fermions”, Nature Physics 11, 261 (2015)].

3) The Abraham-Minkowski controversy refers to two different expressions for the momentum density of light inside a dielectric medium derived separately by Abraham (1909) and Minkowski (1908). Although these are classical effects it is often convenient to express them in terms of the photon momentum: $p_A = p_0/n$ and $p_M = p_0 n$, where $p_0 = \hbar k$ is the free space momentum of a single photon and n is the refractive index of the dielectric. This topic was first discussed during and following talks 8 and 9 during the earlier part of the workshop.

The momentum density of light can be extracted from the stress-energy tensor, but it turns out that there is a considerable degree of arbitrariness in writing down stress-energy tensors. Examples include those due to Abraham, Minkowski and also Einstein and Laub [M. Mansuripur, Electromagnetic Force and Momentum, in *Roadmap on Structured Light*, edited by H. Rubinsztein-Dunlop, Journal of Optics 19, 013001, 8 (2017)]. This has led some authors to prefer to focus on the actual forces involved by calculating the Lorentz force on each current (bound and unbound) in the material due to the electromagnetic field [J. P. Gordon, “Radiation forces and momenta in dielectric media”, Phys. Rev. A 8, 14 (1973)].

Experiment is the ultimate arbiter in science, and over the years there have been a large number of experimental investigations that have variously agreed with either Abraham or Minkowski or have come up with their own expressions. The essential difficulty lies in separating the total momentum (which all agree must be conserved) into that due to the optical field and that due to the material. In other words, the polarization excitation induced by the optical field that propagates inside a medium is part atom, part electromagnetic field. Also, the result one finds depends upon precisely what measurements are made. One school of thought [championed by Loudon and collaborators: C. Baxter, M. Babiker, and R. Loudon, “Canonical approach to photon pressure”, Phys. Rev. A 47, 1278 (1993)] is that the Abraham expression corresponds to kinetic momentum of the light and the Minkowski expression to a canonical momentum. For example, if an experiment could measure the recoil velocity of a transparent block of dielectric as a pulse of light enters it, then using the concept of the uniform motion of the centre-of-mass of energy [a principle emphasized by Einstein: A.

Einstein, "The principle of conservation of motion of the center of gravity and the inertia of energy", Ann. Phys. 20, 627 (1906)] leads to the expectation that the momentum of the light in the block takes the Abraham value [S. M. Barnett and R. Loudon, "The enigma of optical momentum in a medium", Phil. Trans. R. Soc. A 368, 927 (2010)]. By contrast, in experiments involving diffraction it can be argued that it is more likely that the canonical (Minkowski) momentum sill be found because the wavelength, and hence the wave vector, is associated with canonical momentum.

During the discussions, participant Masud Mansuripur gave a tutorial on the classic Balazs thought experiment where a pulse of light enters a dielectric block [N. L. Balazs, "The energy-momentum tensor of the electromagnetic field inside matter", Phys. Rev. 91, 408 (1953)] and using only a knowledge of the Fresnel reflection coefficients obtained the result that the momentum density is the average of the Abraham and Minkowski results, $p = p_0(n+n^{-1})/2$ (in contrast to the Abraham momentum density result found using the principle of uniform motion of the centre-of-mass energy). Mansuripurs result is also obtained if one instead calculates the Lorentz force on the bound currents and charges in the material [M. Mansuripur, "Radiation pressure and the linear momentum of the electromagnetic field", Opt. Express 12, 5375 (2004)]. Mansuripur went on to discuss the case of the recoil of a mirror immersed in a fluid, investigated in the classic experiment by Jones [R. V. Jones and J. C. S. Richards, "The pressure of radiation in a reflecting medium", Proc. Roy. Soc. A 221, 480 (1954)] whose results support the Minkowski result, the idea being that the mirrors recoil is a direct measure of the momentum of light in a medium. One issue that Mansuripur highlighted, yet to be tested experimentally, is the effect of varying the Fresnel reflection coefficients of the mirror. This controls the interference fringes that form in the medium between the incident light and that reflected from the mirror. These intensity fringes lead to forces on the molecules in the fluid as momentum transfers between light and medium.

A number of suggestions for future investigations were made: firstly, to repeat the Jones experiment as discussed above, with varying mirror reflectivity. Secondly, it was suggested to repeat Ashkin's surface bulge experiment in which the reaction of a fluid surface to an incident beam of light is measured (the surface can either bulge outward or be pressed inward) [A. Ashkin and J.M. Dziedzic, "Radiation pressure on a free liquid surfac", Phys. Rev. Lett. 30, 139 (1973)] as the interpretation of this experiment is controversial. Thirdly, there has been a suggestion [E. A. Hinds and S. M. Barnett, "Momentum Exchange between Light and a Single Atom: Abraham or Minkowski?", Phys. Rev. Lett. 102, 050403 (2009)] for measuring the recoil of a single atom due to a laser pulse – which could potentially be attempted with cold atomic gases, provided a clever way is found to circumvent the problems associated with the random recoil due to spontaneous emission.

In *Group C* discussions centred on: 1) Degenerate down conversion; 2) Number-phase uncertainty; 3) Information content of photons, and its limits.

1) Discussion was launched by considering the consequence of a recent publication [K. A. Forbes, J. S. Ford and D. L. Andrews "Nonlocalized generation of correlated photon pairs in degenerate down-conversion", Phys. Rev. Lett. 118, 133602 (2017)]. This suggests that each photon pair produced by spontaneous parametric down conversion (SPDC) can originate from spatially distinct locations. Interest centred on whether this would affect the information content of the down-converted biphoton, and how any such change of information might manifest itself. It was agreed that considering only plane waves as excitation during SPDC is a limiting case, which led to the question of whether a superposition of wave-vectors is necessary to explain and measure this effect.

2) Number-phase quantum uncertainty remains a vexed issue, and despite the special significance of phase in connection with structured light it seems to have received little attention in the literature. It is well known that there is no simple or completely defensible form for an optical phase operator, partly due to the cyclical nature of its parameter space. In the course of the discussions on number-phase uncertainty, participant Iwo Bialynicki-Birula gave a brief tutorial on a negentropic representation, cast in terms of limits on azimuthal measurability. This leads to questions about the phase of a single photon, which it was agreed should be rephrased with regard to some form of reference. Phase is only relevant relative to another photon, or material oscillation, begging the question of whether there is an operator which could realise this and how it could best be implemented.

3) This group then discussed that the extent of accessible information of a photon, recognising that it is subject to boundary condition of the measurement process, so that in a sense every measurement of information relies on a restricted Hilbert space. This in turn should the affect the fundamental uncertainty

relation and leads to the question whether there is a correction required to account for this. Michael Mazilu pointed to his work on optical eigenmodes as offering a possible way forward [M. Mazilu et al., “Optical eigenmodes; exploiting the quadratic nature of the energy flux and of scattering interactions”, Opt. Express 19, 933 (2011)].

Other, informal group discussions focused on the C points occurring in photonic crystal waveguide modes, as observed in experiments by Kopus – a relatively unexplored area of research at the intersection between structured light and topological phases of light and matter. This group identified pressing questions that should be addressed in the near future:

1) What is the relation between the C points observed experimentally in real space modal profiles, and momentum space C points occurring in models of photonic topological insulators?

2) Present designs for photonic topological insulators are basis on silica-rod photonic crystals. For experimental probes of near-field properties, it would be better to employ a design utilising air-hole photonic crystals. Accordingly, there is a need to investigate whether such a design supports topological edge modes.

3) Once an appropriate design is identified, it would be interesting to study the near field structure of the topological edge modes, and compare against the occurrence of C points in previously-studied photonic crystal waveguides.

5 Outcomes and future plans

In the closing session, in addition to many voices of satisfaction with the meeting there were calls for a follow-up workshop, perhaps in two or three years time. In the meanwhile there will be a follow-up to another suggestion, that proceedings of work initiated by this workshop might be combined with content from this summers now-biennial International Conference on Optical Angular Momentum [Capri, 18-22 September 2017] in a special issue of some suitable journal. The organisers of that conference have now been approached, to explore this possibility.

Numerous new collaborations were instigated directly through the workshop; those that have been reported thus far involve groupings of individuals who together represent more than a third of the workshop participants, specifically including Alonso, Andrews, Bliokh, Fernandez-Corbaton, Galvez, Jones, Karimi, Khanikaev, Leader, Leykam, Loeffler, Mazilu, ODell and Salam.

6 Press release

At this early point in the twenty-first century it is already apparent that a paradigm shift is taking place in numerous areas of technology, as the new science of photonics increasingly outperforms and displaces twentieth century electronics. Prominent examples are high throughput nanoscale connectivity in IT systems, secure and enhanced telecommunications, and quantum computing. Some of the latest advances, aiming to exploit the distinctive quantum properties of light, convey information in the form of highly unconventional structured beams of light. This potentially transformative field is revolutionising many areas of optical physics, yet it has raised numerous questions in urgent need of address at the level of fundamental theory. This workshop brought top experts together in the aim of resolving these issues, laying new mathematical foundations to secure the ground for further progress.

Acknowledgement

Thanks to all other organisers and participants: above all, and on behalf of all, hearty thanks to BIRS for making this workshop possible, intellectually profitable, and thoroughly enjoyable.