Turbulent cascades in incompressible fluid systems in two dimensions

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The term "α turbulence" refers to a general class of incompressible fluid turbulence in two dimensions, in which the conserved scalar $(-\Delta)^{\alpha/2}\psi$ is advected in a velocity field with stream function ψ. This family includes surface quasi-geostrophic ($\alpha = 1$) and Navier–Stokes (NS) ($\alpha = 2$) turbulence. The advective nonlinear transfer conserves two quadratic invariants (energy and enstrophy in the NS case): $\Psi_\alpha = \int k^\alpha \Psi(k)\, dk$ and $\Psi_{2\alpha} = \int k^{2\alpha}\Psi(k)\, dk$, where $\Psi(k)$ is the streamfunction power spectrum and k is the wavenumber. According to the Kraichnan–Leith–Batchelor theory [1-3], which was originally formulated for NS turbulence and subsequently believed to apply to α turbulence in general, if the system is driven around a (forcing) wavenumber s, virtually all the injection of Ψ_α cascades toward ever-lower wavenumbers (inverse cascade), down to wavenumber zero, and virtually all the injection of $\Psi_{2\alpha}$ cascades toward a high wavenumber $k_\nu \gg s$ (direct cascade), around which $\Psi_{2\alpha}$ is dissipated. This extreme nonlinear transfer by the advective term is known as the dual cascade. For NS turbulence, analyses based on Kolmogorov's phenomenology predict that the energy (enstrophy) cascades via a $k^{-5/3}$ (k^{-3}) inertial range.

Despite numerical evidence comfirming the realization of an inverse energy cascade and of the Kolmogorov–Kraichnan $k^{-5/3}$ spectrum in NS turbulence [4-8], the dual cascade remains conjectural. No convincing evidence exists of a direct enstrophy cascade. Rather, there exist some negative results with respect to this problem. In particular, for a bounded system in equilibrium, a direct enstrophy cascade is excluded [9-11]. For an unbounded system, a direct enstrophy cascade is prohibited for all weak inverse energy cascades [7,8]. By "weak", it is understood that the inverse cascades do not carry virtually all of the energy input to the large scales. This is presumably the dynamical behaviour of finite-Reynolds number turbulence. The questions then arise whether the inverse energy cascades can become "strong" in the limit of infinite Reynolds number and whether a direct enstrophy cascade can then be realizable.

This talk reviews the dual-cascade hypothesis and explores the possibility of non-direct-cascade dynamics in α turbulence, for arbitrarily large Reynolds numbers.

References

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