

# A Level Set Approach for Surfactant Problem

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Many applications in fluids, materials and biology involve multiphase phenomena. In many multiphase problems the boundary between different phases can be formulated as a sharp interface. In such problems the interactions and dynamics of different phases determine the geometry and dynamics of the interface and vice versa. For a class of these sharp interface problems – clean interface problems – only the geometry of the interface is involved in the whole system. For instance the surface tension is present at the interface and is proportional to the local curvature of the interface. In many other multiphase problems there is more complicated physics involved on the interface. For instance, when there are surface-active agents (so-called surfactants) presented at the interface, the concentration of the surfactants is both advected by the ambient flow and diffused along the moving interface. In this case, the surface tension depends on both the geometry of the interface and the surfactant concentration. Hence the motion and geometry of the moving interface, the advection and diffusion of the surfactant concentration and the dynamics of the bulk flows are all coupled together. Physically the evolution of the surfactant concentration along the interface results in the Marangoni effect on the fluid dynamics and leads to many interesting physical phenomenon. It has drawn extensive attention of applied mathematicians and chemical engineering scientists.

The level set method, invented by Osher and Sethian, has been successfully applied to a broad range of problems in fluids, materials, image processing and computer vision. The interface is implicitly captured as the zero level set of a level set function. A geometrical problem of the interface is turned into a PDE for the level set function, and hence efficient and robust numerical schemes for PDEs can be easily adopted to deal with discontinuities and nonlinearities. However, most of the previous application of the level set method focused on capturing the location and geometry of the interface only. No physical or other quantities associated to the interface.

We propose a level set approach for solving the surfactant problem. A dual function orthogonal to the level set function defined in a neighborhood of the interface is used to represent the surfactant concentration. Then a semi-implicit Crank-Nicholson method is used to remove the stiffness of the surfactant convection-diffusion equation caused by surface diffusion. Immersed interface method is used for solving the fluid equations. Some computational results are given.