BIRS Workshop: Conformal Geometry
July 31 – August 5, 2004

TITLES and ABSTRACTS

Spyros Alexakis: TBA

Helga Baum: Essential Conformal Structures on Lorentzian Manifolds
It is well known that a compact Riemannian manifold with a group of essential conformal transformations or with a noncompact conformal group has to be the standard sphere with its canonical conformal structure. In the first part of the talk I will explain results on Lorentzian Kleinian spaces (Ch. Frances 2002/03) and on noncompact isometry groups of compact Lorentzian spaces (S. Adams/G. Stuck, A. Zeghib '95) that show, that the situation in the Lorentzian case is much more difficult. In the second part I will explain how parallel spinors on Lorentzian manifolds are related to essential conformal transformations. The second part will include results of F. Leitner and T. Leistner.

Andreas Čap & Jan Slovák (joint presentation): Parabolic Geometries
Parabolic geometries form a large class of geometric structures including conformal and quaternionic structures, CR structures, path geometries, and quaternionic contact structures. These structures can be studied in a surprisingly uniform manner, with elementary finite dimensional representation theory being an important ingredient. In the talk we will present an overview of the tools which are available for dealing with parabolic geometries. We will show the advantages of a uniform treatment by outlining constructions relating geometries of different types. These include the construction of correspondence and twistor spaces, the classical Fefferman construction and some generalizations.

Louise Dolan: Conformal Operators for Partially Massless States
The AdS/CFT correspondence is explored for "partially massless" fields in AdS space (which have fewer helicity states than a massive field but more than a conventional massless field). Such fields correspond in the boundary conformal field theory to fields obeying a certain conformally-invariant differential equation that has been described by Eastwood and others. The first descendant of such a field is a conformal field of negative norm. Hence, partially massless fields may make more physical sense in de Sitter as opposed to Anti de Sitter space.

Maciej Dunajski: Paraconformal Structures and Instantons

Robin Graham: Dirichlet-to-Neumann Map for Poincaré-Einstein Metrics

Rod Gover: A Conformally Invariant Operator Yielding the Fefferman-Graham Ambient Obstruction Tensor
The conformally invariant Bach tensor can be obtained by applying a conformally invariant operator to the Weyl curvature. The FG obstruction tensor is a natural higher order analogue of the Bach tensor for even dimensional conformal manifolds. An obvious question is whether there exists a conformally invariant operator which yields this upon its application to the Weyl curvature. In this talk I will show that there is, although the situation is significantly more subtle in the general setting than for the case of the Bach tensor. I will construct a conformally invariant differential operator with domain the space of tensor fields with Weyl tensor type algebraic symmetries, which takes values in a reducible bundle, and which has the property that upon application to the Weyl tensor it yields the FG obstruction tensor. Along the way we expose a new ambient formula for the obstruction, give an algorithm for calculating the FG obstruction tensor, obtain new proofs of its properties - including that it is an obstruction to (conformally) Einstein metrics as well as some related observations. This is collaborative work with Larry Peterson.

Kengo Hirachi: Volume Renormalization of Strictly Pseudoconvex Domains
There has been much work done about the volume renormalization of conformally compact Einstein manifolds and its relation to the conformal structure on the boundary at the infinity. We here consider the analogous concept for strictly pseudoconvex domains in a complex manifold. For such domains, we can formulate the volume expansion with respect to the complete Einstein-Kaehler metric or the Bergman volume form. Then we show that some coefficients of the expansion have intimate relation to the geometry of the boundary as a CR manifold. Part of these results are obtained by Neil Seshadri.

Doojin Hong: Intertwining Relation on Spinors
We consider the spectrum generating relation on spinors over $S^1 \times S^{n-1}$, n even with a Lorentzian signature on S^1 and the standard Riemannian on S^{n-1}.

Claude LeBrun: Optimal Metrics on 4-Manifolds

Felipe Leitner: Conformal Holonomy
In conformal geometry there exists a canonical connection, which has as structure the Mobius group. This connection gives rise to an invariant notion of conformal holonomy. We will discuss certain properties of the conformal holonomy on a smooth manifold with conformal structure, in particular, its relation to parallel tractors and the conformal Einstein condition. For homogeneous conformal spaces we find an explicit formula for the holonomy algebra and we calculate holonomy groups for bi-invariant metrics on a Lie group.

Lionel Mason: Transparent Connections on the Plane and Sphere
Zoll manifolds material redone for connections, except that the Blaschke conjecture is not true for connections and zero holonomy connections on the projective plane can be written down explicitly in terms of algebraic data for holomorphic vector bundles on $\mathbb{CP}^2$. Links with Ward's chiral model and its lump solutions, topological 'lump number' and relations to other solitons and integrable systems explained to a small extent.

Vladimir Matveev: Projective Lichnerowicz-Obata Conjecture
During the talk I will explain the proof of Theorem: Let a connected Lie group acts on a closed connected Riemannian manifold by projective transformations (recall that a diffeomorphism of a Riemannian manifold is called a projective transformation, if it takes (unparametrized) geodesics to geodesics). Then, it acts by isometries, or the metric has constant positive sectional curvature.

Gestur Olafsson: The Complex Horospherical Transform
The Radon transform or horospherical transform on Riemannian symmetric spaces is well known, and can be thought of as a tool to decompose $L^2(G/K)$ into irreducible representations. Litle has been done in this direction for non-Riemannian symmetric spaces G/H. We will assume that G/H is contained in a complex symmetric space G_C/H_C. We then define complex horospheres in G_C/H_C. An important subclass are those horosphers, that do not intersect G/H, i.e., do not have any real points. We will discuss our joint work with S. Gindikin and B. Kroetz on how to use this concept in harmonic analysis on G/H, in particular the realization of certain parts of $L^2(G/H)$. This is joint work with S. Gindikin and B. Kroetz.

Don Page: Compact Kerr-de Sitter Einstein Metrics in All Higher Dimensions
G. W. Gibbons, H. Lu, C. N. Pope and I have given in hep-th/0404008 the general Kerr-de Sitter metric in all higher dimensions, $D > 4$. From the Euclidean-signature versions, we derived the regularity conditions to give complete non-singular compact Einstein metric on associated S^{D-2} bundles over S^2. For each even D, we showed there is just one nontrivial example, given previously by Y. Hashimoto et al, hep-th/0402199. But for each odd $D > 4$, there are infinitely many examples, including infinitely many compact Einstein metrics on the topological product of S^2 and S^{2n+1} for each positive integer n.

Larry Peterson: Formulas for the Fefferman-Graham Ambient Obstruction Tensor
In even dimensions the construction of the Taylor series for the Fefferman-Graham ambient metric is, in general, obstructed by a certain trace-free symmetric 2-tensor B. We will discuss an algorithm one can use to construct an explicit formula for B in each even dimension. This is joint work with Rod Gover.

Gerd Schmalz: CR-manifolds of Engel Type
The invariants of CR manifolds with non-degenerate Levi form have been intensely studied by many authors. A general theory was developed in the case when the structure algebra is semi-simple. CR-manifolds of Engel type are 4-dimensional CR-manifolds of CR-dimension 1 with solvable structure algebra. Such CR manifold M cannot be non-degenerate in the usual sense by dimension reasons. One requires instead that the second derived differential system of the CR-distribution D coincides with the whole tangent bundle TM, i.e. (M, D) is an Engel manifold. A canonical Cartan connection of CR Engel manifolds can be derived in a very explicit way. For embedded real-analytic CR-manifolds of this type a normal form was constructed as well. This is joint work with Beloshapka and Ezhov in progress.

Neil Seshadri: Volume Renormalisation in CR Geometry
A look at how the notions of renormalised volume and the associated conformal anomaly, renormalised Chern–Gauss–Bonnet formulae, etc. have analogues in the setting of a strictly pseudoconvex domain in a complex manifold.

Vladimír Souček: Subcomplexes in BGG Sequences
BGG sequences of invariant operators on manifolds with a given parabolic structure do not form, in general, a complex. Sources and targets of individual operators in the BGG sequence can be split into irreducible components. Hence the composition of two consecutive BGG operators can be also split into components acting between the irreducible pieces. Assuming a suitable restriction on the Cartan curvature, it is possible to derive an algebraic criterion for vanishing of pieces of the composition. This general scheme will be applied in special cases of quaternionic and CR geometries. As a result, we shall describe a series of subcomplexes for these geometries. An interesting geometric interpretation can be given to a subcomplex of the BGG sequence corresponding to the adjoint representation. This is a joint work with Andreas Čap.

George Sparling: CR Geometry, Twistor Theory and Physics
I will discuss the quantum fluid approach to quantum twistor theory and its unexpected links with the geometry of ordinary differential equations.

Paul Tod: Conformal Gauge Singularities in General Relativity
Penrose has suggested that the singularities of space-time should fall into two classes: initial ones, at which the conformal curvature is non-singular while the Ricci tensor is singular, and final ones, which don't have this restriction. One can produce singularities of the first kind by taking a non-singular conformal metric and choosing a conformal factor which vanishes at an initial hypersurface. These are just singularities due to the choice of conformal gauge. How can one recognise when a curvature singularity is just a conformal gauge singularity, so that the conformal structure can be extended through the singularity?

William Ugalde: Conformal Invariants Using Wodzicki's Residue
For an even dimensional, compact, conformal manifold M without boundary we use the Wodzicki residue to construct conformal invariants. The first one is a symmetric, bilinear, differential functional B_n acting on $C^\infty(M)$. The second one is a differential operator P_n like the critical GJMS operator. The main relation is:

$$\mathrm{Wres}([2\mathcal{D}-1,f][2\mathcal{D}-1,h]) = \int_M B_n(f,h)\,dx = \int_M f P_n(h)\,dx,$$

where $\mathcal{D}$, acting on the square integrable sections of middle dimension forms of M, is the orthogonal projection on the space of exact forms.

Alfredo Villanueva: TBA