Banff International Research Station
Critical Scaling for Polymers and Percolation
May 28 to June 2, 2005

MEALS

Breakfast (Continental): 7:00 – 9:00 am, 2nd floor lounge, Corbett Hall, Sunday – Thursday
***Lunch** (Buffet): 11:30 am – 1:30 pm, Donald Cameron Hall, Sunday – Thursday
***Dinner** (Buffet): 5:30 – 7:30 pm, Donald Cameron Hall, Saturday – Wednesday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each lunch and dinner.**

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by bridge on 2nd floor of Corbett Hall). Hours: 6 am – 12 midnight. LCD projector, overhead projectors and blackboards are available for presentations. *Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155-159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.*

SCHEDULE

Saturday

16:00 Check-in begins (Front Desk – Professional Development Centre - open 24 hours)
Lecture rooms available after 18:00 (if desired)

17:30-19:30 Buffet Dinner, Donald Cameron Hall

20:00 Informal gathering in 2nd floor lounge, Corbett Hall (if desired)
Beverages and small assortment of snacks available on a cash honour-system basis.

Sunday

7:00-8:45 Breakfast
8:45-9:00 Introduction and Welcome to BIRS by BIRS Station Manager, Max Bell 159
9:00-9:50 Rohde
10:00-10:50 Lawler
10:50-11:20 Coffee Break, 2nd floor lounge, Corbett Hall
11:20-11:45 Kennedy
11:45-13:30 Lunch
13:30-14:30 Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:30-15:20 Whittington
15:25-16:00 Coffee Break, 2nd floor lounge, Corbett Hall
16:00-16:25 van Rensburg
16:30-16:55 Soteros
17:30-19:30 Dinner

Monday

7:00-9:00 Breakfast
9:00-9:50 Duplantier
10:00-10:50 Werner
10:50-11:10 Coffee Break, 2nd floor lounge, Corbett Hall
11:10-12:00 Camia
12:00-12:10 Group Photo; meet on the front steps of Corbett Hall
12:10-13:30 Lunch
13:30-15:30 free
15:30-16:00 Coffee Break, 2nd floor lounge, Corbett Hall.
16:00-16:50 Borgs
17:00-17:25 Sakai
17:30-17:55 Biskup
18:00-19:30 Dinner

Tuesday

7:00-9:00 Breakfast
9:00-9:50 den Hollander
10:00-10:50 van der Hofstad
10:50-11:10 Coffee Break, 2nd floor lounge, Corbett Hall
11:10-12:00 Bolthausen
12:00-13:30 Lunch
Free Afternoon
17:30-19:30 Dinner

Wednesday

7:00-9:00 Breakfast
9:00-9:50 Cardy
10:00-10:50 Sheffield
10:50-11:10 Coffee Break, 2nd floor lounge, Corbett Hall
11:10-12:00 Aizenman
12:00-13:30 Lunch
13:30-13:55 Angel
14:00-14:25 Holroyd
14:30-15:00 Coffee Break, 2nd floor lounge, Corbett Hall
15:00-15:25 Rolles
15:30-15:55 Jarai
17:30-19:30 Dinner

Thursday

7:00-9:00 Breakfast
9:00-9:25 Wilson
9:30 -10:20 Peres
10:25-10:55 Coffee Break, 2nd floor lounge, Corbett Hall
11:30-13:30 Lunch

Checkout by 12 noon.

** 5-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 4 pm on Thursday, although participants are still required to checkout of the guest rooms by 12 noon. **

ABSTRACTS:

M. Aizenman (Princeton): "On some lingering questions concerning the effects of periodic boundary conditions on the scaling limits"

O. Angel (UBC): "Percolation on random maps"

Abstract: Random planar maps (also known as 2-d quantum gravity) have a random geometry that makes percolation on them amenable to direct analysis. This allows us to prove that the scaling limits of various probabilities exist. In some cases explicit formulas can be derived. Generally exploration in the scaling limit of critical percolation is related to stable processes with increments having the Airy distribution.

M. Biskup (UCLA): "Scaling limit of simple random walk on supercritical percolation clusters"

Abstract: We consider a simple random walk on the (unique) infinite cluster $\mathcal C_\infty$ of bond percolation in $\mathbb Z^d$, where $d\ge2$. At each unit of time, the walk picks one of its $2d$ neighbors in $\mathbb Z^d$ at random and attempts to move to it, but the move is suppressed if the corresponding bond is not present in $\mathcal C_\infty$. We will show, under the ``usual'' scaling of space and time, that in almost every realization of $\mathcal C_\infty$ the path distribution of this walk converges weakly to that of an isotropic $d$-dimensional Brownian motion. The proof is based on analysis of the ``harmonic embedding'' of $\mathcal C_\infty$ on which the corresponding walk is an L^2-martingale. Based on joint work with Noam Berger.

E. Bolthausen (Zurich): "Exit distributions for random walks in random environments"

Abstract: We present an alternative method for discussing random walks in random environments in dimensions $d\geq3.$ The method is based on proving that the exit distributions from large sets are close to the exit distributions of standard random walk. A difficulty arises from the fact that the statement must be wrong when measuring the distance in total
variation. Therefore, a careful smoothing procedure for the exit distributions is required.

The approach is simpler than the earlier one by Bricmont and Kupiainen [BK]. For recent results for continuous time diffusions, see also Sznitman and Zeitouni [SZ].

This is joint work with Ofer Zeitouni, University of Minnesota.

[BK] Bricmont, J. and Kupiainen, A.: Random walks in asymmetric random environments. Comm. Math. Phys. 142, 345-420 (1991).

[SZ] Sznitman, A.-S. and Zeitouni, O.: An invariance principle for isotropic diffusions in random environment.

C. Borgs (Microsoft): "A proof of the local REM-Conjecture for Number Partitioning"

Abstract: The number partitioning problem (NPP) is a classical combinatorial optimization problem: Given n numbers or weights, one is faced with the problem of partitioning this set of numbers into two subsets to minimize the discrepancy, defined as the absolute value of the difference in the total weights of the two subsets. Here we consider random instances of this problem where the n numbers are i.i.d. random variables.

In this version, the NPP naturally maps into a disordered spin system, with partitions encoded by spin configurations, and discrepancies corresponding to energies. In spite of the fact that the energies of the 2^n possible spin configurations are clearly correlated, a surprising recent conjecture states that energies near any given threshold become asymptotically independent, and that the partitions corresponding to these energies become uncorrelated. In statistical physics terms, the conjecture states that the random energy model (REM) becomes exact in the thermodynamic limit.

In this talk, I describe our recent proof of this conjecture. This is joint work with J. Chayes, S. Mertens, C. Nair.

F. Camia (EURANDOM): "The Continuum Scaling Limit of 2D Critical Percolation"

Abstract: I will discuss some aspects of the convergence of critical percolation interfaces on the triangular lattice to their continuum scaling limit described in terms of SLE(6) and related random fractal curves in the plane. (Joint work with C.M. Newman)

John Cardy (Oxford): "Another percolation formula"

Abstract: Consider critical site percolation in the upper half plane. All sites on the boundary are constrained to be white, except that at the origin which is conditioned to be in the infinite black cluster. This defines two curves γ_- and γ_+ which are the boundaries of the white clusters containing the negative and positive real axes respectively.

We give formulae for the probabilities that a given point in the upper half plane lies to the left of γ_-, to the right of γ_+, or in between the two curves, in the scaling limit. These are based on conformal field theory but we suggest how they might be proved using ideas of generalised SLE. This work was jointly carried out with A. Gamsa.

Bertrand Duplantier (Saclay): " Conformal random geometry''

Abstract: We describe the fractal geometry of random conformally-invariant (CI) scaling curves in the plane. We focus on critical exponents associated with interacting random paths, by exploiting an underlying quantum gravity structure, within the framework of conformal field theory (CFT), with applications to well-recognized critical models, like $O(N)$ and Potts models, and to the Stochastic Loewner Evolution.

The multifractal function $f(\alpha,\,c)$ of the harmonic measure (i.e., electrostatic potential, or diffusion field) near any conformally invariant fractal boundary, is given as a function of the central charge c of the associated CFT. It represents the Hausdorff dimension of the set of points where the potential varies with the distance r to the fractal frontier as r^{α}. The universal mixed multifractal spectrum $f(\alpha,\lambda;c)$ describing the local winding rate λ and singularity exponent α of the harmonic measure near any CI scaling curve is given.

R. van der Hofstad (Eindhoven): " The incipient infinite cluster for percolation models above the critical dimension"

Abstract: The incipient infinite cluster (IIC) describes the infinite cluster which is on the verge of arising in critical percolation models. Kesten (1986) first constructed the incipient infinite cluster for two-dimensional percolation. We discuss his results, as well as the extension by Jarai for the two-dimensional case and the IIC constructions for percolation above the upper critical dimension. We will give 2 different constructions for the IIC for percolation above six dimensions, and 3 different constructions for the oriented percolation IIC above 4+1 dimensions. The fact that the IIC can be constructed in several natural ways indicates that it is a natural object. We will also discuss properties of the IIC, such as its dimension and, in the oriented case, its scaling limit.
This is joint work with Frank den Hollander, Antal Jarai and Gordon Slade.

F. den Hollander (EURANDOM): "Copolymers near interfaces"

Abstract: This talk is a sequel to the talk by Stu Whittington. I will describe some recent developments for a model of a two-dimensional directed copolymer near a one-dimensional flat interface. The copolymer is a random concatenation of hydrophobic and hydrophilic monomers. The interface separates oil and water. I will focus on the properties of the critical curve at which a localization/delocalization transition takes place, plotted in the plane of temperature versus affinity bias. In addition, I will describe some recent developments for a model where the interface is random, namely, where oil and water are arranged in a percolation-type fashion. The critical curve for this model is rather hard to analyze and depends on greedy percolation characteristics.

A.E. Holroyd (UBC): "Extra Heads and Stable Marriage"

Abstract: Given a discrete set of points in the plane, the well-known Voronoi tesselation allocates a polygon (of different area) to each point by assigning every location in the plane to its closest point.

The geometry of "fair" allocations in which all points are assigned equal area is richer and more mysterious. When the set of points arises from an invariant point process, it turns out that there is a unique such allocation which is "stable" in the sense of Gale and Shapley (I will explain what this means). For a picture see www.math.ubc.ca/~holroyd/stable/. Every point is assigned a bounded region with finitely many components, but it is an open problem to find ANY quantitative upper bound on the diameter of a typical region! This model can be regarded as the critical case in a family of models indexed by a parameter called the "appetite".

One application concerns shift-coupling. When viewed from the random point assigned to the origin, the law of the point process becomes that of the Palm measure.

A. Jarai (Carleton): " Infinite volume limit of high-dimensional sandpile models"

Abstract: The Abelian sandpile, also known as the Bak-Tang-Wiesenfeld model, is one of the basic examples of self-organized criticality (SOC). The general feature of SOC models is that they possess a stochastic dynamics driving them towards a stationary state characterized by power laws. A beautiful observation of Majumdar and Dhar establishes a correspondence between the sandpile model and the uniform spanning tree. We illustrate how this correspondence can be used to study properties of the thermodynamic limit of the model. (Joint work with S. Athreya and F. Redig)

T. Kennedy (Arizona): "Monte Carlo comparisons of the self-avoiding walk and SLE as parameterized curves"

Abstract: Monte Carlo simulations of the two dimensional self-avoiding walk (SAW) have given support to the conjecture that its scaling limit is SLE with parameter 8/3. These past simulations treated the SAW and SLE as subsets of the plane, i.e., the parameterizations of the curves were ignored. The SLE curve is usually parameterized by a its capacity. The SAW can be parameterized by its length, and this parameterization should lead to a natural parameterization for the scaling limit, assuming it exists. With these parameterizations the SLE curve and the SAW do not have the same distribution. We consider how to reparameterize the curves so that they do have the same distribution.

G. Lawler (Cornell): "The polymer measure in two dimensions"

Abstract: We now have a very good understanding of what the scaling limit of self-avoiding walks (SAW) SHOULD be in two dimensions. I will give a review of this process (SLE with parameter8/3) focusing on the the "infinite two-sided SAW" or "infinite self-avoiding polymer" case. I will also discuss some of the major challenges in trying to produce rigorous results about discrete SAWs.

Y. Peres (Berkeley): "Scaling limits for uniform spanning trees and for rotor-router aggregation"

Abstract: Jim Pitman conjectured that the scaling limit of the uniform spanning tree in a high-dimensional discrete torus is Aldous' continuum random tree. With D.Revelle we proved this conjecture in dimension at least 5; open problems remain in dimensions 3 and 4. The rotor-router model is a deterministic analogue of random walk invented by Jim Propp. It can be used to define a deterministic aggregation model in Z^d analogous to internal diffusion limited aggregation. I will outline the proof (obtained with L. Levine) that the scaling limit of of this model is a ball, and the mystery on the rate of convergence. Simulations indicate a much better approximation than we can prove, see e.g. http://stat-www.berkeley.edu/~peres/router/router.html *Spanning trees and Random walks are common threads in both parts of the talk.*

B. van Rensburg (York): "Square and Rectangular Vesicles in the Cubic Lattice"

Abstract: In this talk I shall briefly discuss tricritical scaling behaviour in models of square and cubical lattice vesicles. The tricritical crossover exponent in the area-perimeter model of two dimensional square vesicles is known to be $\phi = 1/2$, and in the volume-area model of three dimensional cubic vesicles it takes the value $\phi = 2/3$. Rectangular vesicles in two and three dimensions are similarly examined, and the tricritical nature of these models in area-perimeter and volume-area ensembles in two and three dimensions will be given. In particular, the crossover exponent for two dimensional rectangular vesicles in the area-perimeter ensemble is $\phi = 1$, but it takes considerable more work to determine the scaling in the three dimensional counterparts of this model.

S. Rohde (Washington): " Introduction to SLE"

Abstract: The stochastic Schramm Loewner Evolution SLE appears naturally as the scaling limit of various critical lattice processes such as percolation. It has become a powerful tool to tackle outstanding questions (dimension of Brownian frontier, self avoiding walk), as well as an object of independent interest. I will give an introduction to SLE, beginning with the (deterministic) Loewner differential equation from complex analysis.

S. Rolles (Bielefeld): "Reinforced random walks"

Abstract: I will give an overview over recent work on linearly edge-reinforced random walks. On finite graphs, the process has the same distribution as a random walk in a random environment. This representation has been a powerful tool in analyzing the process on infinite graphs. Furthermore, it has applications in Bayesian statistics. The mentioned results were obtained in joined work with Persi Diaconis, Mike Keane, and Franz Merkl.

A. Sakai (EURANDOM): "Lace expansion for the Ising model"

Abstract: The lace expansion has been successfully applied to investigate mean-field behavior for self-avoiding walk, percolation, lattice trees and lattice animals in high dimensions. In this talk, I will briefly explain the lace expansion for the Ising model, as well as its consequence for the ferromagnetic case above four dimensions assuming certain bounds on the expansion coefficients.

S. Sheffield (Berkeley): "Random geometries and conformal loop ensembles"

Abstract: We define the conformal loop ensembles, CLE_kappa, for 8/3 <= kappa <= 8; these are the conjectural scaling limits of the loops in the critical and supercritical O(n) models. CLE_{8/3} is almost surely empty. CLE_4 is a set of contour lines of the Gaussian free field. CLE_6 is the scaling limit of percolation cluster boundaries on the triangular lattice. CLE_8 consists almost surely of a single loop---and is the scaling limit of the boundary of the uniform spanning tree.

We discuss various properties of and methods of constructing these loop ensembles, which include loop soups, branching SLE(kappa, kappa-6), and a family of (non-metric) "random geometries" in which the boundaries of the loops are all straight lines (more precisely, autoparallels of a random connection). The north-going autoparallels of these connections can be viewed as flow lines of the complex vector field e^{ih}, where h is a multiple of the Gaussian free field.

This talk includes joint work with Oded Schramm, with Oded Schramm and David Wilson, and with Wendelin Werner.

C. Soteros (Saskatchewan): "Knot Probabilities of Self-avoiding Polygons After a Local Strand Passage"

Abstract: On the macroscopic scale, circular DNA can be viewed simply as a ring polymer. Experimental evidence indicates that enzymes (known as topoisomerases) act locally in DNA allowing two strands of the DNA which are close together to pass through one another (ie enabling a ``local'' strand passage) in order to disentangle the DNA to allow for replication to occur. Thus these enzymes are apparently able to act locally on the DNA and efficiently change a global property (the knot type) of the molecule. Determining what local information is important and to what extent randomness is involved in this process are open questions. This has motivated investigation of the following question about self-avoiding polygon (SAP) models of ring polymers: For the set of n-edge SAPs with a fixed knot type, how does the distribution of knots after a strand passage depend on the initial SAP knot type, SAP length n, and on the specific details of the strand passage such as where the strand passage occurs and the number of edges altered in the strand passage? As a first step towards investigating this, PhD student M. Szafron has studied a model of unknotted ring polymers in dilute solution for which it is assumed that two segments of the polymer have already been brought close together for the purposes of performing a strand passage. The conformations of the ring polymer are represented by n-edge unknotted SAPs containing a specific pattern (designed to facilitate a strand passage in which exactly two segments of the polygon pass through each other) on the simple cubic lattice. We assume that each such SAP conformation is equally likely. We have investigated, both theoretically and numerically, the distribution of knots after a strand passage has been performed at the location of the special pattern. Note that the only knots which can be obtained after such a strand passage are those which have unknotting number at most one. Given any such knot K, we expect that the proportion of SAPs having knot type K after a strand passage goes to a value lying strictly between 0 and 1 as n goes to infinity. We have proved that the rate of approach to the limit is less than exponential. In order to investigate the knot distribution as a function of polygon length, a Multiple Markov Chain Monte Carlo BFACF algorithm has been developed and proved to be ergodic for the set of polygons in question by M. Szafron. This model, theoretical results and recent estimates of the limiting after-strand-passage knot probabilities will be presented.

W. Werner (Orsay): ``Loop-soups and loop-ensembles''

Abstract: I plan to describe the Brownian loop-soups, their conformal invariance properties (joint work with Greg Lawler), and the close connections of their geometries with the Gaussian Free Field, the conjectured scaling limits of O(N) and Ising-type models (ongoing joint work with Scott Sheffield).

S. Whittington (Toronto): "Self-avoiding walk models of adsorption and localization of copolymers"

Abstract: Random copolymers are an example of a quenched random system since the sequence of comonomers is determined by a random process but, once determined, it is fixed. These molecules can adsorb at an impenetrable surface and localize at an interface between two immiscible liquids. The polymer can be modelled in various ways and this talk will focuss on the self-avoiding walk model of the conformational properties of the polymer. Quite a lot of rigorous results are available and these will be reviewed. There are still many open questions and these will be introduced and discussed.

D. Wilson (Microsoft): "SLE, Quadrangles, and Curvilinear Triangles"

Abstract: This is joint work with Oded Schramm.