



# BIRS Workshop Order, Disorder, and Transport: Recent Advances in Schrödinger Operator Theory 17–22 September 2005

# MEALS

Breakfast (Continental): 7:00 - 9:00 am, 2nd floor lounge, Corbett Hall, Sunday - Thursday
\*Lunch (Buffet): 11:30 am - 1:30 pm, Donald Cameron Hall, Sunday - Thursday
\*Dinner (Buffet): 5:30 - 7:30 pm, Donald Cameron Hall, Saturday - Wednesday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
\*Please remember to scan your meal card at the host/hostess station in the dining room for each lunch and dinner.

# MEETING ROOMS

All lectures are held in the main lecture hall, Max Bell 159. Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155-159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.

# SCHEDULE

TIME	SUNDAY	MONDAY	TUESDAY	WEDNESDAY	THURSDAY
9:00-50	Welcome; Damanik	Germinet	Denissov	Warzel	Spitzer
10:00	Coffee/Tea	Coffee/Tea	Coffee/Tea	Coffee/Tea	Coffee/Tea
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10:15-40	Kostrykin	Nakano	Leschke	Aizenman	Hundertmark
10:45-11:10	Klopp	(Open)	Kirsch	Bellissard	(Open)
			Group Photo		
11:30	LUNCH	LUNCH	LUNCH	LUNCH	LUNCH
3:45	Coffee/Tea	Coffee/Tea	Outing	Coffee/Tea	End of Program
0.10			- O uting		
4:00-50	Schenker	Müller	Outing	Stoiciu	
5:00	Break	Break	Outing	Break	
5.00	Dicak	Dietak	- Cating	Dicak	
5:15-40	Figotin	Graf	Outing	Killip	
5:45-6:10	Herbst	Erdos	Outing	Simon	
6:30	Dinner	Dinner	Dinner	Dinner	
0.50	Dimer	Dimer	Dimer	Diffier	

<sup>&</sup>lt;sup>1</sup>Long Talks: 50 minutes; Short Talks: 25 minutes. Long talks begin at 9:00 and 4:00. Last minute changes will be announced and posted in the lounge.

 $<sup>^{2}</sup>$ A group photo will be taken on Tuesday at 11:15 am, directly after the last lecture of the morning. Please meet on the front steps of Corbett Hall.

 $<sup>^{3}</sup>$ A free guided tour of The Banff Centre is offered to all participants and their guests on Sunday starting at 1:30 pm. The tour takes approximately 1 hour. Please meet in the 2nd floor lounge in Corbett Hall.





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# ABSTRACTS (in alphabetic order by speaker surname)

## Speaker: M. AIZENMAN

### Title: On the spectral statistics of random operators on trees

Abstract: The talk will address the nature of the statistics of the spectral levels for Schroedinger operator on regular tree graphs with iid potentials, resolved down to the scale of the mean level spacing. The operators are known to have both pure-point and, at weak enough disorder, absolutely-continuous spectra. It is shown that in the vicinity of randomly selected energies within the singular spectrum the corresponding level statistics is Poisson. Presented will also be some partial results concerning the nature of the level statistics for the ac spectral regions. Joint work with S. Warzel.

# Speaker: J. BELLISSARD

Title: On the Kubo Formula Abstract:

# Speaker: **D. DAMANIK**

### Title: Applications of Kato's formula to quantum dynamics

Abstract: The Plancherel Theorem allows one to relate the unitary group generated by a self-adjoint operator to the resolvent. This explicit formula, due to Kato, has recently played an important role in the study of anomalous transport and quantum dynamics associated with singular continuous spectral measures. We will explain some applications of Kato's formula to one-dimensional quantum systems that show how transfer matrix bounds give rise to bounds on transport exponents. As a consequence, we are able to prove anomalous transport for the Fibonacci Hamiltonian. This is joint work with Serguei Tcheremchantsev.

### Speaker: S. DENISSOV

### Title: Scattering for multidimensional Schrodinger operator

Abstract:We will discuss recent results in scattering theory for multidimensional Schrodinger operator with slow (random slow) decay. The so-called Simon's conjecture on the preservation of a.c. spectrum will be proved for the Caley tree. Some other cases will also be discussed. The technique used has roots in approximation theory.

# Speaker: L. ERDOS

### Title: Quantum diffusion: subtleties of the discrete model

Abstract: The long time evolution of the Schrodinger equation with a weak random potential converges

to a space diffusion in an appropriate scaling limit. This is a large scale phenomenon and physically it should not matter whether one works with the continuous or discrete Anderson model. Unlike in the localization proofs, where typically the continuous model is technically more demanding, here the situation is the opposite: the discrete model has more complications. In this talk I will explain some of them.

### Speaker: A. FIGOTIN

#### Title: Principle of linear superposition in nonlinear wave dynamics

Abstract: We study nonlinear systems of hyperbolic PDE's and difference equations on multidimensional lattices describing wave propagation. We show that a wide class of such systems, including nonlinear Schrodinger and Maxwell equations, Fermi-Pasta-Ulam model and many other not completely integrable systems, satisfy a principle of "approximate linear modal superposition". The essence of the principle is that the nonlinear evolution of a wave composed of generic wavepackets (defined as almost monochromatic waves) reduces with very high accuracy to independent nonlinear evolution of the involved wavepackets. Such an independence of wavepackets in the course of evolution persists for times long enough to observe fully developed nonlinear phenomena. An essential common property of systems obeying the principle of approximated superposition is the absolute continuity of the spectrum of underlying linear component. Such systems are not covered by either the classical complete integrability theory, including Birkhoff separation of variables theorem, or the KAM theory, nor by the Lax pairs method. In particular, our approach provides a simple justification for numerically observed effect of almost non interaction of solitons passing through each other without any recourse to the complete integrability. The mathematical framework developed for establishing the approximate linear superposition principle includes the theory of analytic functions of infinite-dimensional variable and the asymptotic theory of oscillatory integrals.

#### Speaker: F. GERMINET

#### Title: Localization for Schrödinger operators with Poisson random potentials

Abstract: We prove exponential localization for the random Poisson Schrödinger operator, in any dimension, at the bottom of the spectrum. We also deal with the large density regime. We further prove dynamical localization and finite multiplicity of the eigenvalues. In collaboration with A. Klein and P. D. Hislop.

#### Speaker: G. M. GRAF

#### Title: Equality of bulk and edge Hall conductances in a mobility gap

Abstract: (Joint work with A. Elgart, J. Schenker.) I will present two interpretations of the integer quantum Hall effect on the basis of either bulk or edge currents, introduce the corresponding conductances, and heuristically show that they are equal. In a more strict part, I will consider quantum Hall systems for which the Fermi energy falls in a band where bulk states are localized, this being the prerequisite for a Hall plateau. After defining the edge conductance in a manner appropriate to this situation, I will prove its equality to the bulk value.

### Speaker: I. HERBST

#### Title: Particle in a long range magnetic fieldParticle in a long range magnetic field

Abstract: I will talk about the temporal asymptotic behavior of states of a quantum particle in  $\mathbb{R}^2$  in a magnetic field which is asymptotically homogenous of degree -1. It turns out that because of the long range nature of the field, the asymptotic behavior does not resemble in the least the behavior of a particle which is asymptotically free. Some of the results and techniques of proof will be described. This is work in collaboration with H. Cornean and E. Skibsted.

### Speaker: **D. HUNDERTMARK**

#### Title: The Strichartz inequality

Abstract: We have a nice interpretation of this thing in dimension oner and two now (for Schr"odinger) and in dimension two and three (for wave). It boils down to finding the largest eigenvaule of a linear operator. That linear operator happens to be a multiple of a projection operator, so things are really easy.

## Speaker: R. KILLIP

#### Title: Eigenvalue statistics with decaying potentials

Abstract: I will describe some on-going work with Irina Nenciu and Mihai Stociu about the eigenvalue spacings for CMV matrices with random decaying potentials.

### Speaker: W. KIRSCH

### Title: Anderson localization and Lifshitz tails for surface potentials

Abstract: We investigate random potentials concentrated near a hyperplane  $\mathbb{R}^{d_1}$  in  $\mathbb{R}^d$ . Under mild conditions we prove that the density of surface states for such a potential has Lifshitz asymptotics near the bottom of the spectrum with Lifshitz exponent  $d_1/2$ . This result is used as an initial scale estimate to prove localization at low energies. The main new technical ingredience are results on potentials which are periodic in some directions. In particular, we prove a gap estimate for these partially periodic potentials. The talk is based on a joint work with Simone Warzel. Analogous, in fact stronger, results for the discrete case were obtained in a joint work with Frederic Klopp.

## Speaker: F. KLOPP

# Title: Lifshitz Tails in Constant Magnetic Fields

Abstract: (joint with G. Raikov) We consider the 2D Landau Hamiltonian H perturbed by a random alloy-type potential, and investigate the Lifshitz tails, i.e. the asymptotic behavior of the corresponding integrated density of states (IDS) near the edges in the spectrum of H. If a given edge coincides with a Landau level, we obtain different asymptotic formulae for power-like, exponential sub-Gaussian, and super-Gaussian decay of the one-site potential. If the edge is away from the Landau levels, we impose a rational-flux assumption on the magnetic field, consider compactly supported one-site potentials, and formulate a theorem which is analogous to a result obtained by F. Klopp and T. Wolff in the case of a vanishing magnetic field.

## Speaker: V. KOSTRYKIN

## Title: Lipschitz continuity of the IDS for sign-indefinite potentials

Abstract: The talk is devoted to the study of spectral properties of random Schrödinger operators with signindefinite single-site potentials. Using a finite section method for Toeplitz matrices, we prove a Wegner estimate and Lipschitz continuity of the integrated density of states for distributions with density of bounded total variation. In certain disorder regimes we are able to prove localization near spectral edges. This is a joint work with Ivan Veselić (Chemnitz).

### Speaker: H. LESCHKE

# Title: Ballistic transport in random magnetic fields with anisotropic long-ranged correlations

Abstract:(joint work with Simone Warzel and Alexandra Weichlein) We present energetic and dynamic properties of a charged quantum particle in the Euclidean plane subject to a perpendicular random magnetic field of Gaussian or Poissonian type with non-zero mean. Our results refer simply and solely to the limiting case in which the realizations of the magnetic field depend (almost surely) only on one of the two Cartesian coordinates for the plane. They are therefore certain "random analogs" of results obtained by IWATSUKA in 1985 and by MANTOIU and PURICE in 1997.

# Speaker: P. MÜLLER

# Title: On Mott's formula for the ac-conductivity in the Anderson model

Abstract: We study the ac-conductivity in linear response theory in the general framework of ergodic magnetic Schrödinger operators. For the Anderson model, if the Fermi energy lies in the localization regime, we prove that the ac-conductivity is bounded by  $C\nu^2(\log \frac{1}{\nu})^{d+2}$  at small frequencies  $\nu$ . This is to be compared to Mott's formula, which predicts the leading term to be  $C\nu^2(\log \frac{1}{\nu})^{d+1}$ .

## Speaker: F. NAKANO

Title: "Repulsion" of localization center in Anderson model

Abstract: We consider a definition of the localization center of localized states, and show that they are repulsive in some sense, as the corresponding energies being close together.

## Speaker: J. SCHENKER

Title: A first step toward localization for interacting electrons: weak Hartree-Fock interactions at large disorder and positive temperature

Abstract: I will discuss localization for interacting electrons in the context of a two band Hubbard type model with Hartree-Fock ("mean field") interactions. A proof of localization for this model can be accomplished in two steps: 1) solving a temperature dependent non-linear fixed point equation for an effective correlated random potential and 2) establishing spectral and dynamical localization for the resulting effective one particle Hamiltonian. We have carried out this program for weak interactions at large disorder and positive temperature by showing that the solution to the fixed point problem satisfies the requirements of the Aizenman-Molchanov fractional moment technique. (Joint work S. Chiesa.)

### Speaker: B. SIMON

## Title: Perturbations of Periodic Jacobi Matrices

Abstract: I will describe work in progress with Damanik and Killip that extends some of the "big theorems" for slowly decaying perturbations of the free Jacobi matrix (for example, Denisov-Rakhmanov and Killip-Simon) to perturbations of periodic Jacobi matrices.

## Speaker: W. SPITZER

### Title: A geometric proof of extended states for the Anderson model on a tree graph

Abstract: We study the standard Anderson model on a tree graph. In order to prove the existence of absolutely continuous spectrum for small disorder we introduce a new geometric approach to control the Green's function. The proof is based on a contraction property of a transformation of hyperbolic spaces, and reduces to an estimate of a single function whose singularities we resolve using a series of blow- ups familiar in algebraic geometry. This preliminary report is based on joint work with Richard Froese and David Hasler.

### Speaker: M. STOICIU

# Title: The Distribution of the Eigenvalues of Random CMV Matrices

Abstract: Recent developments in the theory of orthogonal polynomials on the unit circle have emphasized the importance of the CMV matrices (five-diagonal unitary matrices of a particular form). We consider certain classes of random CMV matrices and we prove that the asymptotic local statistical distribution of their eigenvalues is Poisson. This means that, as in the case of random Schrödinger operators, there is no local correlation between the eigenvalues of large random CMV matrices.

# Speaker: S. WARZEL

Title: Stability of the absolutely continuous spectrum of random Schroedinger operators on tree graphs Abstract: The subject of this talk are discrete or continuum Schroedinger operators on regular tree graphs with weak but extensive disorder in the form of a random potential or random edge length. For such operators we present a new method for establishing the persistence of absolutely continuous spectrum in the presence of disorder.