A dynamical approach to rigidity of automorphisms

Alex Furman (Univ. Ill. at Chicago),
Barak Weiss (Ben-Gurion University)

June 24, 2006 – July 1, 2006

1 Overview of the Field

Let Γ be a lattice in a second countable locally compact topological group G, and let H, F be closed subgroups of G. We are interested in the action of Γ on G/H, and in the measurable Γ-automorphisms $G/H \to G/F$. The main question to consider is:

Does any Γ-map extend to a G-map between these spaces?

The special case $H = F$ is known as the Γ-centralizer question. For certain choices of G, H, Γ, it was proved in recent work of Shalom-Steger [2] and Furman [1] that any measurable measure class preserving Γ map $G/H \to G/H$ coincides almost everywhere with a G-map. Note there is a natural measure class on G/H obtained by decomposing haar measure on G with respect to haar measure on H. Also note that, since the G-action is transitive, the G-maps are easily described algebraically, for example in the case $G/H \to G/H$ they are parametrized by the group $N_G(H)/H$. Similar questions may be asked for joinings for the Γ actions on such spaces, self-joinings, quasi-factors, etc.

A far-reaching conjecture is:

Main conjecture: *Suppose G is a locally compact second countable group, Γ a lattice in G, and H and F closed subgroups such that Γ acts ergodically on G/H. Then any measure class preserving Γ-map $G/H \to G/F$ coincides almost everywhere with a G-map.*

2 Scientific Progress Made

During the workshop the main conjecture was proved under the following additional conditions:

- G, H, F are the k-points of an algebraic group $\mathbf{G}$ defined over k, and its k-subgroups $\mathbf{H}, \mathbf{F}$, where k is a local field of characteristic zero.
- Γ is a lattice in G.
- H has no proper normal cocompact subgroups.
- The one-dimensional subgroups of H acting ergodically on G/Γ topologically generate H.

In view of the Howe-Moore theorem, the above conditions are satisfied for a very large class of subgroups, e.g. whenever $\mathbf{G}$ is semisimple with no compact factors and $\mathbf{H}$ is generated by unipotent and k-split semisimple elements. The result vastly extends the previous results of [2, 1], and the proof is significantly easier.

The proof discovered during the workshop is flexible and there is hope that it could be adapted to other setups, for example when Γ is not necessarily assumed to be a lattice. Also more general results in the framework of factors, quasi-factors, and joinings were obtained, too technical to list here.

3 Outcome of the Meeting

We are very grateful to BIRS for ideal working conditions for this project. We are very pleased with the mathematical fruit of our labor and expect to submit it for publication in the near future.

References

[1] A. Furman, Rigidity of group actions on infinite volume homogeneous spaces II, preprint (2006) submitted.

[2] Y. Shalom and T. Steger, Rigidity of group actions on infinite volume homogeneous spaces, unpublished.