BIRS Workshop 06w5039
Nonlinear diffusions: entropies, asymptotic behavior and applications
April 15 - April 20, 2006

MEALS

Breakfast (Continental): 7:00 - 9:00 am, 2nd floor lounge, Corbett Hall, Sunday - Thursday
*Lunch (Buffet): 11:30 am - 1:30 pm, Donald Cameron Hall, Sunday - Thursday
*Dinner (Buffet): 5:30 - 7:30 pm, Donald Cameron Hall, Saturday - Wednesday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each lunch and dinner.**

MEETING ROOMS

All lectures are held in the main lecture hall, Max Bell 159. *Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155-159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.*

SCHEDULE

	Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday
7:00-9:00		Continental Breakfast, 2nd floor lounge, Corbett Hall				
9:00-9:45		Daskalopoulos	Mazón	Markowich	Jüngel	Hamilton
9:45-10:30		Ambrosio	Lee	Gamba	Kurganov	Vázquez
10:30-11:00		Coffee Break, 2nd floor lounge, Corbett Hall				
11:00-11:45		Denzler	Ghoussoub	Carrillo	Sturm	
11:45-12:05		Aronson	Laurençot	Panferov	Cáceres	
12:05-12:15			Group Photo[1]			
11:30-13:30		Buffet Lunch, Donald Cameron Hall				
13:15-14:15			Guided Tour[2]	free afternoon		
14:45-15:30		Chow	Agueh Andreu	free afternoon	Chertock Matthes	
15:30-16:00		Coffee Break, 2nd floor lounge, Corbett Hall (except Tues.)				
16:00-16:45		DiFrancesco Kim	Arnold Fellner	free afternoon	Slepcev	
16:45-17:30		Nazaret Gentil	Puel Gualdani	free afternoon	Loeper	
17:30-19:30	Buffet Dinner, Donald Cameron Hall					

[1]A group photo will be taken on Monday, directly after the last lecture of the morning. Please meet on the front steps of Corbett Hall.

[2]A free guided tour of The Banff Centre is offered to all participants and their guests on Monday starting at 1:15 pm. The tour takes approximately 1 hour. Please meet in the 2nd floor lounge in Corbett Hall.

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ABSTRACTS
(in alphabetic order by speaker surname)

AGUEH Martial

Title: SHARP GAGLIARDO-NIRENBERG INEQUALITIES AND OPTIMAL TRANSPORTATION THEORY

Abstract: It is known that best constants and optimal functions of many geometric inequalities can be obtained via the Optimal transportation theory. But so far, this approach has been successful for a special subclass of the Gagliardo-Nirenberg inequalities, namely, those for which the optimal functions involve only power laws. In this work, we explore the link between Optimal transportation theory and all the Gagliardo-Nirenberg inequalities. We show that the optimal functions can be explicitly derived from a specific nonlinear ordinary differential equation, which appears to be linear for a subclass of the Gagliardo-Nirenberg inequalities or when the space dimension reduces to 1. In these cases, we give the explicit expressions of the optimal functions, along with the sharp constants of the corresponding Gagliardo-Nirenberg inequalities.

AMBROSIO Luigi

Title: CONVERGENCE OF ITERATED TRANSPORT MAPS AND NONLINEAR DIFFUSION EQUATIONS

Abstract: We analyze the asymptotic behaviour of iterated transport maps arising in the implicit time discretization of nonlinear diffusion equations, modelled on the porous medium equation. This analysis allows to answer affermatively to a question raised in a recent paper by Gangbo, Evans and Savin, in connection with gradient flows of a class of polyconvex energy functionals.

ANDREU Fuensanta

Title: RENORMALIZED AND WEAK SOLUTIONS FOR A DEGENERATE ELLIPTIC-PARABOLIC PROBLEM WITH NONLINEAR DYNAMICAL BOUNDARY CONDITIONS

Renormalized and Weak Solutions for a Degenerate Elliptic-parabolic Problem with Nonlinear Dynamical Boundary Conditions

F. Andreu[a], N. Igbida[b], J. M. Mazón[a] and J. Toledo[a]

[a]Dept. de Análisis Matemático, Universitat de Valencia.

[b]Université de Picardie Jules Verne

We are interested in the following degenerate elliptic-parabolic problem with nonlinear dynamical boundary conditions

$$P_{\gamma,\beta}(f,g,z_0,w_0) \quad \begin{cases} z_t - \operatorname{div} \mathbf{a}(x,Du) = f, \ z \in \gamma(u), & \text{in } Q_T :=]0,T[\times\Omega \\ w_t + \mathbf{a}(x,Du)\cdot\eta = g, \ w \in \beta(u), & \text{on } S_T :=]0,T[\times\partial\Omega \\ z(0) = z_0 \ \text{ in } \Omega, \ w(0) = w_0 \ \text{ in } \partial\Omega. \end{cases}$$

The nonlinear elliptic operator div $\mathbf{a}(x,Du)$ is modeled on the p-Laplacian operator $\Delta_p(u) = \operatorname{div}\,(|Du|^{p-2}Du)$, with $p > 1$, γ and β are maximal monotone graphs in $\mathbb{R}^2$ such that $0 \in \gamma(0)$ and $0 \in \beta(0)$. Particular instances of this problem appear in various phenomena with changes of phase like multiphase Stefan problem and in the weak formulation of the mathematical model of the so called Hele Shaw problem. Also, the problem with non-homogeneous Neumann boundary condition is included.

Under certain assumptions on γ, β and $\mathbf{a}$, we prove existence and uniqueness of renormalized solutions of problem $P_{\gamma,\beta}(f,g,z_0,w_0)$ for data in L^1, and also that these renormalized solutions are weak solutions if the data are in $L^{'p}$.

ARNOLD Anton

Title: Improved decay rates for the large time behavior of parabolic equations

Abstract: It is well known that the solution to heat equation behaves for large time like the Gaussian with the same moments of order 0, 1, and 2. And the "distance" from the solution to this Gaussian can be measured or estimated conveniently in terms of the relative entropy. Surprisingly, the known estimates can be improved by using the relative entropy of the initial function with respect to a Gaussian with a smaller second moment.

ARONSON Donald G.

Title: Some aspects of the focusing problem for the porous medium equation

Abstract: I will discuss the role of self-similar solutions in the analysis of the focusing problem for the porous medium equation.

CÁCERES Maria José

Title: Long time behavior of linearized fast diffusion equations using a kinetic approach

Abstract: We study the long time behavior of linearized fast diffusion equations showing that their rate of convergence towards the self–similar solution can be related to the number of moments of the initial datum that are equal to the moments of the self–similar solution at a fixed time. As a consequence, we find an improved rate of convergence to self–similarity in terms of a Fourier based distance between two solutions.

The key idea to prove the results is the asymptotic equivalence of a collisional kinetic model of Boltzmann type with a linear Fokker-Planck equation with nonconstant coefficients, for which the recovering of the rate of decay in terms of the Fourier based distance is immediate. (Joint work with G. Toscani)

CARRILLO José A.

Title: Tanaka Theorem for Inelastic Maxwell Models

Abstract: We show that the Euclidean Wasserstein distance is contractive for inelastic homogeneous Boltzmann kinetic equations in the Maxwellian approximation and its associated Kac-like caricature. This property is as a generalization of the Tanaka theorem to inelastic interactions. Even in the elastic classical Boltzmann equation, we give a simpler proof of the Tanaka theorem than the ones by Tanaka (1978) and Villani (2002). Consequences are drawn on the asymptotic behavior of solutions in terms only of the Euclidean Wasserstein distance.

CHERTOCK Alina

Title: Strongly Degenerate Parabolic Equations with Saturating Diffusion

Abstract: We first consider a nonlinear diffusion equation used to describe propagation of thermal waves in plasma or in a porous medium, endowed with a mechanism for flux saturation, which corrects the nonphysical gradient-flux relations at high gradients. We study the model both analytically and numerically, and discover that in certain cases the motion of the front is controlled by the saturation mechanism. Instead of the typical infinite gradients, resulting from the linear flux-gradients relations, we obtain a discontinuous front, typically associated with nonlinear hyperbolic phenomena. We prove that if the initial support is compact, independently of the smoothness of the initial datum inside the support, a shock discontinuity at the front forms in a finite time, and until then the front does not expand.

Adding a nonlinear convection enhances the conditions for a breakdown. In fact, the most interesting feature is the effect of criticality, that is, unlike small amplitude solutions that remain smooth at all times, large amplitude solutions may develop discontinuities. This feature is easily seen via the analysis of traveling waves: while small amplitude kinks are smooth, in large amplitude kinks part of the upstream-downstream transition must be accomplished via a discontinuous jump (subshocks). Thus induced discontinuities may persist indefinitely since the traveling waves represent a forced motion. Unlike the classical Burgers case, here, due to the saturation of the diffusion flux, the viscous forces have a bounded range. When the inertial forcing exceeds a certain threshold, the disparity between the inertial and dissipative forces is resolved by formation of a discontinuity.

Short talk title: Same.

CHOW Bennett

Title: Combinatorial Curvature Flows

Abstract: We will discuss geometric flows of both simplicial surfaces and polygons in the plane. The combinatorial Ricci flow of surfaces takes triangulated surfaces which are piecewise hyperbolic, euclidean, or spherical and tries to make the curvatures at the vertices constant. It is related to Thurston's circle packing metrics. At the moment, very little seems to be known about combinatorial flows of planar polygons. We start with a linear equation which can be analyzed.

DASKALOPOULOS Panagiota

Title: Type II collapsing of maximal Solutionscenterlinelarge f to the Ricci flowType II collapsing of maximal Solutions to the Ricci flow

Abstract: We consider the initial value problem $u_t = Deltalogu$, $u(x,0) = u_0(x)ge0$ in $BbbR^2$, corresponding to the Ricci flow, namely conformal evolution of the metric $u(dx_1^2 + dx_2^2)$ by Ricci curvature. It is well known that the maximal solution u vanishes identically after time $T = frac14piint_{BbbR^2} u_0$. We provide upper and lower bounds on the geometric width of the solution and on the maximum curvature. Using these estimates we describe precisely the Type II collapsing of u at time T: we show the existence of an inner region with exponentially fast collapsing and profile, up to proper scaling, a soliton cigar solution, and the existence of an outer region of persistence of a logarithmic cusp. This is the only Type II singularity which has been shown to exist, so far, in the Ricci Flow in any dimension.

DENZLER Jochen

Title: Delocalized source type solutions for fast diffusion and porous medium

Abstract: We describe a family of explicit solutions to the porous medium and fast-diffusion equations, which are not radially symmetric, and we study their asymptotic behavior. Simarly as for the Barenblatt solution, there is reason to hope that these new solutions shed light on the asymptotics for general solutions to PME and FDE. This is joint work with Robert McCann.

DI FRANCESCO Marco

Title: The Keller-Segel model for chemotaxis with prevention of overcrowding: linear vs nonlinear diffusion.

Abstract: We shall discuss the effects of linear and nonlinear diffusion in the large time asymptotic behavior of the Keller-Segel model of chemotaxis prevention of overcrowding. In the linear diffusion case we provide several sufficient condition for the diffusion part to dominate and yield decay to zero of solutions. We also provide an explicit decay rate towards self–similarity. Moreover, we prove that no stationary solutions with positive mass exist. In the nonlinear diffusion case we prove that the asymptotic behavior is fully determined by whether the diffusivity constant in the model is larger or smaller than the threshold value $e = 1$. Below this value we have existence of non-decaying solutions and their convergence (along subsequences) to stationary solutions. For $e > 1$ all compactly supported solutions are proved to decay asymptotically to zero, unlike in the classical models with linear diffusion, where the asymptotic behavior depends on the initial mass.

FELLNER Klemens

Title: Entropy Methods for Systems Combining Diffusion and Nonlinear Reaction

Abstract: Reaction-diffusion systems and coagulation and fragmentation of polymers are examples of models which combine diffusion and nonlinear reactions in terms of an entropy (free energy) functional. We present entropy methods (i.e. the idea how a functional inequality relates the entropy relative to equilibrium with the entropy-dissipation accounting the conserved quantities) to study global existence and long-time behaviour.

In a first part, we discuss in particular a reaction-diffusion system modelling four chemical substances with individual diffusion coefficients, which react by reversible mass-action kinetics within a bounded domain. In this case - up to our knowledge - global L^∞ bounds are unknown, but for which, at least in 1D, a polynomially growing L^∞ bound can be established due to the decay of the entropy. We improve the existing theory in 1D by getting 1) almost exponential convergence in L^1 to the steady state via a precise entropy-entropy dissipation estimate, 2) an explicit global L^∞ bound via interpolation of a polynomially growing H^1 bound with the almost exponential L^1 convergence, and 3), finally, explicit exponential convergence to the steady state in all Sobolev norms.

In a second part, we present work in progress on the Aizenman-Bak model of coagulating and fragmenting polymers with non-degenerate size-dependent diffusion coefficients. Again in 1D, we prove a-priori estimates which show immediately smoothing in time and space while in size-distribution solutions are decaying faster than any polynomial. Moreover, we are very positive to be able to establish a sharp enough entropy entropy-dissipation estimate, which will imply - similar to the strategy above - explicit exponential convergence towards the steady state.

KURGANOV Alexander

Title: Effects of Saturating Diffusion

Abstract: I will talk about strongly degenerate parabolic PDEs with a saturating diffusion flux. The simplest model is:

$$u_t = Q(u_x)_x, \qquad (1)$$

where Q is a bounded increasing function. Such a nonlinear diffusion is "weaker' than the linear one present in the "standard' heat equation,

$$u_t = u_{xx}.$$

The effect of the saturating diffusion in (1) is manifested in a possible "delayed diffusion" phenomenon: initial discontinuities may be smeared out only after a certain (finite) time.

In the past 10 years, Philip Rosenau (Tel-Aviv University) and I together with several of collaborators of ours have been studying various effects of saturating diffusion on convection-diffusion equations,

$$u_t + f(u)_x = [u^n Q(u_x)]_x, \quad n \geq 0,$$

porous media type equations,

$$u_t = [u^n Q(u_x)]_x, \quad n > 0,$$

and reaction-diffusion equations,

$$u_t = Q(u_x)_x - f(u).$$

We have obtained several interesting, sometimes rather surprising results, and I will present some of them, including the most recent ones.

GAMBA Irene

Title: Self-similar asymptotics for generalized non-linear kinetic Maxwell models

Abstract: We study long time dynamics to solutions of initial value problems to a rather general multi-linear kinetic models of Maxwell type which may describe qualitatively different processes in applications, but have many features in common. In particular we focus in the existence, uniqueness and asymptotics to self-similar (or dynamical scaling) solutions. We use a relationship of spectral properties of the problem in Fourier space to the existence and asymptotic behavior of the solution of the original initial value problem as well as the characterization of the domain of attraction to self-similar states. In particular we show that the self-similar asymptotic dynamics imply that the solutions of these type of problems evolve to 'infinitely divisible 'process from the probabilistic viewpoint, where the tails and time decay laws are classified from the spectral properties related to the original problem.par Examples are models of Maxwell type in classical space homogeneous, elastic or inelastic Boltzmann equation, and the elastic Boltzmann equation in the presence of a thermostat, all with finite or infinite initial energy, as well as Pareto distributions models in economy, and Smoluckowski type of equations.

This is work in collaboration with A. Bobylev and C. Cercignani.

GENTIL Ivan

Title: About modified logarithmic Sobolev inequalities and applications to concentration inequalities

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GHOUSSOUB Nassif

GUALDANI Maria Pia

Title: Discontinuous Galerkin method for dissipative quantum models

Abstract: The motion of a particle ensemble interacting with an environment can be described with a Wigner approach, where the interaction mechanisms are taken into account by a Fokker-Planck scattering term. Solutions to such kind of models are characterized by an oscillatory behavior; a modified Discontinuous Galerkin method based on non-polynomial function space is used for the numerical approximation to this problem. The choice in the scheme of trigonometric functions for the finite element space allows for a better approximation to the highly oscillatory solutions.

ILLNER Reinhard

JUENGEL Ansgar

Title: Algorithmic derivation of entropy-entropy dissipation inequalities by solving polynomial decision problems

Abstract: The proof of analytical and numerical properties of solutions to nonlinear evolution equations is usually based on appropriate a priori estimates and monotonicity properties of Lyapunov functionals, which are called here entropies. These estimates can be shown by subtle integration by parts. However, such proofs are usually skillful and not systematic. In this talk a systematic method for the derivation of a priori estimates for a large class of nonlinear evolution equations of even order in one and several variables with periodic boundary conditions is presented. This class of equations contains the thin-film equations, for instance.

The main idea is the identification of the integrations by parts with polynomial manipulations. The proof of a priori estimates is then formally equivalent to the solution of a decision problem known in real algebraic geometry, which can be solved algorithmically. The method also allow us to prove the non-existence of entropies and to derive new logarithmic Sobolev inequalities.

KIM Yong Jung

Title: Potential comparison and long time asymptotics of convection, diffusion and p-Laplacian in one space dimension

Abstract: Recently a potential comparison technique has been developed for solutions to a nonlinear diffusion equation. This method can be applied to other problems after a suitable modification. In this talk this technique will be discussed for the cases in the title. The convergence order of the magnitude of the solution itself is shown in L^1 norm when the three terms are together. Convergence order $1/t$ is shown when only one of them exists under extra conditions for the initial value.

KOCH Herbert

LAUGESEN Richard S.

LAURENÇOT Philippe

Title: CONVERGENCE TO STEADY STATES FOR A ONE-DIMENSIONAL VISCOUS HAMILTON-JACOBI EQUATION WITH DIRICHLET BOUNDARY CONDITIONS

Abstract: The convergence to steady states of solutions to the one-dimensional viscous Hamilton-Jacobi equation $\partial_t u - \partial_x^2 u = |\partial_x u|^p$, $(t,x) \in (0,\infty) \times (-1,1)$ with homogeneous Dirichlet boundary conditions is investigated for $p \in (0,1)$. For that purpose, a Liapunov functional is constructed by the approach of Zelenyak (1968). Instantaneous extinction of $\partial_x u$ on a subinterval of $(-1,1)$ is also shown for suitable initial data.

LEDOUX Michel

LEE Ki Ahm

Title: GEOMETRIC PROPERTIES IN ELLIPTIC AND PARABOLIC PROBLEMS

Abstract: In this talk, we are going to discuss the geometric properties in parabolic flows, for example porous medium equations, parabolic p-Laplace equations, and free boundary problems. And the study of the asymptotic behavior of these flows will give us another promising method to find the geometric properties of solutions in elliptic problems

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LOEPER Grégoire

Title: REGULARITY OF MAPS SOLUTIONS OF OPTIMAL TRANSPORTATION PROBLEMS

Abstract: Given two probability measures μ, ν and a cost function $c(x,y)$, one seeks to minimize $\int c(x,T(x))d\mu(x)$ among all maps T that push forward μ onto ν. This work is concerned with the contintuity of the minimizers. Based on the Monge-Kantorovitch duality, the minimizers are expressed though the gradient of a "c-convex" potential ϕ (c-convexity being the appropriate generalization of convexity for general cost c instead of $c(x,y) = |x-y|^2$). This potential will solve a Monge-Ampère type equation of the form $\det(M(x,\nabla\phi) + D^2\phi) = f(x,\nabla\phi)$. Ma, Trudinger and Wang found a sufficient condition on the cost function so that for smooth positive measures, the optimal T is smooth. I will show that this condition is actually a necessary condition for regularity, and that it is equivalent to the connectedness of the c-subdifferential of c-convex functions. Finally, I will show that when the Ma, Trudinger and Wang condition is satisfied in a strict sense, one can obtain continuity of the optimal T (i.e. C^1 regularity for the potential ϕ) under lower requirements than what is needed for the usual Monge-Ampère equation $\det D^p hi = f$.

MARKOWICH Peter

Title: NONLINEAR DIFFUSIONS AS DIFFUSION LIMITS OF KINETIC EQUATIONS WITH RELAXATION COLLISION KERNELS Peter Markowich (joint work with J. Dolbeault, D. Ölz and C. Schmeiser)

(peter.markowich@univie.ac.at)

Faculty of Mathematics, University of Vienna, Nordbergstr. 15, A-1090 Wien, Austria

At the kinetic level, it is easy to relate the parameters with simple physical quantities, but the price to pay is the high dimensionality of the phase space. On the other hand, hydrodynamical equations or parabolic models are in principle simpler to compute, but their direct derivation is far less intuitive. This motivates the study of hydrodynamic or diffusion limits and in our approach, local or global Gibbs states will be considered as basic input for the modeling. This is a very standard assumption for instance in semiconductor theory when one speaks of Fermi-Dirac distributions, or when one considers polytropic distribution functions in stellar dynamics. It is the purpose of this work to provide a justification of nonlinear diffusions as limits of appropriate simple kinetic models.

Let us mention that in astrophysics, power law Gibbs states are well known (see, *e.g.*, [1], and [7] for some mathematical properties of such equilibrium states).

In our approach [2], [3] we say nothing about the physical phenomena responsible for the relaxation towards the local Gibbs state and, on the long time range, towards the global Gibbs state. We introduce at the kinetic level a caricature of a collision kernel, which is simply a projection onto the local Gibbs state with the same spatial density, thus introducing a local Lagrange multiplier which will be referred to as the pseudo Fermi level.

We prove existence and uniqueness of solutions to the kinetic model under the assumption of boundedness of the initial datum and prove the convergence to a global equilibrium. With the parabolic scaling we rigorously prove the convergence of the solutions to a macroscopic limit using compensated compactness theory. Most notably, we are able to reproduce non-linear diffusion equations $\partial_t \rho = \Delta(\rho^m) + \nabla \cdot (\rho \nabla V)$, ranging from porous medium equation to fast diffusion, $0 < m < \frac{5}{3}$, as macroscopic limits by employing the appropriate energy profiles.

In the mathematical study of diffusion limits for semiconductor physics, more results are known, starting with [4],[5]. Other reference papers are [6] and [8].

Refernces [1] Binney, J. and Tremaine, S. (1997). *Galactic dynamics*, Princeton university press

[2] Dolbeault, J., Markowich, P., Oelz, D. and Schmeiser, C. (submitted) *Nonlinear diffusions as diffusion limits of kinetic equations with relaxation collision kernels*

[3] Dolbeault, J., Markowich, P., Oelz, D. and Schmeiser, C. (to be submitted) *Asymptotic regimes of kinetic equations with generalized relaxation collision kernels*

[4] Golse, F. and Poupaud, F. (1988). *Fluid limit of the Vlasov-Poisson-Boltzmann equation of semiconductors.* In: BAIL V (Shanghai, 1988), Boole Press Conf. Ser., **12**

[5] Golse, F. and Poupaud, F. (1992). *Limite fluide des équations de Boltzmann des semi-conducteurs pour une statistique de Fermi-Dirac.* In: Asymptotic Anal., **6**, 135–160

[6] Goudon, Thierry and Poupaud, Frederic (2001). Approximation by homogenization and diffusion of kinetic equations. Comm. Partial Differential Equations 3-4 **26**, 537–569.

[7] Guo, Yan and Rein, Gerhard (2003). *Stable models of elliptical galaxies* In: on. Not. R. Astronom.,

[8] Poupaud, Frédéric and Schmeiser, Christian (1991). Charge transport in semiconductors with degeneracy effects. Math. Methods Appl. Sci **14**, 301–318.

MATTHES Daniel

Title: Two applications of an algebraic method for entropy construction

Abstract: This short presentation outlines two recent extensions and applications of the algebraic method for the construction of entropy functionals as introduced by Jüngel and the speaker.

First, a variant of the method is used to estimate the rate of entropy dissipation in the logarithmic fourth order (DLSS) equation in arbitrary space dimensions.

Second, a particular (linear) Fokker-Planck equation is considered. Although the Bakry-Emery-criterion fails in this example, the algebraic approach still yields entropy dissipation estimates. These estimates give rise to a family of Beckner-type interpolation inequalities. Explicit values for the appearing constants are calculated.

MAZON RUIZ Jose M.

Title: Finite Propagation Speed for Limited Flux Diffusion Equations

Finite Propagation Speed for Limited Flux Diffusion Equations

F. Andreu[a], V. Caselles[b], J. M. Mazón[a] and S. Moll[b]

[a]Dept. de Análisis Matemático, Universitat de Valencia.

[b]Universitat Pompeu Fabra (Barcelona)

To correct the infinite speed of propagation of the classical diffusion equation Ph. Rosenau proposed the tempered diffusion equation

$$u_t = \nu \operatorname{div} \left(\frac{uDu}{\sqrt{u^2 + \frac{\nu^2}{c^2}|Du|^2}} \right). \tag{1}$$

Equation (1) was derived by Y. Brenier by means of Monge-Kantorovich 's mass transport theory and he named it as the *relativistic heat equation.* We prove existence and uniqueness of entropy solutions for the Cauchy problem for the quasi-linear parabolic equation

$$\frac{\partial u}{\partial t} = \operatorname{div}\, \mathbf{a}(u, Du), \tag{2}$$

where $\mathbf{a}(z,\xi) = \nabla_\xi f(z,\xi)$ and f being a function with linear growth as $\|\xi\| \to \infty$, satisfying other additional assumptions. In particular, this class includes the relativistic heat equation (1) and the flux limited diffusion equation

$$u_t = \nu \operatorname{div} \left(\frac{uDu}{u + \frac{\nu}{c}|Du|} \right) \tag{3}$$

used in the theory of radiation hydrodynamics.

We study the evolution of the support of entropy solutions of relativistic heat equation. For that purpose, we give comparison principles between sub-solutions (or super-solutions) and entropy solutions of the Cauchy problem and then using suitable sub-solutions and super-solutions, we establish the following result.

"Let C be an open bounded set in $\mathbf{R}^N$. Let $u_0 \in (L^1(\mathbf{R}^N) \cap L^\infty(\mathbf{R}^N))^+$ with support equal to $\overline{C}$. Assume that given any closed set $F \subseteq C$, there is a constant $\alpha_F > 0$ such that $u_0 \geq \alpha_F$ in F. Then, if $u(t)$ is the entropy solution of the Cauchy problem for the equation (1) with u_0 as initial datum, we have that

$$\operatorname{supp}(u(t)) = \overline{C} \oplus \overline{B_{ct}(0)} \qquad \text{for all} \quad t \geq 0."$$

NAZARET Bruno

Title: Optimal Sobolev trace inequalities on the half space

Abstract: Using a mass transportation method, we study optimal Sobolev trace inequalities on the half space and prove a conjecture made by Escobar in 1988 about the minimizers

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NI Lei

PANFEROV Vladislav

Title: Strong solutions of the Boltzmann equation in one-dimensional spatial geometry

Abstract: We study the nonlinear Boltzmann equation in the setting of one-dimensional (plane wave) solutions, in the assumption of bounded microscopic collision rate, satisfying certain cutoffs. Using the estimates of the relative entropy and of the quadratic functional introduced by Bony we show that the 'strong'bounds ensuring L^1 stability propagate globally in time.

PUEL Marjolaine

SLEPCEV Dejan

Title: Coarsening in thin liquid films

Abstract: Thin, nearly uniform, layers of some liquids can destabilize under the effects of intermolecular forces. After the initial phase, the liquid breaks into droplets connected by an ultra-thin liquid film. As the droplets interchange mass, the configuration of droplets coarsens over time. The characteristic distance between droplets and their average size grow, while their number is decreasing.

This physical process can be modeled by an equation for the height of the fluid — the thin-film equation. The evolution is a gradient flow, that is the steepest descent in an energy landscape. I will describe how information on the geometry of the energy landscape yields a rigorous upper bound on the coarsening rate.

The mass exchange between droplets can be mediated by two mechanisms: exchange through the connecting ultra-thin layer and droplet collisions. I will discuss the relative importance of the two mechanisms.

This is joint work with Felix Otto and Tobias Rump.

STURM Karl-Theodor

Title: Optimal Transportation and Ricci Curvature for Metric Measure Spaces

Abstract: We introduce and analyze generalized Ricci curvature bounds for metric measure spaces (M, d, m), based on convexity properties of the relative entropy $Ent(.|m)$. For Riemannian manifolds, $Curv(M, d, m) \geq K$ if and only if $Ric_M \geq K$ on M. For the Wiener space, $Curv(M, d, m) = 1$.
One of the main results is that these lower curvature bounds are stable under (e.g. measured Gromov-Hausdorff) convergence.

Moreover, we introduce a curvature-dimension condition $\mathrm{CD}(K, N)$ being more restrictive than the curvature bound $Curv(M, d, m) \geq K$. For Riemannian manifolds, $\mathrm{CD}(K, N)$ is equivalent to $\mathrm{Ric}_M(\xi, \xi) \geq K \cdot |\xi|^2$ and $\dim(M) \leq N$.
Condition $\mathrm{CD}(K, N)$ implies sharp version of the Brunn-Minkowski inequality, of the Bishop-Gromov volume comparison theorem and of the Bonnet-Myers theorem. Moreover, it allows to construct canonical Dirichlet forms with Gaussian upper and lower bounds for the corresponding heat kernels.

VÁZQUEZ Juan Luis

Title: log-diffusion of measures

Abstract: We discuss the diffusion of Dirac measures surrounded by a locally integrable distribution according to the log-diffusion equation in two space dimensions. The point masses trickle into the medium at a rate of 4pi units per unit time and mass location.

WUNSCH Marcus