Report on "Modular Forms and String Duality" (06w5041) June 3–8, 2006

The workshop was a huge success. Altogether thirty-seven mathematicians and physicists converged at the BIRS for the five day workshop. There were 26 one hour talks presented. Some were introductory lectures by mathematicians designed to prepare physicists in modular forms, quasimodular forms, modularity of Galois representations, and toric geometry. Vice versa, introductory lectures by physicists were intended toward educating mathematicians about some aspects of mirror symmetry, string theory in connection with number theory. These introductory lectures were scheduled in the mornings of early days of the workshop. Research talks were scheduled in the afternoons and later days. They covered the recent advances on various aspects of modular forms, differential equations, conformal field theory, topological strings and Gromov–Witten invariants, holomorphic anomaly equations, motives, mirror symmetry, homological mirror symmetry, construction of Calabi–Yau manifolds, among others. More detailed descriptions of scientific activities will be reported on in Section 4.

Though number theorists and string theorists have been working on modular forms, quasimodular forms and more general modular forms in their respective fields, there have been very little interactions between the two sets of researchers with few exceptions. In other words, both camps have been living in parallel universe. This workshop brought together researchers in number theory, algebraic geometry, and string theory whose common interests are modular forms. We witnessed very active and intensive interactions of both camps from early mornings to late nights. We all felt that all things modular have came together at BIRS from both sides: number theory and string theory. At the end of the workshop, there was a strong urge of having this kinds of workshops more frequently. Accordingly, a follow-up of this workshop is in the planning in the year 2008 at BIRS! with the current organizers plus Sergei Gukov from String Theory and Don Zagier from Mathematics.

The Proceedings of the workshop is currently under negotiation with Cambridge University Press, most likely to be published in the London Mathematical Society Lecture Notes Series.

Contents

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2. Press Release: Number Theory and String Theory at the Crossroads

Modular forms have long played a key role in the theory of numbers, including most famously the proof of Fermat's Last Theorem. Through its quest to unify the spectacularly successful theories of quantum mechanics and general relativity, string theory has long suggested deep connections between branches of mathematics such as topology, geometry, representation theory, and combinatorics. Less well-known are the emerging connections between string theory and number theory the subject of next weeks workshop 06w5041: Modular Forms and String Duality at the Banff International Research Centre, June 3 - 8, 2006. Mathematicians and physicists alike will converge on the Banff Centre for a week of both introductory lectures, designed to educate one another in relevant aspects of their subjects, and research talks at the cutting edge of this rapidly growing field. The event also coincides with the introduction of a new journal, "Communications in Number Theory and Physics" (http://www.intlpress.com/CNTP) published by International Press, which will provide a venue for dissemination of results at this crossroads well into the future. An expository proceedings for the workshop itself is under consideration for publication by Cambridge University Press.

3. Summary of scientific and other objectives

Physical duality symmetries relate special limits of the various consistent string theories (Types I, II, Heterotic string and their cousins, including F-theory) one to another. By comparing the mathematical descriptions of these theories, one reveals often quite deep and unexpected mathematical conjectures. The best known string duality to mathematicians, Type IIA/IIB duality also called **mirror symmetry**, has inspired many new developments in algebraic and arithmetic geometry, number theory, toric geometry, Riemann surface theory, and infinite dimensional Lie algebras. Other string dualities such as Heterotic/Type II duality and F-Theory/Heterotic string duality have also, more recently, led to series of mathematical conjectures, many involving elliptic curves, K3 surfaces, and modular forms. Modular forms and quasi-modular forms play a central role in mirror symmetry, in particular, as generating functions counting the number of curves on Calabi–Yau manifolds and describing Gromov–Witten invariants. This has led to a realization that time is ripe to assess the role of number theory, in particular that of modular forms, in mirror symmetry and string dualities in general.

One of the principal goals of this workshop is to look at modular and quasi-modular forms, congruence zeta-functions, Galois representations, and L-series for dual families of Calabi–Yau varieties with the aim of interpreting duality symmetries in terms of arithmetic invariants associated to the varieties in question. Over the last decades, a great deal of work has been done on these problems. In particular it appears that we need to modify the classical theories of Galois representations (in particular, the question of modularity) and modular forms, among others, for families of Calabi–Yau varieties in order to accommodate "quantum corrections".

As dictated by the research interests of the participating members, the research activities will be focused on the following themes:

(A) Arithmetic of Calabi–Yau varieties defined over number fields: Arithmetic of elliptic curves, K3 surfaces, Calabi–Yau threefolds, and higher dimensional Calabi–Yau varieties defined over number fields in connection with string dualities. These will include the following topics and problems: Interpretation of string duality phenomena of Calabi–Yau varieties in terms of zeta-functions and L-series of the varieties in question, the modularity conjectures for Calabi–Yau varieties, the conjectures of Birch and Swinnerton-Dyer for elliptic curves and Abelian varietes, the conjectures of Beilinson-Bloch on special values of L-series and algebraic cycles, and intermediate Jacobians of Calabi–Yau threefolds. Calabi-Yau varieties of CM (complex multiplication) type and their possible connections to rational conformal field theories and, in the elliptic fibered case, behavior under F-Theory/Heterotic string duality.

(B) Mirror symmetry for families of Calabi–Yau varieties: Characterization of mirror maps in connection with the mirror moonshine phenomenon and, via Fourier-Laplace transform, the classification of Q-Fano threefolds. In particular, differential equations associated to modular and quasi-modular forms related to GKZ-hypergeometric systems and, more generally, to Picard-Fuchs differential systems will be investigated.

(C) Modular and quasi-modular forms in string duality: Modular forms and quasi-modular forms have appeared frequently in mirror symmetry contexts, e.g., in the generating functions counting the number of simply ramified covers of elliptic curves with marked points, in Gromov–Witten invariants, and also as mirror maps. The appearance of modular and quasi-modular forms in string dualities, e.g., in the Harvey-Moore conjectures of Heterotic-Type II duality, will be investigated. Understanding why modular and quasi-modular forms play central roles in string dualities is one of our goals.

4. Summary of scientific activities

Talks presented at the workshop may be classified into not clearly disjoint sets of the following seven subjects. They are:

(a) Modular, quasimodular, bimodular forms, and their applications
(b) Topological string theory and modular forms
(c) Modularity, and arithmetic questions
(d) Mirror symmetry: various versions
(e) Toric geometry
(f) Differential equations
(g) Miscellaneous topics

The workshop's kick-off talk was delivered by Don Zagier about modular forms and differential equations. This talk sat the tone of the entire workshop. Modular forms (of one variable) are the key players of the workshop. The standard references on modular forms are the classical four proceedings volumes (I,II,III,IV) of the Antwerp conference (LNM 320, 349, 350 and 476) [B1], plus the volumes (V, VI) (the proceedings of the Bonn conference) (LNM 601, 627) [B2], entitled "Modular Functions of One Variable". The most recent lecture notes are the proceedings of the Nordfjordeid Summer School on "Modular Forms and their Applications", where they cover modular forms of one variable, Hilbert modular forms, and Siegel modular forms. (This is yet to appear [B3].)

The most recent references on String Theory and Mirror Symmetry are the Clay Math. Monographs *Mirror Symmetry* [B4] which covers recent advances on mathematics and physics about mirror symmetry, *Mirror Symmetry V*, AMS/IP Advanced Studies in Math. [B5] (this is the proceedings of the BIRS workshop 03w5061, December 2003), and *Calabi–Yau Varieties and Mirror Symmetry*, the Fields Institute Communications [B6].

These references served as the cornerstone for the rapidly developing topics discussed at the workshop.

(a) Modular, quasimodular, bimodular forms, and their applications

Zagier's lecture started with the definition of modular forms of one variable, and then quickly formulated the "Problem" that the derivative of a modular form is NOT a modular form, and addressed the strategy for attacking the problem. Let f be a modular form of weight $k \in \mathbf{Z}$ for a subgroup $\Gamma \subset SL_2(\mathbf{R})$, and let Df denote the derivative of f, i.e., $Df := f' = \frac{1}{2\pi i}\frac{df}{dz}$ (with $z \in \mathbf{H}$ where $\mathbf{H} := \{\, z = x + iy,\ y > 0 \,\}$ denotes the upper-half complex plane). Method 1: Change the definition of modular forms (which led to *quasimodular forms*); Method 2: Change the definition of derivative (which led to the new derivative $\partial f := Df - \frac{k}{4\pi y} f$ and *almost holomorphic modular forms*); Method 3: Eliminate the problem (which led to the first *Rankin–Cohen brackets*); Method 4: Avoid the problem (which led to *Eichler integrals, theory of periods*); Method 5: Enjoy the problem (which led to *differential equations of modular forms*). **Theorem**: *(1) A modular form f of weight k satisfies an autonomous differential equation of order 3, that is, there is a polynomial P such that $P(f, f', f^{(2)}, f^{(3)}) = 0$. (2) A modular form f of integral weight $k > 0$ satisfies a linear differential equation of order $k + 1$ with respect to a meromorphic modular function t. That is, write $f = \phi(t)$ locally, then $L\phi = 0$ for some linear differential operator L of order $k+1$ with polynomial coefficients.* His second lecture discussed in detail quasimodular forms, differential equations, Rankin–Cohen brackets and related algebraic structures, as enjoyments! The structure of Rankin–Cohen algebra as a commutative associative algebra was one of the main points of discussion. Also bimodular forms are introduced at the end of his lecture. The vector space of quasimodular forms is contained in that of bimodular forms.

Kaneko's talk was a continuation of Zagier's lectures, and covered modular forms and quasimodular forms and their applications. He reported on joint works with M. Koike, and D. Zagier. Consider an order-two differential equation of the form $f''(z) - \frac{k+1}{6}E_2(z)f'(z) + \frac{k(k+1)}{12}E_2'(z)f(z) = 0$ where $k \in \mathbf{Q}$, $z \in \mathbf{H}$ and $' = \frac{1}{2\pi i}\frac{d}{dz} = q\frac{d}{dq}$ with $q = e^{2\pi iz}$. Solutions are explicitly described which depend on congruence conditions of k. Some of these solutions appear also in physics literature (e.g., conformal field theory). The generating function of the (weighted) number of simply ramified covers of genus g over an elliptic curve with marked points is shown ([1]) to be expressed in terms of quasi-modular forms. This work provided a mathematical proof to Dijkgraaf's "theorem" [2] about mirror symmetry for elliptic curves.

Both Zagier and Kaneko were concerned with modular and quasimodular forms for congruence subgroups. **Ling Long**'s talk was about modular forms for *noncongruence* subgroups $\Gamma \subset SL_2(\mathbf{Z})$. The Fourier coefficients of modular forms for noncongruence subgroups have unbounded denominators. Long reported on her joint work with Atkin, W. Li and Z. Yang [3] about a refinement of Atkin and Swinnerton-Dyer congruence property satisfied by the Fourier coefficients of modular forms concentrating on some examples of noncongruence subgroups.

J. Stienstra reported on his joint work with Zagier on bimodular forms and holomorphic anomaly equation. Let $f_1, f_2, \cdots, f_n$ be quasimodular forms and ∂ be the derivation on quasimodular forms introduced by Zagier's talk. A holomorphic anomaly equation (HAE) is the differential equation $\partial f_n = -\frac{n}{2}\sum_{i=1}^{n-1} f_i f_{n-i}$. The main result is that solutions to HAE are given by bimodular forms.

(b) Topological string theory and modular forms

E. Scheidegger reported on a joint work with A. Klemm, M. Kreuzer and Riegler [4]. Let Σ_g be a Riemann surface of genus g and X be a smooth Calabi–Yau threefold. The problem addressed here is the enumerative properties of holomorphic maps from $\Sigma_g \to X$, which are concocted into the generating functions of the Gromov–Witten invariants (the genus g topological string amplitude on X). There are various approaches to the problem; this talk concentrated on the interpretation of the generating functions in terms of modular forms for Calabi–Yau threefolds X with specific $K3$-fibrations. Assuming that X are complete intersections in toric varieties with $h^{1,1} = 2$ with $K3$ fibrations and that $H_2(X, \mathbf{Z}) = i_*M \oplus [\beta]\mathbf{Z}$ where $M =< 2n >$, $n = 1, 2, 3, 4$, the authors construct 29 topologically inequivalent $K3$ fibrations, and show that the generating functions are weakly holomorphic modular forms of weight $k = -3/2$ on $\Gamma_0(8n)$.

A. Clingher discussed the correspondence between elliptically fibered $K3$ surfaces with section and elliptic curves endowed with certain flat connections. The correspondence is purported to describe the dualities in string theory, the F-theory/heterotic duality in eight dimensions. He reported on a joint work with C. Doran about geometric ways of exhibiting the above correspondence. Let $G = (E_8 \times E_8) \rtimes \mathbf{Z}_2$ be a Lie group where $\mathbf{Z}_2$ switches E_8. Elliptic curves with G-flat connections are classified. Moving to the other side, elliptically fibered $K3$ surfaces X with section are shown to correspond in one-to-one manner to H-polarizations $H \subset Pic(X)$ (H being the rank 2 hyperbolic lattice). The moduli space of elliptically fibered $K3$ surfaces with section is known by Dolgachev to be isomorphic to a type IV symmetric domain (the quotient of the period domain of X by the group Γ of isometries). The correspondence underlying the duality has to do with a partial compactification of this space, e.g., Mumford's partial compactification. The talk ended with examples of elliptically fibered $K3$ surfaces such that $Pic(X) \supset H \oplus E_8 \oplus E_8$, which correspond via Shioda–Inose structure to pairs of elliptic curves $\{(E, E')\}$.

A. Klemm discussed topological strings (TS) and modular forms. He used the holomorphic anomaly equation to solve the gravitational corrections to Seiberg–Witten theory. He constructed propagators that give recursive solution in the genus modulo a holomorphic ambiguity. The gravitational corrections can be expressed in closed form as quasimodular function on $\Gamma(2)$. (Cf. his recent preprint [5] for part of his talk.)

V. Bouchard reported on his joint work with A. Klemm and M. Aganagic in progress. The philosophy behind their work is to use symmetries to solve the topological strings. Steps involved are mirror symmetry, quantum mechanics (which led to modular forms) and modular forms (via monodromy groups). Gromov–Witten invariants arise from A-model on a Calabi–Yau threefold, which via mirror symmetry, correspond to holomorphic anomaly equations from B-model side. B-model side is easier computation wise. The inverse propagators are claimed to have modular property that they should be almost holomorphic modular forms of weight 0 on some subgroup Γ of $SL_2(\mathbf{Z})$. They show, depending on the choice of polarization, the genus g topological string amplitude is either a holomorphic quasimodular form, or an almost holomorphic modular form of weight 0 on Γ. (See [17] for details.)

(c) Modularity, and arithmetic questions

Ron Livné, in his first lecture, gave an overview on modularity of Galois representations. He defined the Galois representations associated to algebraic varieties over $\mathbf{Q}$, and those arising from modular forms. Several strategies (Faltings, Serre, Livné) to establish the modularity of a Galois representation were discussed. The modularity of 2–dimensional Galois representations are illustrated by examples, elliptic curves over $\mathbf{Q}$, singular $K3$ surfaces defined over $\mathbf{Q}$ and rigid Calabi–Yau threefolds over $\mathbf{Q}$.

In his second talk, **Livné** addressed the question of how to determine defining equations explicitly for abelian varieties of dimension > 1. This is really a challenging problem and there are no methods known to tackle this problem. He reported on a joint work with A. Besser about the explicit construction of universal families of Kummer surfaces over Shimura curves over $\mathbf{Q}$.

Sergei Gukov discussed in two lectures about interactions between number theory and physics illustrating with several examples. (In fact, his second lecture had to be scheduled in haste on strong demands by mathematicians.) The main point of his talks was that various partition functions arising in physics have modular properties. Example 1: Conformal Field Theory. Index/Elliptic genus $Z(\tau, z)$ ($\tau \in \mathbf{H}$, $z \in \mathbf{C}$ is a topological invariant of a Calabi–Yau threefold, and under modular transformation of $\Gamma_1 = SL_2(\mathbf{Z})$, it is a weak Jacobi form of weight 0 and index $k = \hat{c}/2$ where $\hat{c}$ is the central charge. Example 2: Guage Theory. Consider a smooth 4-manifold X with Chern characters $c_1 = 0$, $c_2 = k$. The electro-magnetic duality implies that the partition function Z^X_{VW} of Vafa and Witten is a modular form for an index 2 subgroup of $SL_2(\mathbf{Z})$. Example 3: Black Hole Attractors. Let X be a Calabi–Yau threerfold. Given charge $\gamma \in H^3(X, \mathbf{Z})$, look for solutions to the attractor equations. Solutions are isolated points in the moduli space. According to Greg. Moore, attractor Calabi–Yau threefolds are defined over number fields. In particular, rigid Calabi–Yau threefolds are attractive. Example 4: Rational Conformal Field Theory (which are defined as exactly solvable Conformal Field Theory). Conjecture (Friedan–Shenker): RCFTs are dense. Conjecture: RCFT if and only if Calabi–Yau manifold X has complex multiplication (CM). Complex tori and Fermat Calabi-Yau orbifolds have CM. Example 5: The Chern–Simons Theory. Let M be a 3-manifold, and A a connection on principal G-bundle. The Chern–Simons invariant is defined by $CS(A) := \frac{k}{4\pi}\int_M Tr(A \wedge dA + \frac{2}{3}A \wedge A \wedge A)$. The goal is to compute the partition function $Z(M, G) = \int \mathcal{D}Ae^{-CS(A)} = exp(\sum_{\ell=0}^{\infty}(\frac{2\pi}{k})^{\ell-1}S_\ell) = \exp(\frac{k}{2\pi}S_0 + S_1 + \sum_{n\geq 1}(\frac{2\pi}{k})^n S_{n+1})$ where the coefficients are interesting invariants. When G is compact and $\ell > 1$, then $S_\ell \in \mathbf{Q}$. However, if $G = SL(2, \mathbf{C})$ (non-compact) and $\ell > 1$, then S_ℓ are not of finite type, in particular, not necessarily in $\mathbf{Q}$. When M is a hyperbolic 3-manifold of finite type, e.g., $M = \mathbf{H}^3/\Gamma$ (where $\Gamma \subset PSL(2, \mathbf{C})$, then S_ℓ are contained in the number field, $\mathbf{Q}(tr(\gamma))$, $\gamma \in \Gamma$. If $M = \mathbf{S}^3 \setminus K$ where K is a knot, then Z is related to a colored Jones polynomial. Example 6: (Zagier–Lawrence and Hikami and others.) Here $M = \mathbf{S}^3/\Gamma$ with $\Gamma \subset SU(2)$ (finite subgroup). Then $Z(M, SU(2))$ is expressed in terms of the Eichler integral of a modular form of weight $3/2$.

Noriko Yui introduced motives to describe topological mirror symmetry for certain classes of Calabi–Yau threefolds constructed from Fermat hypersurfaces in weighted projective 4-spaces by orbifolding. Motives which are invariant under mirror maps are determined. Also one-to-one correspondence between motives and monomial classes is established at the Fermat (the Landau–Ginzburg) point in the moduli space. These results are obtained in the paper [7]. Modularity question (of Galois representations) becomes more manageable at motivic level, and the modularity of many motives are established. There appear rank 4 motives, which are conjecturally modular in the sense that there should be Siegel modular forms of weight 3 on some congruence subgroups of $Sp(4, \mathbf{Z})$ that determine the L-series of such motives. How can one determine the modular groups in the conjecture? For this, consider the 14 Calabi–Yau threefolds whose Picard–Fuchs differential equations are of hypergeometric type. In a joint work with Yifan Yang and his student [8], it is shown that the monodromy group is contained in the congruence subgroup $\Gamma(d_1, d_2) \subset Sp(4, \mathbf{Z})$ of finite index. (However, it is still open if the monodromy group itself is of finite index in $Sp(4, \mathbf{Z})$.)

Rolf Schimmrigk addressed the modularity of three Calabi–Yau threefolds over $\mathbf{Q}$, focusing on their connection to physics of the string worldsheet. He discussed three specific examples of Calabi–Yau threefolds, which are built up from lower dimensional Calabi–Yau manifolds, e.g., elliptic curves, and $K3$ surfaces. Motives arising from his examples are already discussed in Yui's talk. Also he constructed explicitly mirror maps for certain mirror pairs of Calabi–Yau orbifolds in weighted projective 4-spaces.

(d) Mirror symmetry: various versions

Shinobu Hosono gave an introductory talk on "What is the mirror symmetry?" SYZ's version: Every Calabi–Yau manifold has a T^3-fibration, up to some singular fibers. Konstevich's version: There exists an equivalence of categories $D^b(coh(X))$ and $D^b(Fukaya(Y))$ for a mirror pair (X, Y) of Calabi–Yau manifolds. He explained the two versions of mirror symmetry focusing on the quintic Calabi–Yau threefold and its mirror partner. Picard–Fuchs differential equations for these mirror pair are explicitly determined. (Originally, **Bong H. Lian** was scheduled to give an introduction to mirror symmetry. But due to unforeseen development, he had to cancel his participation and his talk. Bong Lian sent in his lecture notes [B9] about mirror symmetry. At the last minutes, Hosono kindly agreed to give an introductory talk about mirror symmetry.)

Johannes Walcher reported on his recent work [8] on the determination of the number of holomorphic disks of degree d ending on the real Lagrangian L in the quintic Calabi–Yau threefold. For $d = 1$, 30 such disks, 1530 for $d = 3$, and 1088250 for $d = 5$ and so on. These numbers are coming from a generalized Picard–Fuchs differential equation. String theory motivates study of $2d$ field theories on worldsheets of higher genus, with boundary and possibly unoriented. Pick boundary conditions (D-branes). On A-model side, one has Lagrangian submanifold $\subset X$, and on B-model side, one gets holomorphic submanifold with holomorphic vector bundle on the mirror Y. Tension of the domain wall is the topic of his talk. Computing the open string instanton expansion (normalized appropriately), one reach at the numbers listed above. This is an open analogue of the paper [9] where the numbers of rational curves of degree d on the quintic Calabi–Yau threefold are computed using mirror symmetry.

M. Aldi reported on a joint work with E. Zaslow [10] about homological mirror symmetry. The construction of Seidel's mirror map for abelian surfaces and Kummer surfaces is discussed computing explicitly twisted homogeneous coordinate rings from the Fukaya category of symplectic mirrors. The computation of Seidel's map, however, depends on a symplectomorphism representing the large complex structure monodromy. Several examples of mirror maps are presented for abelian surfaces, and Kummer surfaces.

(e) Toric geometry

Helena Verrill gave an introductory talk on toric geometry focusing on the Batyrev–Borisov construction of toric Calabi–Yau hypersurfaces and complete intersections. For general references about toric varieties are Fulton [B7] and Oda [B8]. Batyrev [11] starts with a polytope Δ and associates a family $\mathcal{F}(\Delta)$ of hypersurfaces. A special class of polytopes that play the key role in Batyrev–Borisov mirror symmetry is the class of reflexive polytopes. A polytope Δ is *reflexive* if the origin is in Δ and Δ and its dual Δ^* are lattice polytopes. In dimension 4, there are $473, 800, 776$ reflexive polytopes. Associated to a pair of reflexive polytopes (Δ, Δ^*), there correspond families $(\mathcal{F}(\Delta), \mathcal{F}(\Delta^*))$ of Calabi–Yau hypersurfaces. Batyrev gave formulas for the Hodge numbers $h^{2,1}(\Delta)$ and $h^{1,1}(\Delta^*)$. and then showed that that $\mathcal{F}(\Delta)$ and $\mathcal{F}(\Delta^*)$ are mirror pairs in the sense of topological mirror symmetry, that is, the Hodge numbers $h^{2,1}(\Delta) = h^{1,1}(\Delta^*)$ and $h^{2,1}(\Delta) = h^{1,1}(\Delta^*)$. The Batyrev–Borisov construction produces singular Calabi–Yau manifolds, and one needs to consider MPCP (maximal projective crepant partial) desingularization (i.e., triangulation of polytopes). Verrill illustrated the Batyrev–Borisov construction by a number of examples. She remarked that most Calabi–Yau families can contain rigid Calabi–Yau threefolds.

C. Doran reported on his joint work with J. Morgan [12] on the computation of integral homology, the topological K-theory, and the rational Hodge structure on cohomology of Calabi–Yau hypersurfaces and complete intersections in toric varieties. The Doran–Morgan approach to these problems is purely topological. The geometric representatives of integral cohomology classes for H^2 and H^3 are explicitly described. The rational Hodge structures of weight 3 are determined from the geometric representatives.

(f) Differential equations

G. Almkvist reported on his joint work with W. Zudilin and D. van Straten [13]. (Incidentally, both were invited to the workshop but could not take part by respective reasons). The talk was concerned with the Apéry like limits arising from recursions of Calabi–Yau differential equations of order 4. Let $\sum_{n=0}^{\infty} A(n)x^n$ be the analytic solution of a Calabi–Yau differential equation of order 4. The $A(n)$ satisfy a recursion formula with polynomial coefficients with $A(n) = 0$ for $n < 0$ and $A(0) = 1$. Suppose that $B(n)$ satisfy the same recursion but with $B(n) = 0$ for $n \leq 0$ and $B(1) = 1$. The main concern of his talk was what happens to the limit $B(n)/A(n)$ when n tends to ∞. This is a generalization of the Apéry limit for a third order differential equation (discussed in Zagier's first talk). These limits are conjectured to be expressed by special values of L-functions at $s = 2$ or $s = 3$. Several examples in support of the conjecture are presented.

Masahiko Saito reported in his recent work about Painlevé property of ordinary differential equations. In particular, he and his collaborators analyzed movable branching points and gave a necessary condition for Painlevé property to hold by means of geometry of logarithmic symplectic varieties.

Ahmed Sebber reported on differential theta relations and Galois theory for Riccati equations. He discussed several types of ordinary differential equations: Riccati equation, algebraic hypergeometric equation, Chazy equation, and non-linear differential equations satisfied by theta functions. The Riccati equation is of the form: $\frac{du}{dx} + u^2 = q(x)$. The problem addressed here was : For which potential q, the Riccati equation has algebraic solutions? That is, u satisfies an algebraic equation of degree n over some fixed (differential) field. There happened to be only four values for n, namely, $n = 3, 4, 6, 12$. Some applications to physics were mentioned at the end of his lecture.

(g) Miscellaneous topics

Nam-Hoon Lee discussed a method of constructing Calabi–Yau manifolds using the method of smoothing normal varieties developed by Kawamata and Namikawa [14]. This construction yielded 22 new examples of Calabi–Yau threefolds with Picard rank 1 [15]. Also he gave a counterexample to the conjecture of Tyurin that very Calabi–Yau manifold is constructible or is birational to a variety that is a deformation of constructible Calabi–Yau manifolds. Such a counterexample is a Calabi–Yau threefold $X(10) \subset \mathbf{P}(1, 1, 1, 2, 5)$ in the weighted projective 4-space with weight $(1, 1, 1, 2, 5)$.

J. Stienstra discussed a recent new interpretation of mirror symmetry, namely, AdS_5/CFT theory. Classically, string theory lives on 10-dimensional space (Minkowski 4-space × Calabi–Yau threefold). Recently, Anti-de-Sitter 5-space AdS_5 and Sasaki–Einstein 5 manifold came into the mirror symmetry picture. The boundary at infinity of AdS_5 is Minkowski 4-space. Sasaki–Einstein 5 manifold is a manifold Y whose metric cone $CY := \mathbf{R}_{\geq 0} \times Y$ is Kähler and Ricci-flat, i.e. CY is a singular and non-compact Calabi–Yau threefold. Then AdS_5/CFT correspondence is interpreted as the correspondence between 3-dimensional Calabi–Yau singularity and quiver gauge theory (cf. McKay correspondence). Several examples of Sasaki–Einstein 5-manifolds and their metric CY cones are presented. Dimension 3 CY singularities are constructed from toric data, e.g., multi-grid.

C. Herzog addressed the following question: What is the low energy gauge theory description of a set of D-branes at a Calabi–Yau singularity? Let X be a singular Calabi–Yau manifold. Claim: a gauge theory can be constructed from a set of objects in the derived category $D^b(X)$ of coherent sheaves, called fractal branes. An exceptional collection of objects in $D^b(X)$ is defined as an ordered set of sheaves satisfying special mapping properties, and it gives a convenient basis of D-branes. It is conjectured that a simple exceptional collection always exists for any Deligne–Mumford stack with coarse moduli space which is "mildly" singular variety with positive curvature. Some examples of such collections are constructed. This talk reported on his joint work with R. Karp ([16]).

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6. Particpants

We had in total 37 participants for the workshop, ten were either graduate students or postdoctoral fellows, and two were official observers. We had five last minutes cancellations (Terry Gannon (Alberta, Canada), Amer Iqbal (Washington), Bong H. Lian (Brandeis, USA), John McKay (Concordia, Canada), and Andrey Todorov (Santa Cruz, USA and MPIM Bonn, Germany) by various reasons.

1. Marco Aldi (Northwestern University, USA) (Ph.D. student)
2. Gert Almkvist (Lunds University, Sweden)
3. Maiia Bakhova (Louisiana State University, USA) (Ph.D. student)
4. Matthew Ballard (University of Washington, USA) (Ph.D. student)
5. Vincent Bouchard (MSRI, USA) (Postdoctoral fellow)
6. Adrian Clingher (Stanford University, USA)
7. Chuck F. Doran (University of Washington, USA)
8. George Elliott (University of Toronto, Canada) (Obserber)
9. Sharon Frechette (College of the Holy Cross, USA)
10. Sergei Gukov (California Institute of Technology, USA)
11. Christopher Herzog (University of Washington, USA)
12. Sinobu Hosono (University of Tokyo, Japan)
13. Simon Judes (Columbia University, USA) (Ph.D. student)
14. Shabnam Kadir (University of Hannover, Germany) (Postdoctoral fellow)
15. Masanobu Kaneko (Kyushu University, Japan)
16. Albrecht Klemm (University of Wisconsin, USA)
17. Edward Lee (UCLA, USA) (Postdoctoral fellow)
18. Nam-Hoon Lee (KIAS, Korea) (Postdoctoral fellow)
19. Ron Livné (Hebrew University of Jerusalem, Israel)
20. Ling Long (Iowa State University, USA)
21. Stephen Lu (Universite de Quebec a Montreal, Canada)
22. Richard Ng (Iowa State University, USA) (Observer)
23. Matthew Papanikolas (Texas A & M University, USA)

24. Mike Roth (Queen's University, Canada)
25. Masahiko Saito (Kobe University, Japan)
26. Emanuel Scheidegger (Universita del Piemonte Orientale Amadeo Avogadro, Italy)
27. Rolf Schimmrigk (Indiana University South Bend, USA)
28. Abdellah Sebber (University of Ottawa, Canada)
29. Ahmed Sebber (Univeristy of Bordeaux I, France)
30. Jan Stienstra (University of Utrecht, the Netherlands)
31. Hiroyuki Tsutsumi (Osaka University of Health & Sport Sciences, Japan)
32. Helena Verrill (Louisiana State University, USA)
33. Johannes Walcher (Institute for Advanced Study, USA)
34. Ursula Whitcher (University of Washington, USA) (Ph.D. student)
35. Jeng-Daw Yu (Harvard University, USA) (Postdoctoral fellow)
36. Noriko Yui (Queen's University, Canada)
37. Don Zagier (Max-Planck Institute for Mathematics Bonn, Germany/ College de France, France)

7. Titles and Abstracts of Talks at the Workshop

JUNE 4, 2006

9:00am–10:00am **Don Zagier** (MPIM Bonn and College de France)

Introduction: Modular Forms and Differential Equations

10:15am–11:15am **Ron Livné** (Hebrew and IAS Princeton)

Modularity of Galois Representations: Overview

Given a two dimensional Galois representation of geometric origin, over a field, it is conjectured, and often known, that it arises from a modular form for GL(2). Even when general theorems do not exist (yet), it is sometimes possible to prove a given instance of the conjecture by making a guess and verifying it. In this survey talk we will give a miscellany of results in these directions.

11:30am–12:30pm **Helena Verrill** (Louisiana State)

An Introduction to the Batyrev–Borisov Construction of Toric Calabi–Yau Varieties

This talk will give an overview of the Batyrev-Borisov construction of toric Calabi-Yau hypersurfaces and complete intersections. It will start with a quick summary of the definition of toric varieties and reflexive polytopes, discuss properties of toric Calabi-Yaus, and Kreutze and Skarke's method of enumerating reflexive polyhedra. Some examples will be discussed, which have been used to find new modular Calabi-Yau threefolds, though the topic of modularity will be left to another speaker. This talk will not introduce new results, and is particularly aimed at graduate students.

3:00pm–4:00pm **Shinobu Hosono** (Tokyo)

Introduction to Differential Equations in Mirror Symmetry

4:15pm–5:15pm **Emanuel Scheidegger** (TU Vienna)

Topological Strings on $K3$ Fibrations and Modular Forms

We explain that motivated by heterotic-type II duality, certain Gopakumar-Vafa invariants (and hence, conjecturally, Gromow-Witten invariants) for Calabi-Yau manifolds that admit a K3 fibration can be collected in a generating function. This function is in general an automorphic form determined by the topology of the fibration. In the class of K3 fibrations in toric varieties in which the Picard lattice of the fiber has rank one, we show how this automorphic form can be determined explicitly.

5:30pm–6:30pm **Jan Stienstra** (Utrecht)

From multi-grid to multi-helix; remarkable geometries from AdS/CFT

A 1-grid with parameters a, b, c is the system of parallel equidistant lines in the plane described by the equation $ax+by+c=$ integer. An N-grid is a system of N 1-grids. The 'dual' of an N-grid can be presented as a tessellation of the plane by rhombi. We are interested in grids with integral (a, b)-parameters. These give rise to periodic rhombus tilings. Moreover, viewing (a, b) as a vector the tiles can be equipped with extra pictures. There are simple 'braiding rules' to transform a given rhombus tessellation into a new one. It turns out that by repeated application of the braiding rules one can reach a tiling in which the extra pictures form an interesting pattern, like the projection of a link with a spanning Seifert surface and embedded into the Seifert surface is a bi-partite graph. In the talk this will be illustrated with many pictures. We will also indicate how this relates to the AdS/CFT correspondence, Mirror Symmetry and the McKay correspondence.

JUNE 5, 2006

9:00am–10:00am **Don Zagier** (MPIM Bonn and College de France)

Quasimodular forms, Rankin–Cohen brackets and related algebraic structures

10:15am–11:15am **Masanobu Kaneko** (Kyushu)

Modular and Quasimodular Forms and their Applications

I shall review works with Don Zagier on "mirror symmetry in dimension one" and with Masao Koike on modular and quasimodular solutions of certain differential equation, with a brief mention to their possible connection to conformal field theory.

11:30am–12:30pm **Johannes Walcher** (IAS Princeton)

Opening Mirror Symmetry on the Quintic

Aided by mirror symmetry, we determine the number of holomorphic disks ending on the real Lagrangian in the quintic threefold. The tension of the domainwall between the two vacua on the brane, which is the generating function for the open Gromov–Witten invariants, satisfies a certain extension of the Picard–Fuchs equation governing periods of the mirror quintic. We verify consistency of the monodromies under analytic continuation of the superpotential over the entire moduli space. We reproduce the first few instanton numbers by a localization computation directly in the A-model, and check Ooguri–Vafa integrality. This is the first exact result on open string mirror symmetry for a compact Calabi–Yau manifold.

2:30pm–3:30pm **Gert Almkvist** (Lund)

Apéry-like limits connected with Calabi-Yau differential equations

Let $\sum_{n=0}^{\infty} A(n)x^n$ be the analytic solution of a Calabi-Yau differential equation (4-th order). Then $A(n)$ satisfies a recursion formula with polynomial coefficients with starting values $A(n) = 0$ for $n < 0$ and $A(0) = 1$. Let $B(n)$ satisfy the same recursion with $B(n) = 0$ for $n <= 0$ and $B(1) = 1$. Very often the limit of $B(n)/A(n)$ exists when $n \to \infty$. Usually the limit is a rational linear combination of values of L-functions at $s = 2$ or $s = 3$. This is a joint work with van Straten and Zudilin.

4:00pm–5:00pm **Christopher Herzog** (Washington)

How Exceptional Collections Stack Up

I would attempt to give a broad overview of my two recent papers with Robert Karp, hep-th/0507175 and hep-th/0605177. The papers advance the program of using exceptional collections of objects in the derived category of coherent sheaves to understand the low energy gauge theory description of a D-brane probing a Calabi-Yau singularity.

5:15pm–6:15pm **Nam-Hoon Lee** (KIAS)

Constructing Calabi–Yau Manifolds

A smoothing theorem for normal crossings to Calabi-Yau manifolds was proved by Y. Kawamata and Y. Namikawa. This talk is about a study of the observation that the Picard groups and Chern classes of these Calabi-Yau manifolds are constructible from the normal crossings in such smoothings. Various applications will be discussed, including the construction of many new examples of Calabi-Yau 3-folds with Picard number one. With this construction as a starting point, I hope to convince audience that smoothing normal crossings is a promising method of constructing Calabi-Yau manifolds. This talk is based on my recent preprint (math.AG/0604596).

8:00pm–9:00pm **Ling Long** (Iowa State)

Modular Forms for Noncongruence Subgroups

Majority of finite index subgroups of the modular groups are noncongruence subgroups. In the 1970, Atkin and Swinnerton-Dyer have pioneered the investigation on modular forms for noncongruence subgroups. Some of their important observations have been verified by Scholl. Despite that, modular forms for noncongruence subgroups still remains to be very mysterious. In this talk, we will discuss some recent results on the arithmetic properties of modular forms for noncongruence groups.

JUNE 6, 2006

9:00am–10:00am **Adrian Clingher** (Stanford)

Geometry underlying the F-Theory/Heterotic String Duality in Eight Dimensions

One of the dualities in string theory, the F-theory/heterotic string duality in eight dimensions, predicts an interesting correspondence between two seemingly disparate geometrical objects. On one side of the duality there are elliptically fibered K3 surfaces with section. On the other side, one finds elliptic curves endowed with certain flat connections. I will discuss the basic Hodge theoretic framework underlying the duality as well as its consequences in algebraic geometry and number theory.

10:15am–11:15am **Sergei Gukov** (CalTech)

Strings, Fields, and Arithmetic, Part I

String theory and quantum field theory are known to have many deep connections and applications to geometry and topology. In recent years, new connections between string/field theory and number theory started to emerge. Examples include a relation between elliptic genera and Jacobi forms, complex multiplication and black hole attractors/RCFTs, etc. In this talk, I will review some the known relations and in the end present new ones.

11:30am–12:30pm **Noriko Yui** (Queen's)

Motives, Mirror Symmetry and Modularity

We consider certain families of Calabi–Yau orbifolds and their mirror partners constructed from Fermat hypersurfaces in weighted projective spaces. We use Fermat motives to interpret the topological mirror symmetry phenonemon. These Calabi–Yau orbifolds are defined over $\mathbf{Q}$, and we can discuss the modularity of the associated Galois representations. We address the modularity question at motivic level. We give some examples of modular Fermat motives. We then formulate a modularity conjecture about rank 4 Fermat motives that there exist Siegel modular forms on some congruence subgroups of $Sp(4, \mathbf{Z})$.

8:00pm–9:00pm **Sergei Gukov** (CalTech)

Strings, Fields, and Arithmetic, Part II

JUNE 7, 2006

9:00am–10:00am **Albrecht Klemm** (Wisconsin)

Modular, Quasimodular Forms and Gromov–Witten Invariants

10:15am–11:15am **Ron Livné** (Hebrew and IAS Princeton)

Explicit descriptions of universal $K3$ families over Shimura curves

It is quite hard to give explicit algebraic description of abelian varieties of dimension > 1. However, the Kummer surfaces of abelian surfaces - and sometimes related $K3$ surfaces - do have useful projective models. In joint work with A. Besser we give some instances where this can be done universally over Shimura curves over $\mathbf{Q}$. While the universal families of abelian surfaces exist only when rather high level is added to the moduli problem, our universal $K3$ fibrations exist over very low level Shimura curves - often level < 1, allowing particularly simple descriptions.

11:30am–12:30pn **Chuck Doran** (Washington)

Algebraic Topology of Calabi–Yau Threefolds in Toric Varieties

We compute the integral homology (including torsion), the topological K-theory, and the Hodge structure on cohomology of Calabi-Yau threefold hypersurfaces and complete intersections in Gorenstein toric Fano varieties. The methods are purely topological. This is joint work with John Morgan.

2:30pm–3:30pm **Marco Aldi** (Northwestern)

Twisted homogeneous coordinate rings of abelian surfaces via Mirror Symmetry

Seidel's mirror map reconstructs the homogeneous coordinate ring of a given projective CY or Fano variety in terms of Lagrangian intersection data on its mirror. We discuss the computation of Seidel's mirror map for abelian and Kummer surfaces and related work on integrality and noncommutative geometry.

4:00pm–5:00pm **Masahiko Saito** (Kobe)

Painlevé Property of ODEs and Deformation of Logarithmic Symplectic Varieties

We will analyze movable branching points of algebraic ordinary differential equations, and give a necessary condition for Painlevé property by means of geometry of logarithmic symplectic varieties. The result establishes the reason why the condition of Okamoto-Painlevé pairs is necessary for Painlevé equations. Furthermore, we can give a very simple proof of a result of Fuchs', Poincaré, Malmiquist and M. Matuda

5:15pm–6:15pm **Rolf Schimmrigk** (Indiana, South Bend)

String Modular Ω–Motives and Aspect of Mirror Symmetry

The purpose of this talk is to describe some new extensions of recent string modularity results for elliptic curves and $K3$ surfaces to higher dimensional varieties of both Calabi-Yau and Fano type. The resulting examples establish that it is possible to construct Calabi–Yau varieties from the physics of the string worldsheet in all physically interesting dimensions. These constructions also provide arithmetic checks for ideas in mirror symmetry. Implications for the mirrors of rigid Calabi–Yau threefolds, as well as elliptic curves, will be discussed. The results concerning special types of Fano varieties provide checks for a conjecture of Serre concerning the type of modular forms associated to generalized Calabi–Yau Hodge structures.

8:00pm–9:00pm **Ahmed Sebber** (Bordeaux)

Differential theta relations and Galois theory for Riccati equations

JUNE 8, 2006

9:00am–10:00am **Jan Stienstra** (Utrecht)

Bimodular Forms and Holomorphic Anomaly Equation

10:15am–11:15am **Vincent Bouchard** (MSRI/Perimeter)

Topological Strings, Holomorphic Anomaly, and (Almost) Modular Forms

8. Appendix

Several people had to cancel their participation to the workshop at the last minutes by various reasons. They were

• Terry Gannon (Alberta): He and his wife had twins on May 28th. Accordingly, Terry had to cancel his participation in the workshop and his talk.

The Monster, Modular Functions and RCFT

Rational conformal field theory 'explains' the modularity of lattice theta series, affine Kac-Moody algebra characters, Monstrous Moonshine functions, etc. We try to identify the essence of this argument, and use this to speculate on the nature of a more conceptual second proof of the Monstrous Moonshine conjectures.

• Amer Iqbal (Washington): He was not able to obtain visa to enter Canada.

• Bong H. Lian (Brandeis): He had to go to Singapore for an urgent matter on June 2nd. He had to cancel his participation and his talk. He sent in a pdf file of his intended talk.

Introduction to Mirror Symmetry

This will be a mathematical survey of both the history and development of mirror symmetry beginning with the early suggestions from physics. Topics may include
0) Two dimensional super conformal field theory
1) Basic examples of mirror manifold constructions
2) Mathematical predictions of mirror symmetry
3) Fourier-Mukai transforms

• John McKay (Concordia): He got sick and in hospital.

• Andrey Todorov (Santa Cruz/MPIM Bonn): He arrived at Calgary International Airport on June 3rd. Upon his arrival, he found out that his uncle in Bulgaria died, and had to take a flight back immediately to Europe. He promised to send in notes of his intended talk.

Regularized Determinants of CY Metrics and Applications to Mirror Symmetry

We will prove the existence of the analogue of the Dedekind Eta Function for CY threefolds and K3 surfaces with unimodular Picard Group. We will discuss the combinatorial properties of the Generalized Dedekind Eta function in the A and B model and its relations with Harvey–Moore–Borcherds Product Formulas.