

# Optimization and Engineering Applications

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## 1 Objectives

This workshop had several objectives:

- to bring together outstanding researchers from the fields of optimization and engineering;
- to establish new collaborations; and
- to expose both sides to the challenge and opportunities in the respective fields, through identifying new problems and either matching these with potential solution approaches or developing new models and methodologies.

Among the participants were optimization researchers that work on engineering optimization problems and vice versa. The cross fertilization of experts in both fields is crucial for the development of new methods in both communities. Although there is an increasing awareness of common interests, until recently there have been only very few opportunities for the two sides to meet at conferences. In the past two years there have been a few occasions where optimizers, e.g., Z.Q. Luo and S. Boyd have been invited to give tutorials in signal processing and electrical engineering conferences. The MOPTA conference series at McMaster University, University of Waterloo, and University of Windsor, organized by T. Terlaky, H. Wolkowicz and their colleagues, has featured several engineers who presented their work on the application of optimization in their fields. However, there has not been a workshop specifically targeting the exchange of cutting edge expertise in mathematical optimization and engineering optimization, where an elite club of expertise from both fields was formed to pave the road for future directions.

A particular fact that should be mentioned here is the MITACS project “High Performance Optimization: Theory, Algorithm Design and Engineering Applications” led by T. Terlaky, which will have been in operation for five years by 2007. The workshop provided a good opportunity to review the achievements

of the entire project, discuss future research plans for the project, and identify the areas in engineering for which optimization research has a major impact in the future and new challenges of engineering optimization problems offer to the mathematical, algorithmic optimization community.

## 2 Overview of the Field

Optimization is a subject that deals with the problem of minimizing or maximizing a certain function in a finite dimensional Euclidean space over a subset of that space, which is usually determined by functional inequalities. During the past century, optimization has been developed into a mature field that includes many branches, such as linear conic optimization, convex optimization, global optimization, discrete optimization, etc. Each of such branch has a sound theoretical foundation and is featured by an extensive collection of sophisticated algorithms and software. Optimization, as a powerful modelling and problem solving methodology, has a broad range of applications in management science, industry and engineering.

The application of optimization in engineering has a very long history. It is well known that two special classes of optimization problems, linear least squares and linear optimization problems, have been widely used in a tremendous number of application areas, such as transportation, production planning, design and data fitting. Since 1990, the appearance of highly efficient interior point method (IPM) based algorithms and software for convex optimization has motivated people to apply convex optimization models in several new areas, such as automatic control systems, signal processing, communications and networks, product and shape design, truss topology design, electronic circuit design, data analysis and modelling, statistics and financial engineering. A celebrated example of successful application of optimization in electrical engineering is the company, Barcelona Design, established by Prof. S. Boyd of Stanford University, which sells custom designed electronic circuits and uses convex optimization as the core technology to optimize its circuit designs. The application of convex optimization in engineering enables engineers to design and solve problems more efficiently. In recognition of the contribution of optimization to engineering, S.S. Rao wrote: "Optimization is now viewed as an indispensable tool of the trade for engineers working in many different industries, especially the aerospace, automotive, chemical, and manufacturing industries."

## 3 Recent Developments and Open Problems

Starting in the middle 1980's, the field of optimization has experienced a revolutionary development, in particular in the area of convex optimization. This was sparked by Karmarkar's ground-breaking work on IPMs for linear optimization, and later accomplished by many excellent optimization experts. A remarkable work in the IPM stream is the self-concordant theory by Nesterov and Nemirovski that shows several important classes of optimization problems, such as second-order conic and semidefinite optimization (SDO) problems, can be solved efficiently by IPMs. Several high quality optimization packages based on IPMs, such as LOQO, MOSEK, SeDuMi and McIPM, have been developed and they are widely used problem solving tools today. The IPM revolution has brought new theoretical and practical powerful tools for solving classes of convex optimization problems and led to new research areas such as SDO and robust optimization (RO). These novel optimization models and methodologies provide powerful approaches for attacking problems in size and complexity that could not be handled before.

The IPM revolution has brought not only the celebrated theory and powerful tools, but more challenges to the optimization community. This is particularly true in the new areas of SDO and RO. In many scenarios such as signal processing and data analysis, the original physical model is a highly nonlinear and non-convex optimization problem, which is typically NP-hard. The SDO approach can be used only as a relaxed approximation to the original problem. It is very important to investigate under what conditions, the approximate solution obtained from the SDO relaxation can match the exact solution of the original problem, or to what degree it can approximate the exact solution. When we use RO approach to attack optimization problems under uncertainty, it is crucial to investigate under what distribution of the data, the resulting RO model is robust and can be solved efficiently. There are also challenges from an algorithmic perspective. For example, all the current SDO solvers can solve only SDO problems in size up to a couple of thousands variables unless the underlying problem has a special data structure, while there is a huge number of problems with larger size in real applications that cannot be efficiently solved by the existing solvers. Some optimization

problems, such as SDO problem with nonnegative constraints on its elements, are theoretically polynomially solvable. However, current SDO solvers can only solve this kind of problems up to sizes of 100 variables. It is important to develop new efficient methodologies for these theoretically solvable but practically extremely hard problems and analyze the properties of these new methods.

The application of optimization in engineering not only helps engineers in their design and analysis of systems, but also leads to significant advances and new discoveries in optimization theory and techniques. For example, numerous engineering design problems need to deal with noisy data, manufacturing error or uncertainty of the environment during the design process. Such engineering optimization problems, like the antenna synthesis problem, lead to the birth of robust optimization, a new emerging research area in the context of convex optimization. Further, the application of optimization in chemical engineering has resulted in several powerful optimization packages in that field and these packages have also been proved to be useful tools for solving general optimization problems.

The interaction between engineering and optimization communities has led to numerous significant achievements in both fields. Besides convex optimization, other optimization techniques, such as integer programming, dynamic programming, global optimization and general nonlinear optimization, have also been successfully applied in engineering. On the other hand, the broad application of optimization methodology in engineering yields a strong stimulus to develop new optimization models and algorithms to meet the increasing demand from engineering practice.

In spite of the broad application of optimization in engineering and occasional close collaboration between individual optimizers and engineers, there have been few chances for experts from both sides to meet to discuss new challenges in the subject area, and explore potential opportunities for fruitful collaboration from both sides.

## 4 The Talks

**Speaker:** M.F. Anjos

**Title:** Finding Nash Equilibria in Electricity Markets: An AC-Network Approach

**Coauthors:** G. Bautista and A. Vannelli.

**Abstract:** Using an AC transmission network, oligopolistic competition in power markets is formulated as a Nonlinear Programming (NLP) problem, and characterized by a multileader single-follower game. The follower is composed of a set of competitive suppliers, demands and the system operator, while the leaders are the dominant suppliers. The transmission network is modeled with a detailed nonlinear system. This approach allows one to capture the strategic behavior of suppliers regarding not only active but also reactive power. With this setting, the impact of voltage and apparent power flow constraints can be easily explored. Based on a three-node system, an illustrative study case is used to highlight the features of the formulation. A larger system is also used to describe computational issues.

**Speaker:** A. d'Aspermont

**Title:** Smooth Semidefinite Optimization and Applications

**Abstract:** We describe an application of Nesterov's optimal first-order method on large scale semidefinite optimization problems arising in multivariate statistics. The method's key numerical step involves computing a matrix exponential, and we show that this can be done very efficiently for favorable matrix spectrum structures.

**Speaker:** John T. Betts

**Title:** Planning a Trip to the Moon? ... And Back?

**Abstract:** Designing an optimal trajectory to the moon and back leads to an extremely challenging optimal control problem. This presentation will describe the practical and computational issues associated with the solution of these problems, including treatment of the underlying nonlinear boundary value problem, and nonlinear programming algorithm.

**Speaker:** A.R. Conn

**Title:** An Initial Algorithmic Framework for Convex Mixed Integer Nonlinear Programs

**Coauthors:** P. Bonami, L.T. Biegler, A. Wächter, G. Cornuéjols, I.E. Grossmann, C.D. Laird, J. Lee, A. Lodi, F. Margot, and N. Sawaya

**Abstract:** In my opinion mixed integer nonlinear programming is an area of ever increasing importance and applications that is significantly under researched – no doubt because it presents many difficult challenges. I will present a basic hybrid framework for convex mixed-integer nonlinear programming. In one extreme case, the method becomes the branch-and-bound approach, where a nonlinear optimization problem is solved in each node of the enumeration tree, and in the other extreme it reduces to the polyhedral outer approximation algorithm, which alternates between the solution of a nonlinear optimization problem and a mixed-integer linear program. Numerical results are presented, using an open source software implementation available on <http://www.coin-or.org>. This work results from an on-going research collaboration between IBM and CMU.

**Speaker:** M. Duer

**Title:** Solving Copositive Programs through Linear Approximations

**Coauthor:** S. Bundfuss

**Abstract:** Optimization over convex cones has become increasingly popular in recent years, the most prominent cones being the positive semidefinite cone and the second order cone. Semidefinite programming provides good and efficiently computable relaxations for several hard combinatorial and quadratic problems. However, it is known that these bounds may be improved by solving optimization problems over the copositive cone. The price of this gain in quality is an jump in complexity, as copositive programs are NP-hard. In this talk, we propose new polyhedral approximations of the cone of copositive matrices which we show to be exact in the limit. This gives rise to necessary as well as sufficient criteria for copositivity, and it can also be used to approximate copositive programs. We present an algorithm resulting from this approach, and conclude by presenting preliminary numerical results.

**Speaker:** M.C. Ferris

**Title:** Optimization of Noisy Functions: Application to Simulations

**Coauthor:** G. Deng

**Abstract:** In many real-world optimization problems, the objective function may come from a simulation evaluation so that it is (a) subject to various levels of noise, (b) not necessarily differentiable, and (c) computationally hard to evaluate.

We propose a two-phase approach for optimization of such functions. Phase I uses classification tools to facilitate the global search process. By learning a surrogate from existing data the approach identifies promising regions for optimization. Additional features of the method are: (a) more reliable predictions obtained using a voting scheme combining the options of multiple classifiers, (b) a data pre-processing step that copes with imbalanced training data and (c) a nonparametric statistical method to determine regions for multistart optimizations

Phase II is a collection of local trust region derivative free optimizations. Our methods apply Bayesian techniques to guide appropriate sampling strategies, while simultaneously enhancing algorithmic efficiency to obtain solutions of a desired accuracy. The statistically accurate scheme determines the number of simulation runs, and guarantees the global convergence of the algorithm.

We present results on two practical simulations: a Wisconsin breast cancer simulation and the robust design for a floating sleeve coaxial antenna for hepatic microwave ablation. The use of resampling of particular organ structures in this context will be outlined. Particular emphasis will be on general principles that are applicable to large classes of treatment planning problems. Specific examples will also be detailed showing enormous increase in speed of planning, without detriment to the quality of solutions found.

**Speaker:** R.M. Freund

**Title:** On Efficient Randomized Methods for Convex Optimization

**Abstract:** Randomized methods for convex optimization rely on stochastic processes and random number/vector generation as part of the algorithm and/or its analysis. In this talk we will discuss some recent developments in randomization-based algorithms for convex optimization from both a theoretical and practical point of view. We will show some interesting parallels between one randomization-based method and

interior-point methods, and will forecast some possible trends both in theory and practice and pose some pertinent research questions.

**Speaker:** D.Y. Gao

**Title:** Canonical Duality Theory and Applications in Global Optimization

**Abstract:** This paper presents a canonical (i.e., strong) duality theory for solving nonconvex programming problems subjected to box constraints. It is proved that the dual problems are either concave maximization, or convex minimization problems. Both global and local extrema of the constrained nonconvex problems can be identified by triality theory proposed by the author. Applications to nonconvex integer programming and Boolean least squares problem are discussed. Examples are illustrated. A conjecture on NP-hard problems is proposed.

**Speaker:** D. Goldfarb

**Title:** Total Variation Based Image Restoration by Second-Order Cone Programming and Min-Cuts

**Coauthor:** W. Yin

**Abstract:** The traditional approach for solving total variation based image restoration problems is based on solving partial differential equations. We describe here how to formulate and solve these problems either by interior-point algorithms for second-order cone programs or by parametric max flow algorithms.

**Speaker:** F. Jarre

**Title:** An Augmented Primal-Dual Method for Linear Conic Minimization

**Coauthors:** F. Rendl

**Abstract:** We present a new iterative method for solving linear minimization problems over convex cones. The problem is reformulated as an unconstrained problem of minimizing a differentiable convex function. The method does not use any homotopy parameter but solves the primal-dual problem in one step using a nonlinear conjugate gradient type approach. Some approaches for preconditioning of the algorithm will be illustrated with numerical examples.

**Speaker:** E. Krislock

**Title:** The Nonsymmetric Semidefinite Least Squares Problem and Compliance Matrix Estimation

**Abstract:** An important step in the process of making an interactive computer model of a deformable object is to estimate the compliance matrix at various points on the object. This estimation is accomplished by taking experimental measurements of the object and computing the least squares solution of a linear matrix equation of the form  $AX = B$ . For such a compliance matrix  $X$ , it is required that  $\frac{1}{2}(X + X^T)$  be positive semidefinite, otherwise the computer model may respond to the user touching some contact point by pulling the user's hand further in the direction of the touch. Adding this constraint to the least squares problem we get the nonsymmetric semidefinite least squares NS-SDLS

$$\begin{aligned} \min \quad & \|AX - B\|_F \\ \text{subject to} \quad & 0.5(X + X^T) \succeq 0 \end{aligned}$$

When the matrix  $A$  has linearly independent columns, the solution of the NS-SDLS problem exists and is unique. We will provide the Karush-Kuhn-Tucker equations which characterize this solution and show how these equations can be stated as a semidefinite linear complementarity problem. Finally, we will discuss how interior-point methods can be used for the numerical solution of the NS-SDLS problem.

**Speaker:** K. Kostina

**Title:** Model Based Design of Optimal Experiments for Parameter Estimation and Applications

**Coauthor:** H. G. Bock, S. Koerkel, and J. P. Schloeder

**Abstract:** The development and quantitative validation of complex nonlinear differential equation models is a difficult task that requires the support by numerical methods for sensitivity analysis, parameter estimation, and the optimal design of experiments. We first present particularly efficient "simultaneous" boundary value problems methods for parameter estimation in nonlinear differential algebraic equations, which are based on constrained Gauss-Newton-type methods and a time domain decomposition by multiple shooting. The

method include a numerical analysis of the well-posedness of the problem and an assessment of the error of the resulting parameter estimates. Based on these approaches, efficient optimal control methods for the determination of one, or several complementary, optimal experiments are developed, which maximize the information gain subject to constraints such as experimental costs and feasibility, the range of model validity, or further technical constraints.

Special emphasis is placed on issues of robustness, i.e. how to reduce the sensitivity of the problem solutions with respect to uncertainties - such as outliers in the measurements for parameter estimation, and in particular the dependence of optimum experimental designs on the largely unknown values of the model parameters. New numerical methods will be presented, and applications will be discussed that arise in satellite orbit determination, enzyme kinetics and robotics. They indicate a wide scope of applicability of the methods, and an enormous potential for reducing the experimental effort and improving the statistical quality of the models.

**Speaker:** Z.Q. Luo and S. Zhang

**Title:** Optimization in Resource Management: Complexity, Lyapunov Theorem and Approximation

**Abstract:** We consider a class of nonconvex optimization problems arising from resource (e.g., spectrum) management in multiuser communication. For the discretized version of this problem, we characterize its computational complexity under various practical settings and study the structure of its global optimal solutions. It is shown that this discretized nonconvex optimization problem is NP-hard in general and has a positive duality gap. Surprisingly this duality gap disappears asymptotically as the size of discretization step decreases to zero, thanks to a hidden convexity that can be uncovered by the Lyapunov Theorem in functional analysis. Based on this asymptotic zero duality result and a Lagrangian dual relaxation, we present, for any positive  $\epsilon$ , a polynomial time approximation scheme to compute an  $\epsilon$ -optimal solution for the continuous version of the resource management problem. Finally, we also establish a general min-max theorem for a game theoretic formulation under the continuous framework.

**Speaker:** J. Moré

**Title:** Derivative-Free Methods for Simulation-Based Optimization Problems

**Coauthor:** S. Wild

**Abstract:** We give a brief overview of the current state of the art of derivative-free methods that emphasizes the viewpoint that the performance of derivative-free methods should be measured when there is a constraint on the computational budget, that is, when there is a (small) limit on the number of function evaluations. We discuss how this viewpoint is appropriate for simulation-based optimization problems, and outline current research on new algorithms for this class of optimization problems.

**Speaker:** J. Martins

**Title:** Multidisciplinary Optimization: Current Status and Future Directions

**Abstract:** The objective of this talk is to present an overview multidisciplinary design optimization (MDO), with emphasis on the most significant challenges currently faced by academia and industry in its utilization. All current MDO architectures will be described and a unified mathematical framework for describing these architecture will be proposed. On the more applied side, a new software package that uses these ideas to automatically implement the various MDO architectures will be presented, together with results obtained in the solution of a suite of test problems. The suite itself represents another important focus of the current research and includes scalable problems in order to investigate how the relative merits of each MDO architecture vary. Finally, future research directions will be identified and discussed.

**Speaker:** J. Nie

**Title:** SOS Methods for Sensor Network Localization

**Abstract:** We formulate the sensor network localization problem as finding the global minimizer of a quartic polynomial. Then sum of squares (SOS) relaxations can be applied to solve it. However, the general SOS relaxations are too expensive for practical problems. Exploiting special features of this polynomial, we propose a new Structured SOS relaxation. It works well for large scale problems. At each step of interior-point methods solving the resulting SOS relaxation, the computational cost is  $O(n^3)$ . When distances have

errors and localization is unique, we show that the sensor location given by this SOS relaxation is accurate within a constant factor of the distance error under some technical assumptions.

**Speaker:** A. Ozgadlar

**Title:** Differential Topology for the Uniqueness of Equilibrium in Network Control Models

**Coauthors:** A. Simsek and D. Acemoglu

**Abstract:** In this talk, we first present an extension of the Poincare-Hopf Theorem of index theory to generalized critical points of a function defined on a compact region with nonsmooth boundary, defined by a finite number of smooth inequality constraints. We use the generalized Poincare-Hopf Theorem to present sufficient (local) conditions for the global uniqueness of solutions to finite-dimensional variational inequalities and the uniqueness of stationary points of optimization problems. We finally use our results to establish uniqueness of equilibria in two recent models of communication networks.

**Speaker:** J.D. Pintér

**Title:** Global Optimization in Practice: State-of-the-Art and Perspectives

**Abstract:** Global optimization (GO) – the theory and methods of finding the best solution in multi-extremal models – has become a subject of significant interest in recent decades. The key theoretical results have been followed by software implementations, and a growing range of real-world applications. We present a concise review of these developments, with an emphasis on practical aspects, including modeling environments, software and applications.

**Speaker:** G. Savard

**Title:** Pricing a Segmented Market Subject to Congestion

**Coauthors:** M. Fortin, L. Brothers, and P. Marcotte

**Abstract:** The optimal setting of prices, taxes or subsidies on goods and services can be naturally modeled as a bilevel program. Indeed, bilevel programming is an adequate framework for modeling optimization situations where a subset of decision variables is not controlled by the main optimizer (the leader), but rather by a second agent (the follower) who optimizes its own objective function with respect to this subset of variables. In this presentation we address the problem of setting profit-maximizing tolls on a congested transportation network involving several user classes. At the upper level, the firm (leader) sets tolls on a subset of arcs and strives to maximize its revenue. At the lower level, each user minimizes its generalized travel cost, expressed as a linear combination of travel time and out-of-pocket travel cost. We assume the existence of a probability density function that describes the repartition of the value of time (VOT) parameter throughout the population. This yields a bilevel optimization problem involving a bilinear objective at the upper level and a convex objective at the lower level. Since, in this formulation, lower level variables are flow densities, it follows that the lower level problem is infinite-dimensional. We devise a two-phase algorithm to solve this nonconvex problem. The first phase aims at finding a good initial solution by solving for its global optimum a discretized version of the model. The second phase implements a gradient method, starting from the initial point obtained in the initial phase.

**Speaker:** P. Tseng

**Title:** p-Order Cone Relaxation for Sensor Network Localization

**Abstract:** Building on recent work on 2nd-order cone relaxation for sensor network localization, we discuss extensions to p-order cone relaxation when measured distances are based on p-norm instead of Euclidean norm.

**Speaker:** C. Visweswariah

**Title:** Challenges in Statistical Timing and Optimization of Integrated Circuits

**Abstract:** As transistors and wires on a chip get smaller, they are exhibiting proportionately increasing variability. This variability is changing the design methodology, and the tools and techniques used to analyze and optimize chip designs. The first part of this presentation will give an overview of our research work in statistical timing and optimization. In the second part, some mathematical problems will be formulated that, if solved, would be of tremendous utility to the design automation community.

**Speaker:** H. Wolkowicz

**Title:** Semidefinite Relaxations for Anchored Graph Realization and Sensor Localization

**Abstract:** Many applications use ad hoc wireless sensor networks for monitoring information. Typical networks include a large number of sensor nodes which gather data and communicate among themselves. The location of a subset of the sensors is known; these sensors are called anchors. From the intercommunication, we are able to establish distances between a subset of the sensors and anchors. The sensor localization problem is to find/estimate the location of all the sensors. We study several semidefinite programming relaxations for this numerically hard problem.

**Speaker:** H. Zhang

**Title:** Approximation Algorithms for Routing in VLSI Design and Multicast Networks

**Coauthors:** A. Deza, C. Dickson, T. Terlaky, and A. Vannelli

**Abstract:** Given a computer chip represented as a grid graph and groups of pins as vertices to be connected, the goal of the global routing problem in VLSI design is to find one tree along the channels for each group of pins such that the number of trees crossing a channel is bounded by its capacity and the total cost (a combination of the overall tree length and the total number of bends) is minimized. Global routing is regarded as one of the hardest problems in discrete optimization due to the scales of real instances. We present a concurrent routing algorithm by mathematical programming methods in order to approach the global optimum. Our algorithm runs in a polynomial time and delivers a near-optimal solution with a provably good approximation bound for any instance. Promising numerical results for challenging benchmarks are also reported. We also show that this algorithm can be applied to a multicast routing problem in communication networks.

## 5 Highlights of the Panel Discussions

**Title:** Where we need breakthroughs? What is the next breakthrough?

**Panel:** D. Goldfarb, T. Marlin, J. Pintér, R. Vanderbei

**Moderator:** T. Terlaky

- The dramatic progress in LP technology was highlighted, from the 1947 diet problem (9 constraints, 77 variables, 120 “soldier-days”. The software was 9 people!) through the 1980s (up to few thousands variables and constraints in 6 seconds-300 seconds) and up to 2004 (many large problems can be solved in a few seconds on any decent computer). These improvements were due roughly equally to hardware and software. Preprocessing probably brought the most dramatic improvement! The breakthrough in IPMs for LP came from an obscure, theoretical Soviet mathematician. The question is of course: Where will the next breakthrough come from?

We *really* don't know where the breakthrough will come from! IPMs were very exciting for many years. Now we have symmetric cone optimization, robust optimization... could this have been predicted?

- Dealing with uncertainty is very important and very difficult. The effect of uncertainty on the objective was pointed out: monitor, diagnose, reduce (e.g., control in a chemical plant to maximize the profit). Reduce uncertainty by systematic, focused experimentation to improve the performance of model-based optimization.
- A related point was that for engineers to pay attention, mathematicians must show a genuine interest in the problems. Also, proper modelling is really important, and therefore so is understanding and caring about the problem.
- The really interesting stuff happens in inter-disciplinary research, where the most significant contributions are made (e.g., cancer therapy research needed global optimization).

The only way to succeed is for optimizers and engineers to get together and talk, as is taking place this week at BIRS.

**Title:** What do engineers need? What do optimizers need to know about engineering?

**Panel:** Z.Q. Luo, T. Marlin, J. Martins, J. Moré, C. Viswesvariah, L. Behjat

**Moderator:** R. Vanderbei

- There is tremendous power in optimization algorithms. The challenge is to harness their power.
- What can optimization offer?
  - a suite of good algorithms and software;
  - a theory that helps to characterize the structure of optimal solutions (can serve as a guide to designing heuristics).
- What do engineers need?
  - Education: good optimization, complexity courses, books; course projects (missing some basic ideas and techniques);  
Many engineering students just use the Matlab optimization toolbox blindly. Some engineers develop in-house software without exploiting what is already available. In short, there is a need to train a large number of engineers with a solid understanding of optimization.  
Engineers want to understand the solutions using post-optimality analysis. They often do this visually, e.g., using plots of the result, not analytically.
- Software: robust, efficient, and able to solve large-scale problems.
- Some theory, such as: distributed optimization; game theory over energy-constrained networks with unreliable links.  
Need specialized codes to compete with engineering heuristics. General codes usually not efficient, and high accuracy is typically not needed, it is even useless!
- Key: For optimizers,
  - find a good engineering partner.
  - work closely with engineers to:
    - \* formulate the problem properly;
    - \* suggest good software to use (or develop specialized ones);
    - \* identify interesting theoretical issues for further investigation;
    - \* interpret computational and theoretical results in engineering context.
  - have a lot of patience!
- The real value is in the *interaction* when dealing with really hard problems. Engineers need to appreciate what optimizers can deliver; and the optimizers need to understand the needs of engineers.
- Challenges for the future:
  - Encourage communication between engineers and mathematicians: tutorials? specialized workshops?
  - Need to understand each other's challenges;
  - Start at the undergraduate level, and try to bring optimization into the engineering courses;
  - Better education is really key!

**Title:** Optimization Software

**Panel:** A. Conn, J. Moré, J. Pintér, I. Pólik, K. Toh, R. Vanderbei

**Moderator:** T. Terlaky

- Ease of use more important than quality of algorithm!

- Commercial software often more attractive than open source (the fact they pay for it makes a difference);
- Nonlinear optimization cannot be used like a black-box;
- We publish our theory and build on it. We do not publish our software so others can build on it;
- Software needs good documentation; including simple examples so that users can copy code;
- Where's the reward in writing good software?
- Commercial software does not keep pace with academic breakthroughs;
- Almost all academics and students don't know how to write extensible, maintainable software;
- Lacks: we lack *standards*, we lack *component libraries*, we lack *support*;
- There has been a serious effort in implementing some semi-general purpose SDP solver for large-scale problems;
- Testing is a very important part of developing software; be imaginative on how your code will perform badly!

## 6 Scientific Progress Made

Participants with similar research interests formed several special interest groups. There was thorough discussion about their research work and future plan in each group. Sketch of the discussed topics in each group are as follows:

- **VLSI design** (M. Anjos, L. Behjat, A. Conn, T. Terlaky, C. Visweswariah, H. Zhang): Participants in this group are from both industry and academia.
  - Mathematical models for placement, floorplanning and global routing in VLSI design;
  - Current methods: IPMs, integer nonlinear programming, approximation algorithms;
  - More realistic models fitting the engineering demands needed;
  - Efficient algorithms and fast heuristics desired;
  - Implementation and software packages for industrial benchmark.
- **Digital communication** (Z.Q. Luo, J. Nie, P. Tseng, W. Yu, H. Wolkowicz, S. Zhang):
  - Hidden convexity in resource management in multicast communication;
  - Relaxation for sensor network localization: SDP, Sum of Square, or cone;
  - More application of Lyapunov theorem expected;
  - Alternative proof of Lyapunov theorem?
  - Better approaches for sensor network localization?
  - Link between resource management and fractional packing problems.
- **Bilevel and equilibrium problems** (M. Anjos, M. Ferris, A. Ozdaglar, G. Savard, P. Tseng):
  - Mathematical formulations of equilibria;
  - Computational methods and software for equilibrium problems;
  - Role and importance of congestion in markets;
  - Comparison of different methodologies for bilevel problems.

- **Engineering design and optimization** (J. Betts, E. Kostina, T. Marlin, J. Martins, R. Vanderbei): Participants in this group are from both industry and academia.
  - Challenges for optimization in industry;
  - Importance of MDO;
  - Reduction of uncertainty by systematic, focused experimentation;
  - The challenges of industry-academia interactions.

## 7 Outcomes of the Meeting

There will be a special issue of the journal *Optimization and Engineering*, edited by E. Kostina and J. Martins, dedicated to this scientific meeting.

This meeting clearly generated a significant amount of interaction between members of both communities, and we hope that these discussions have seeded future exciting developments at the interface between optimization and engineering.