Workshop on Operator Spaces and Group Algebras
at Banff International Research Station
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Several aspects of amenability for groupoids
Claire Anantharaman-Delaroche

Abstract : We shall discuss various approaches of amenability for groupoids (mainly equivalence relations and group actions) and some applications. In particular we shall compare global and fiberwise amenability, and relate these notions to asymptotic properties of random walks on groupoids.

Real rank and stable rank of group C*-algebras
Rob Archbold

Abstract : We shall begin with a survey of results concerning real rank and stable rank for the C*-algebras associated with various classes of locally compact groups. In the second part of the talk, we shall describe recent results (joint with E. Kaniuth) on the real rank and stable rank of C*-algebras associated with compact transformation groups.

Operator algebraic rigidity of higher rank lattices
Bachir Bekka

Abstract : The following analogue of Margulis superrigidity theorem in the context of operator algebras was suggested by Connes and Jones. If Γ is a discrete group, view Γ as a subgroup of the unitary group $U(L(\Gamma))$ of its von Neumann algebra $L(\Gamma)$. Assume that Γ is a higher rank lattice. Let M a type II_1 factor and $U(M)$ its unitary group. Then every homomorphism $\pi : \Gamma \to U(M)$ with $\pi(\gamma)'' = M$ extends to a homomorphism $U(L(\Gamma)) \to U(M)$. As we will discuss, the answer is positive for arithmetic lattices like $SL_n(Z)$ for $n \geq 3$. As an application, the full C*algebra of this group has no faithful tracial state; this answers a question of Kirchberg in relation with Connes embedding problem.

Completely positive and completely bounded maps
on Coxeter groups with applications
Marek Bożejko

Abstract : In our talk we present 2 classes of positive denite functions on Coxeter groups (W, S) with applications to constructions of a big class of operator spaces completely isomorphic with $R \cap C$. In this talk we give also the solution of Bessis-Moussa-Villani conjecture (BMV conjecture) for the generalized Gaussian random variables

$$G(f) = a(f) + a * (f),$$

where f is in the real Hilbert space H. The main examples of generalized Gaussian random variables are q-Gaussian random variables, $(-1 \leq q \leq 1)$, related to q- CCR relation and others commutation relations. We will prove that (BMV) conjecture is true for all operators $A = G(f)$, $B = G(g)$; i.e. we will show that the function

$$F(x) = tr(exp(A + ixB))$$

is positive definite on the real line. The case $q = 0$, i.e. when $G(f)$ are the free Gaussian (Wigner) random variables and the operators A and B are free with respect to the vacuum trace, was proved by M.Fannes and D.Petz. Connections with Herz-Schur multipliers on groups i.e.completely bounded multipliers of the Fourier algebra $A(G)$ and Herz- Littlewood multipliers will be done with applications to lacunary sets on groups. We will show that in the Coxeter group (W, S), the set of genera- tors is completely bounded Sidon set and hence also completely bounded $\Lambda(p)$ set for all $p > 2$.

Amenable groups and finite dimensional approximations
Marius Dadarlat

Abstract : We plan to discuss some open questions related to the problem of embedding the C*-algebra of a discrete amenable group into an approximately finite dimensional algebra.

Weighted convolution algebras on the rationals
H. G. Dales

Abstract : Let A be a Banach algebra. Then the second dual space A'' has two Arens products denoted by $\Box$ and $\diamond$. We can define the left and right topological centres of A'' with respect to these products; our algebra is *Arens regular* if these centres are both equal to all of A'', so that $\Box$ and $\diamond$ coincide on A'', and *strongly Arens irregular* if these centres are both equal to A, regarded as a subspace of A''. It is an important theorem of Lau and others that the group algebra $L^1(G)$ of each locally compact group G is strongly Arens irregular; the same result holds for certain semigroup algebras $\ell^1(S)$. There has also been considerable interest in the question: how many elements of A'' are required to determine the topological centres? These results give structural information about group algebras $L^1(G)$ and other algebras.

Let us now consider convolution algebras defined by a weight ω on a semigroup. We shall discuss the new, more complicated situation. In particular, in this talk we shall concentrate on the semigroup $\mathbb{Q}^{+\bullet}$ of strictly positive rational numbers. We shall discuss when the weighted convolution algebras $\ell^1(\mathbb{Q}^{+\bullet}, \omega)$ have the above properties, giving various examples to show that they may be Arens regular, that they may be strongly Arens irregular, and that they may be neither.

Horn's inequalities and Connes' embbeding problem
Ken Dykema
(joint work with Benoit Collins)

Abstract : Connes embedding problem asks whether every separable II_1-factor can be embedded in the ultrapower of the hyperfinite II_1-factor; this is equivalent to asking whether every finite set in every II_1-factor has microstates. We relate this to questions concerning the possible spectral distributions of $a + b$, where a and b are self-adjoint elements in a II_1-factor having given spectral distributions. The finite-dimensional version of the spectral distribution question was solved by Klyatchko, Totaro, Knudson and Tao, in terms of inequalities first formulated by Horn.

Quantized Functional Analysis and Quantum Information Theory
Edward Effros

Abstract : For much of the Twentieth Century, the theory of quantum measurements consisted of various thought experiments which seemed remote from the real world. With the advent of devices that can manipulate individual photons and particles, this situation has drastically changed. Quantum channels are currently realizable both in and out of the laboratory. It is an ideal area for operator algebraists to gain a visceral feeling for the physics of quantum theory. Perhaps the most surprising mathematical aspect of QIT is the depth of the methods that are required. This is reflected in the fact that there are still unsolved problems at the most basic levels.

There have been some truly remarkable applications of quantized functional analysis to this area. This has been in part due to the remarkable work of Marius Junge and Mary Beth Ruskai. I will do my best to give a sketch of the physical and mathematical background for this new area.

Weak amenability of CAT(0) cubical groups
Erik Guentner

Abstract : A CAT(0) cubical complex is a complex in which the cells are Euclidean cubes, the attaching maps are isometries and the intrinsic metric satisfies the CAT(0) inequality. Basic examples are trees and Euclidean spaces. A group acting metrically properly on a CAT(0) cubical complex has the Haagerup property and it is natural to ask whether such a 'CAT(0) cubical group' is weakly amenable. In the talk we shall construct uniformly bounded representations of groups which admit a metrically proper action on a finite dimensional CAT(0) cubical complex. As a consequence, we can conclude that such groups are weakly amenable. The talk is based on joint work with Nigel Higson.

Matricially operator-convex cones and classification of non-simple algebras
Eberhard Kirchberg

Abstract : We consider properties of (point-norm closed) matricially operator convex cones C of completely positive maps, and related C-compatible KK-, Ext- and asymptotic theories (Rordam-groups). Applications to the classifcation of non-simple C*-algebras will be outlined.

Symmetry and the GRS-condition revisited
Michael Leinert
(joint work with G.Fendler and K.Groechenig)

Abstract : In a previous note, with J.Ludwig and C.Molitor-Braun, we showed that symmetry of a weighted L^1 algebra on a compactly generated locally compact group G of polynomial growth is roughly equivalent to the fact that the weight satisfies the GRS-condition. For this, however, we needed an awkward technical assumption: there is a sequence $d_k > 0$ with $\lim d_k^{1/k} = 1$, a generating neighbourhood U, and a natural number l such that for all natural numbers k the weight w satisfies

$$w(x) \geq d_k \sup\{w(y)|y \in U^k\} \text{ for all } x \text{ in } G \setminus U^{kl}.$$

In our present work we show that this assumption is not necessary, so the GRS-condition for the weight really characterizes the symmetry of the corresponding weighted L^1 algebra, no matter what the weight looks like. We also show that the GRS-condition has a bearing on symmetry and inverse-closedness of certain infinite matrix algebras.

Multipdimensional Operator Multipliers
Turowska Lyudmila

Abstract : We introduce multidimensional Schur multipliers and characterise them generalising well known results by Grothendieck and Peller. We define a multidimensional version of the two dimensional operator multipliers studied recently by Kissin and Shulman. The multidimensional operator multipliers are defined as elements of the minimal tensor product of several C*-algebras satisfying certain boundedness conditions. In the case of commutative C*-algebras, the multidimensional operator multipliers reduce to continuous multidimensional Schur multipliers. We show that the multipliers with respect to some given representations of the corresponding C*-algebras do not change if the representations are replaced by approximately equivalent ones. We establish a non-commutative and multidimensional version of the characterisations by Grothendieck and Peller which shows that universal operator multipliers can be obtained as certain weak limits of elements of the algebraic tensor product of the corresponding C*-algebras. If I have time I will also discuss completely compact operator multipliers. This is a joint work with I.G.Todorov and K.Juschenko.

Some properties of character amenable Banach algebras
Mehdi Monfared

Abstract : We show that a natural unital uniform algebra on a compact space X is character amenable if and only if X is the Choquet boundary of the algebra. As a consequence it follows that the analogue of Sheinbergs result on amenable uniform algebras does not hold for character amenable unifrom algebras. We show that a necessary condition for character amenability of a natural, unital Banach function algebra on a compact space X, is that the Choquet boundary of the algebra coincide with X. This condition is not sufficient as the unitization of the Fourier algebra on $SL(2, R)$ shows.

We show that a commutative character amenable Banach algebra is either semisimple or its radical is infinite-dimensional. Further in such an algebra, every ideal of finite co-dimensional is character amenable.

We show that a finite-dimensional algebra is left character amenable if and only if it is semisimple. Hence if A is left character amenable then A/I is semisimple for every ideal of finite co-dimension in A.

We show that left character amenability of a Banach algebra A is equivalent to every continuous derivation $d : A \to E_\phi$, ($\phi \in \Phi_A \cup \{0\}$) being approximately inner. It follows that if $(A'', \Box)$ is left character amenable then so is A. We show the converse of this result is not true by proving that if X is a left introverted subspace of $L^\infty(G)$ containing $AP(G)$, then X' is left character amenable if and only if G is finite. In particular, $L^1(G)''$ is character amenable if and only if G is finite.

We discuss the concept of C-left character amenable Banach algebras, and show that a directed union of C-left character amenable Banach algebras, is itself C-left character amenable.

A representation for locally compact quantum groups
Matthias Neufang

Abstract : Locally compact quantum groups, as presented by J. Kustermans and S. Vaes in 2000, provide the perfect framework for Pontryagin duality beyond locally compact abelian groups, as well as for several non group-like algebras arising in mathematical physics. I shall mainly focus on recent joint work with M. Junge and Z.-J. Ruan which unifies and generalizes earlier work of F. Ghahramani, N. Spronk, E. Stormer, Z.-J. Ruan and myself in the group case, to arbitrary locally compact quantum groups G. We introduce the algebra $M_{cb}^r(L_1(G))$ of completely bounded right multipliers on $L_1(G)$, and prove that it can be identified with the algebra of normal completely bounded $L_1(G)$- bimodule maps on $B(L_2(G))$ leaving $L_1(G)$ invariant. From this representation we deduce that every completely bounded right centralizer of $L_1(G)$ is in fact implemented by an element of $M_{cb}^r(L_1(G))$. We also show that our representation framework allows us to express quantum group Pontryagin duality purely as a commutation relation. We discuss applications to quantum information theory of this natural class of channels, and calculate the cb-entropy in the finite dimensional setting.

Rigidity for profinite actions of free groups

Narutaka Ozawa

Abstract : Let $\mathcal{G} = (\Gamma_n)_{n=1}^{\infty}$ be a decreasing sequence of finite index subgroups of a group Γ such that $\bigcap \Gamma_n = \{1\}$. We write

$$X_{\mathcal{G}} = \varprojlim \Gamma/\Gamma_n$$

for the projective limit of the finite probability spaces Γ/Γ_n with the uniform measures. The ergodic essentially-free probability-measure-preserving action $\Gamma \curvearrowright X_{\mathcal{G}}$ is said to be *profinite*. I will talk on some rigidity results for the profinite actions of the free group $\mathbb{F}_r$, in the framework of von Neumann algebras.

Projectivity and injectivity in topology and analysis

Vern I. Paulsen

Abstract : Because of the well-known contravariant functor between compact, Hausdorff spaces and abelian unital C*-algebras, many results about injectivity in analysis correspond to results on projectivity in topology. We begin by showing how many classical results about injectivity in analysis follow more easily by fully exploiting projectivity. We also use this idea to give a more elementary construction of injective envelopes as the duals of minimal projective covers.

We then turn our attention to what happens in the presence of a group action. We prove that G-projective spaces exist, even when the group is non-amenable, and that G-projectivity is strictly stronger than G-injectivity. We prove that minimal dynamical systems have G-projective covers. But it is not known if the G-injective envelope of the continuous functions on a minimal dynamical system is equal to the continuous functions on its G-projective cover.

This is joint work with Don Hadwin.

Similarity problems, length and amenability

Gilles Pisier

Abstract : We call unitarizable a (locally compact) group such that any (continuous) uniformly bounded representation is unitarizable (i.e. is similar to a unitary representation). Dixmier asked already in 1950 whether unitarizable implies amenable (the converse was proved by him and Day independently). Motivated by this, Kadison (1955) formulated the following conjecture: any bounded homomorphism $u : A \to B(H)$, from a C*-algebra into the algebra $B(H)$ of all bounded operators on a Hilbert space H, is similar to a *-homomorphism, i.e. here is an invertible operator $\xi : H \to H$ such that $x \to \xi x \xi^{-1}$ satisfies $\xi u(x^*)\xi^{-1} = (\xi u(x)\xi^{-1})^*$ for all x in A. These conjectures remain unproved, although many partial results are known. We will survey those as well as more recent results on the closely related notion of length of an operator algebra, which crucially uses completely bounded maps, the Haagerup tensor product and other operator space notions. In particular, we will explain why length equal to 2 characterizes amenable groups or nuclear C*-algebras. Moreover, we will show that if we can always force the similarity to be in the von Neumann algebra generated by the range, then the group (or the C*-algebra) must be amenable.

Normalizers of irreducible inclusions of subfactors
Roger Smith

Abstract : Normalizing unitaries have played a role in von Neumann algebra theory ever since Dixmier used them to classify masas into various types. In this talk we will examine a different situation, an irreducible inclusion $N \subseteq M$ of finite factors. The general problem is to determine the group $\mathcal{N}(N)$ of normalizing unitaries and/or the von Neumann algebras that it generates. We will focus on two situations where it is possible to give complete answers to this. Both arise from discrete groups: the inclusion $L(H) \subseteq L(G)$ for an inclusion of groups $H \subseteq G$ and the inclusion $M^G \subseteq M$ where M^G is the fixed point algebra of a finite group action. In both cases the normalizing unitaries of the subfactor are studied by relating them to certain projections in $N' \cap \langle M, e_N \rangle$ where $\langle M, e_N \rangle$ is the basic construction of subfactor theory.

This is joint work with Stuart White and Alan Wiggins.

The canonical operator space structure for $A_p(G)$
Volker Runde

Abstract : The Fourier algebra $A(G)$ of a locally compact group G is the predual of the group von Neumann algebra; consequently, $A(G)$ has a canonical operator space structure tuning it into a completely contractive Banach algebra. The Fig‘a-Talamanca Herz algebras $A_p(G)$ for $p \in (1, \infty)$ are generalizations of $A(G)$ (we have $A_2(G) = A(G)$) and have many properties in common with $A(G)$. However, for $p \neq 2$, there is no obvious, non-trivial operator space structure on $A_p(G)$.

Recently, A. Lambert extended the notion of column operator space from Hilbert space to general Banach spaces. With the help of his construction, we obtain an operator space structure on $A_p(G)$with the following properties: (1) for $p = 2$, we get the canonical operator space over $A(G)$; (2) $A_p(G)$ is a completely bounded Banach algebra for each p; (3) G is amenable if and only if $A_p(G)$ is operator amenable for one (and equivalently for all) p; (4) the HerzSchur multipliers on $A_p(G)$ are completely bounded (the converse of this assertion is still open).

This is joint work with A. Lambert and M. Neufang.

Convolutions on compact groups and Fourier algebras of coset spaces
Ebrahim Samei

Abstract : In this joint work with Brian Forrest and Nico Spronk, we study two related questions: (1) For a compact group G, what are the ranges of the convolution maps on $A(G \times G)$ given for u, v in $A(G)$ by $u \otimes v \mapsto u \star v'$ ($v'(s) = v(s^{-1})$) and $u \otimes v \mapsto u \star v$? (2) For a locally compact group G and a compact subgroup K, what are the amenability properties of the Fourier algebra of the coset space $A(G/K)$?

In answering the first question, we obtain for compact groups which do not admit an abelian subgroup of finite index, some new subalgebras of $A(G)$. Using those algebras we can find many instances in which $A(G/K)$ fails the most rudimentary amenability property: operator weak amenability. However, using different techniques, we show that if the connected component of the identity of G is abelian, then $A(G/K)$ always satisfies the stronger property that it is hyper- Tauberian.

This will provide us with some new sets of spectral synthesis for $A(G)$. We also establish a criterion which characterises operator amenability of $A(G/K)$ for a class of groups which includes the maximally almost periodic groups.

Operator space structure on Feichtingers Segal algebra
Nico Spronk

Abstract : We extend the definition, from the class of abelian groups to a general locally compact group G, of Feichtingers remarkable Segal algebra $S_0(G)$. In order to obtain functorial properties for non-abelain groups, in particular a tensor product formula, we endow $S_0(G)$ with an operator space structure. With this structure $S_0(G)$ is simultaneously an operator Segal algebra of the Fourier algebra $A(G)$, and of the group algebra $L^1(G)$. We show that this operator space structure is consistent with the major functorial properties: (i) $S_0(G)\hat{\otimes}S_0(H) = S_0(G?H)$ completely isomorphically (operator projective tensor product), if H is another locally compact group; (ii) the restriction map $u \to u|_H : S_0(G) \to S_0(H)$ is completely surjective, if H is a closed subgroup; and (iii) $T_N : S_0(G) \to S_0(G/N)$ is completely surjective, where N is a normal subgroup and $T_N u(sN) = \int Nu(sn)dn$. We also show that $S_0(G)$ is an invariant for G when it is treated simultaneously as a pointwise algebra and a convolutive algebra.

The Haagerup property for wreath products
Alain Valette

Abstract : This is joint work with Y. de Cornulier, F. Haglund and Y. Stalder.

We prove that the Haagerup property (aka a-T-menability) is inherited under wreath products. I will give the proof of this fact for the wreath product of a finite group by the free group on 2 generators, by establishing the stronger property that it acts properly on a space with walls. A recent result by Ozawa-Popa states that this group is not weakly amenable with constant 1. Together, our results disprove a conjecture of Cowling stating that the class of a-T-menable groups coincides with the class of groups which are weakly amenable with constant 1. It also shows that the assumption of finite-dimensionality cannot be removed from the Guentner-Higson result stating that a group acting properly on a finite-dimensional CAT(0) cube complex, is weakly amenable with constant 1 (see Guentner's talk at this conference).

Generalized notions of amenability for Segal algebras
Yong Zhang

Abstract : A Segal algebra $S^1(G)$ on a locally compact group G is a dense left ideal of the group algebra $L^1(G)$ that has a Banach algebra norm $\|\cdot\|_S$ such that $\|f\|_1 \leq \|f\|_S$, $\|L_x f\|_S = \|f\|_S$, and $L_x f$ is continuous in $x(f \in S^1(G)$ and $x \in G)$, where $\|\cdot\|_1$ denotes the L^1-norm on $L^1(G)$ and L_x denotes the left translation operator. A Segal algebra $S^1(G)$ is never amenable except for $S^1(G) = L^1(G)$. We will discuss pseudo amenability, pseudo contractibility and approximate amenability etc for this type of Banach algebras.