Π_1^1 Conservation of COH Over $B\Sigma_2$

(Joint work with Ted Slaman and Yue Yang)

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Hierarchy of the Induction Scheme

Fix $\mathcal{M} = \langle M, \mathbb{X}, +, \cdot, 0, 1 \rangle$ to be a structure in the language of second order arithmetic. $X \subset M$ is M-finite if it is coded in M. Fix $n \geq 1$.

- $\mathcal{M} \models I\Sigma_n$ (Σ_n induction) if it satisfies every Σ_n instance (with parameters in $\mathcal{M}$) of the induction scheme.
- $\mathcal{M} \models B\Sigma_n$ (Σ_n bounding) if every Σ_n definable function maps an M-finite set onto an M-finite set.
- Kirby-Paris: $\cdots \to I\Sigma_{n+1} \to B\Sigma_{n+1} \to I\Sigma_n \to \cdots$
- We take as base theory RCA_0 (Recursive Comprehension Axiom plus $I\Sigma_1$).

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The Combinatorial Principle COH

Definition

Let $R \in \mathbb{X}$ and $R_s = \{t | (s, t) \in R\}$. $C \subset M$ is cohesive for R if for all s, either $C \cap R_s$ is M-finite or $C \cap \bar{R}_s$ is M-finite.

COH: $\mathcal{M} \models$ COH if for all $R \in \mathbb{X}$, there is a $C \in \mathbb{X}$ that is cohesive for R.
An M-extension of $\mathcal{M}$ is a structure $\mathcal{M}^* = \langle M^*, \mathbb{X}^*, +, \cdot, 0, 1\rangle$ such that $M = M^*$ and $\mathbb{X} \subseteq \mathbb{X}^*$.

Theorem

(Cholak, Jockusch and Slaman) *Let $n = 1, 2$. Every countable $\mathcal{M} \models RCA_0 + I\Sigma_n$ has an M-extension $\mathcal{M}^* \models RCA_0 + COH + I\Sigma_n$.*

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$COH + RCA_0 + I\Sigma_n$ is Π^1_1 conservative over $RCA_0 + I\Sigma_n$, i.e. if φ is Π^1_1 and $RCA_0 + COH + I\Sigma_n \vdash \varphi$, then $RCA_0 + I\Sigma_n \vdash \varphi$.

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A Two-Stage Construction for M-Extension

- Stage 1. Build an R'-recursive tree T for which every unbounded path X on T is cohesive for R and GL_1 relative to R, i.e. $X \oplus R' \equiv_T X'$.

Let I be a Σ_2 cut in $\mathcal{M}$ and $g : I \to M$ be Σ_2, increasing and cofinal.

- Build a uniformly R'-recursive nested sequence $\{C_i | i \in I\}$ of $\mathcal{M}$-infinite R-recursive trees such that for all $i \in I$:
  - (i) $C_i \supset C_{i+1}$
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  - (III) Every unbounded path on C_i is 1-generic on C_i for $\exists x \varphi_s$, $s < g(i)$, where φ_s is Δ_0
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A Two-Stage Forcing Construction

- A Cohen-type forcing construction carried out recursively in R' is deployed to achieve GL_1. However,
- For each $i \in I$, need to argue that there is a condition forcing $\exists x \varphi_s$ for all $s < g(i)$.
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- Stage 2. Define a path G (from the *outside*) on T such that $\mathcal{M}[G] \models B\Sigma_2$.
- Define countable sequences $\{T_n\}$ and $\{\sigma_n\}, n < \omega$, such that for each n,
  - $T_n \supset T_{n+1}$ are recursive in R'
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Put $G = \bigcup_n \sigma_n$.

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Ramsey's Theorem For Pairs

Let $\mathcal{M} \models \mathrm{RCA}_0$.

RT^2_2: Every two coloring of $[M]^2$ (pairs of elements of M) has a homogeneous set in $\mathcal{M}$.
SRT^2_2: Every *stable* two coloring of $[M]^2$ has a homogeneous set in $\mathcal{M}$ ($f : [M]^2 \to 2$ is stable if for all x, $\lim_y f(x, y)$ exists).

Hirst: Over RCA_0, $\mathrm{RT}^2_2 \to B\Sigma_2$

Cholak, Jockusch and Slaman: Over RCA_0,
$\mathrm{RT}^2_2 \leftrightarrow \mathrm{COH} + \mathrm{SRT}^2_2$.

Question: Over RCA_0, does $\mathrm{RT}^2_2 \to I\Sigma_2$? Does $\mathrm{SRT}^2_2 \to \mathrm{RT}^2_2$?

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RT^2_2: Every two coloring of $[M]^2$ (pairs of elements of M) has a homogeneous set in $\mathcal{M}$.
SRT^2_2: Every *stable* two coloring of $[M]^2$ has a homogeneous set in $\mathcal{M}$ ($f : [M]^2 \to 2$ is stable if for all x, $\lim_y f(x, y)$ exists).

Hirst: Over RCA_0, $\mathrm{RT}^2_2 \to B\Sigma_2$

Cholak, Jockusch and Slaman: Over RCA_0,
$\mathrm{RT}^2_2 \leftrightarrow \mathrm{COH} + \mathrm{SRT}^2_2$.

Question: Over RCA_0, does $\mathrm{RT}^2_2 \to I\Sigma_2$? Does $\mathrm{SRT}^2_2 \to \mathrm{RT}^2_2$?

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Nonstandard Methods in RT^2_2

- Downey, Hirschfeldt, Lempp and Solomon: There is a Δ_2 $A \subset \omega$ such that neither A nor $\bar{A}$ contains an infinite low Δ_2 set.
- Hirschfeldt, Jockusch, Kjos-Hansen, Lempp and Slaman: Every Δ_2 set $A \subset \omega$ either contains or is disjoint from an infinite *incomplete* Δ_2 set.
- For A Δ_2, call any infinite $X \subset A$ or $\bar{A}$ a solution for A.
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Either $\mathcal{P}$ or $\mathcal{Q}$ is *false*.
Conjecture 1: There is a countable $\mathcal{M} \models RCA_0 + B\Sigma_2$ with an M-extension for the same theory in which every Δ_2 set has a solution.

Corollary (to Conjecture 1): RT^2_2 does not imply $I\Sigma_2$.

Jockusch: There is a recursive two coloring of $[\mathbb{N}]^2$ with no Δ_2 homogeneous set.

Theorem

There is a (first order) $\mathcal{M} \models B\Sigma_2$ with a recursive two coloring of $[M]^2$ having no regular *$\emptyset''$-recursive homogeneous set.*

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Conjectue 2: There is a countable $\mathcal{M} \models \mathrm{RCA}_0 + B\Sigma_2$ with an M-extension for the same theory in which every Δ_2 set has a solution, and in which there is a recursive 2-coloring of $[M]^2$ with no homogeneous set.

Corollary (to Conjecture 2): RT^2_2 does not imply SRT^2_2.

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