Banff International Research Station
for Mathematical Innovation and Discovery

Number Theory and Physics at the Crossroads
September 21–26, 2008

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations. *Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.*

SCHEDULE

Sunday	
16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
	Lecture rooms available after 16:00 (if desired)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall (if desired)
	Beverages and small assortment of snacks available on a cash honour-system.
Monday	
7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome to BIRS by BIRS Station Manager, Max Bell 159
9:00–9:20	S. Gukov: *Overview of the workshop*
9:30–10:30	F. Rodriguez-Villegas: *Mixed Hodge polynomials of character varieties of Riemann surfaces*
10:30–11:00	Coffee Break, 2nd floor lounge, Corbett Hall
11:00–12:00	D. Gaiotto: *Wall-Crossing and Hyperkahler Geometry, Part I*
12:00–13:00	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:30–15:30	A.Neitzke: *Wall-Crossing and Hyperkahler Geometry, Part II*
15:30–16:00	Coffee Break, 2nd floor lounge, Corbett Hall
16:00–17:00	T. Gannon: *Vector-valued modular forms and Moonshine*
17:30–19:00	Dinner
19:00–20:00	D. Zagier and A. Dabholkar: *Mock modular forms and BPS states*
20:00–	Welcoming Party in 2nd floor lounge, Corbett Hall

Tuesday

7:00–9:00	Breakfast
9:30–10:30	V. Bouchard: *Topological open strings on orbifolds*
10:30–11:00	Coffee Break, 2nd floor lounge, Corbett Hall
11:00–12:00	E. Scheideggar: *Noether–Lefshetz Theory and the Yau–Zaslow Conjecture*
12:00	Group Photo: At the Front Steps of Corbett Hall
12:00–13:30	Lunch
14:30–15:30	C. Keller: *Conformal field theory and modular differential operators for weak Jacobi forms*
15:30–16:00	Coffee Break, 2nd floor lounge, Corbett Hall
16:00–17:00	T. Yamazaki: *Degenerate fibers of the Mumford system and rational solutions to the KdV hierarchy*
17:30–19:00	Dinner
19:00–20:00	D. Kazhdan: *Satake isomorphism for Kac–Moody groups*
20:00–21:00	M. Kerr: *The Abel-Jacobi map on the Eisenstein symbol*

Wednesday

7:00–9:00	Breakfast
9:30–10:30	D. Ramakrishnan: *Modular forms and Calabi-Yau varieties*
10:30–11:0	Coffee Break, 2nd floor lounge, Corbett Hall
11:00–12:00	S. Hosono: *BCOV ring and anomaly equations*
12:000–13:30	Lunch
	Free Afternoon
17:30–19:00	Dinner
19:00–20:00	P. Candelas: *Special geometry for CY manifolds over* $\mathbf{C}$ *and* $\mathbf{F}_p$
20:00–21:00	J. Manshot: *Partition functions for supersymmetric black holes*

Thursday

7:00–9:00	Breakfast
9:30–10:30	A. Clingher: *Lattice Polarized K3 Surfaces and Siegel Modular Forms*
10:30–11:00	Coffee Break, 2nd floor lounge, Corbett Hall
11:00–12:00	A. Klemm: *Integrability of the holomorphic anomaly equation*
12:00–13:30	Lunch
14:30–15:30	R. Schimmrigk: *Motivic L-functions in string theory and D-branes*
15:30–16:00	Coffee Break, 2nd floor lounge, Corbett Hall
16:00–17:00	W. Zudilin: *Algebraic transformations of Calabi–Yau differential equations*
17:30–19:00	Dinner
19:00–20:00	J. Lewis: *Normal Forms and Picard-Fuchs Equations for Families of K3 Surfaces over Modular Varieties*
20:00–21:00	P. Gunnells: *Weyl group multiple Dirichlet series*

Friday

7:00–9:30	Breakfast
9:30–10:30	Lecture
10:30–11:00	Coffee Break, 2nd floor lounge, Corbett Hall, Check out during this break
11:00–12:00	Lecture
12:00–13:30	Lunch

Checkout by 12 noon.

** 5-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

Number Theory and Physics at the Crossroads
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ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **V. Bouchard** (Harvard, Physics)
Title: *Topological open strings on orbifolds*
Abstract: Using the new recursive approach to the B-model inspired by matrix models, we study modular properties of topological open string amplitudes on mirrors of toric Calabi-Yau threefolds. As an application, we "modular transform" the large radius amplitudes to the orbifold point. Through mirror symmetry, the resulting amplitudes compute a new type of invariants: open string Gromov-Witten invariants of orbifolds.

Speaker: **A. Clingher** (Missouri–St. Louis)
Title: *Lattice Polarized K3 Surfaces and Siegel Modular Form*
Abstract: This talk will discuss a special family of complex algebraic K3 surfaces polarized by the rank-seventeen lattice $H + E_8 + E_7$. In terms of Hodge theory, these surfaces are naturally related to principally polarized abelian surfaces. I will outline the geometry of the correspondence as well as present an explicit classification of these special K3 surfaces in terms of Siegel modular forms. This is joint work with Charles Doran.

Speaker: **D. Gaiotto and A. Neitzke** (IAS, Physics)
Title: *Wall-Crossing and Hyperkahler Geometry*
Abstract: We will describe recent work on the physical and geometric interpretation of the Kontsevich-Soibelman wall-crossing formula (WCF). We argue that the WCF (in the "non-gravitational" case) expresses the continuity of a certain hyperkahler metric, which arises physically as the moduli space of gauge theory on $R^3 x S^1$, and can be constructed by solving a certain infinite-dimensional Riemann-Hilbert problem. In the first talk we describe our physical setup, the hyperkahler metric and its relation to the WCF. In the second talk we describe a close connection between this construction and the "$tt*$ geometry" of Cecotti and Vafa, and explain some specific examples which arise from D-brane constructions; in these examples the relevant hyperkahler spaces are moduli spaces of ramified Higgs bundles.

This talk has two parts. Part I will be given by D. Gaiotto and Part II by A. Neitzke.

Speaker: **T. Gannon** (Alberta)
Title: *Vector-valued modular forms and Moonshine*
Abstract: The bulk of my talk will review the theory of vector-valued modular forms for subgroups of the modular group, being developed by Peter Bantay and myself. I'll explain applications to conformal field theory and to moonshine. In particular, I'll explain how the 4-point conformal blocks on a sphere, and the 1- and 2-point conformal blocks on a torus, for any rational conformal field theory, fit into this framework, and how this therefore suggests a far-reaching extension of Monstrous Moonshine.

Speaker: **P. Gunnells** (Massachuesetts)
Title: *Weyl group multiple Dirichlet series*

Abstract: Weyl group multiple Dirichlet series are Dirichlet series in several complex variables whose coefficients are constructed from n-th order Gauss sums, with groups of functional equations isomorphic to Weyl groups. Such series in more than one variable first appeared in the work of Siegel, who constructed a series attached to the A2 root system by taking the Mellin transform of a half-integral weight Eisenstein series. Ultimately all such series are expected to be Whitaker coefficients of Eisenstein series on metaplectic groups, although this has only been proved in certain cases.

Unlike the usual Dirichlet series, multiple Dirichlet series do not in general have an Euler product. Instead, they satisfy a "twisted multiplicativity": the coefficients of an n-th order series are multiplicative up to certain products of n-th order power residue symbols. Nevertheless, description of these series boils down to specification of their p-parts.

In this talk we describe a construction of Weyl group multiple Dirichlet series that is uniform for all root systems and for all n. We construct the p-parts using a deformation of the Weyl character formula. The resulting p-parts are fascinating combinatorial objects that resemble characters of representations of simple complex Lie algebras, but with each weight multiplied by a product of n-th order Gauss sums.

This is joint work with Gautam Chinta.

Speaker: **S. Hosono** (Tokyo)
Title: *BCOV ring and anomaly equations*
Abstract: I will introduce a certain differential ring, which I call BCOV ring, defined over the moduli space of Calabi-Yau hypersurfaces. Then, I will write the holomorphic anomaly equation due to Bershadsky-Cecocci-Ooguri-Vafa (BCOV) as an differential equation in this BCOV ring in general.

As an application, I will focus on the modular anomaly equation for $\frac{1}{2}$K3, which is written in the ring of quasi-modular forms. I will show that the BCOV holomorphic anomaly equation in this case is equivalent to the modular anomaly equation.

Speaker: **C. Keller** (Harvard, Physics)
Title: *Conformal field theory and modular differential operators for weak Jacobi forms*
Abstract: For bosonic conformal field theories, there are recursion relations between n-point functions first introduced by Zhu. Applying these relations, one can obtain in a natural way modular covariant differential operators acting on modular forms.

These recursion relations can be generalized to the $N = 2$ supersymmetric case. In this case the object of interest is the elliptic genus, which is a weak Jacobi form of weight 0. The recursion relations motivate the introduction of generalized versions of the Eisenstein series. These generalized Eisenstein series can then be used to construct modular covariant differential operators on the space of weak Jacobi forms.

Speaker: **M. Kerr** (Durham)
Title: *The Abel-Jacobi map on the Eisenstein symbol*
Abstract: In this talk we consider two different constructions of motivic cohomology classes on families of toric hypersurfaces and on Kuga varieties. Under suitable modularity conditions on the former we say how the constructions "coincide", obtaining a complete explanation of a phenomenon observed by Villegas, Stienstra, and Bertin in the context of Mahler measure. (This is where the AJ computation on the Kuga varieties, done using our formula with J. Lewis and S. Mueller-Stach, will be summarized.) We will use this to elducidate the consequences of a conjecture of Hosono and the role played by algebraic K-theory in local mirror symmetry. The material I will cover in my talk is mostly joint work with Charles Doran.

Speaker: **A. Klemm** (Bonn, Physics)
Title: *Integrability of the holomorphic anomaly equation*
Abstract: We show that modularity and the gap condition makes the holomorphic anomaly equation completely integrable for non-compact Calabi-Yau manifolds. This leads to a very efficient formalism to solve the topological string on these geometries in terms of almost holomorphic modular forms. The formalism provides in particular holomorphic expansions everywhere in moduli space including large radius

points, the conifold loci, Seiberg-Witten points and the orbifold points. It can be also viewed as a very efficient method to solve higher genus closed string amplitudes in the $\frac{1}{N^2}$ expansion of matrix models with more then one cut.

Speaker: **J. Lewis** (Washington)
Title: *Normal Forms and Picard-Fuchs Equations for Families of K3 Surfaces over Modular Varieties*
Abstract: We will start with an overview of the Griffiths-Dwork algorithm for using residues to compute Picard-Fuchs equations for families of Calabi-Yau varieties. This algorithm applies to generically smooth families of hypersurfaces in projective space, and has been generalized to families of ample quasi-smooth varieties in simplicial projective toric varieties. We will discuss both difficulties and successes in applying the algorithm to families of singular varieties. The specific families of lattice-polarized K3 surfaces studied are those supported on modular curves, Humbert surfaces, and Shimura curves under the geometric correspondence described in the talk by A. Clingher. This is joint work with A. Clingher, C. Doran, and U. Whitcher.

Speaker: **D. Ramakrishnan** (CalTech)
Title: *Modular forms and Calabi-Yau varieties*
Abstract: This talk will furnish an explanation, with a few key (positive) examples, of the following question which evolved in the speaker's joint work with Kapil Paranjape: Given a regular cusp form f on $GL(n)$ with rational coefficients and (motivic) weight w, is there a Calabi-Yau variety X over the rationals of dimension w, and equipped with an involution not fixing $H^{w,0}(X)$, such that the (rank n) motive $M(f)$ of f occurs in $H^w(X)$? Moreover, can one choose X to be a "bare-bone envelope" of $M(f)$, i.e., whose cohomology contains only Artin-Tate motives besides that of F? The simplest cases to consider are the classical holomorphic newforms for $SL(2, Z)$ of weight $2k$ and rational coefficients, such as the Delta function. Time permitting, the talk will also briefly explore the compatibility of our question with Langlands's principle of functoriality, especially the product structure. The focus of this lecture will be in the converse direction to the usual, deep association of modular forms to Calabi-Yau (and more general)

Speaker: **E. Scheideggar** (Augsburg)
Title: *Noether–Lefshetz Theory and the Yau–Zaslow Conjecturei*
Abstract: The Yau–Zaslow conjecture determines the reduced genus Gromov–Witten invariants of K3 surfaces in terms of the Dedekind η-function. Classical intersections of curves in the moduli space of K3 surfaces with Noether–Lefschetz divisors are related to 3–fold Gromov–Witten theory via the K3 invariants. The classical intersections of these curves and divisors are determined in terms of vector-valued modular forms. The 3–fold invariants are calculated using mirror symmetry. Via a detailed study of the STU model (determining special curves in the moduli space of K3 surfaces), we prove the Yau-Zaslow conjecture for all curve classes on K3 surfaces.

Speaker: **R. Schimmrigk** (Southbend)
Title: *Motivic L-functions in string theory and D-branes*
Abstract: Motivic L-functions have been useful to understand the geometry of string compactifications in terms of the theory on the worldsheet. In this talk this application of L-functions is extended in two ways. The first extension relates L-functions to D-branes, providing a second physical interpretation of this object. The second extension uses L-function to establish relations between bosonic flat string theory and supersymmetric compactified strings.

Speaker: **R. Rodriguez-Villegas** (Texas–Austin)
Title: *Mixed Hodge polynomials of character varieties of Riemann surfaces*
Abstract: Ever since Weil we know that counting points of varieties over finite fields yields topological information about them. In this talk I will describe such a calculation for the varieties of the title (parameterizing representations of the fundamental group of a Riemann surface into GL_n).

I will first discuss the main ingredients of the calculation, which involves an array of techniques from combinatorics and representation theory of finite groups of Lie type. In the process we discover an unexpected relation to certain quiver varieties. I will describe some conjectures that the outcome of the calculation naturally gave rise to. These predict the full mixed Hodge polynomials of the varieties and give a geometric backbone to the connection between the character and quiver varieties.

Besides their intrinsic interest the varieties in question are closely related to the moduli spaces of Higgs bundles on the surface. Somewhat surprisingly we discover a tight connection between the geometry of these character varieties and the Macdonald polynomials of combinatorics.

This is joint work with T. Hausel and E. Letellier

Speaker: **T. Yamazaki** (Tohoku)
Title: *Degenerate fibers of the Mumford system and rational solutions to the KdV hierarchy*
Abstract: This is a joint work with P. Vanhaecke and R. Inoue. We study the structure of a degenerate fiber of the Mumford system in term of the (compactified) Jacobian variety. As an application, we obtain a new algorithm to construct all rational solutions to the KdV hierarchy.

Speaker: **W. Zudilin** (MPIM Bonn)
Title: *Algebraic transformations of Calabi–Yau differential equations*
Abstract: My talk will be based on joint work with Heng Huat Chan, Gert Almkvist and Duco van Straten.

In our study of Picard–Fuchs differential operators of Calabi–Yau type we discover some curious relations of hypergeometric series

$$ {}_mF_{m-1}\left(\begin{matrix} a_1, & a_2, & \dots, & a_m \\ & b_2, & \dots, & b_m \end{matrix} \,\middle|\, z\right) = \sum_{n=0}^{\infty} \frac{(a_1)_n(a_2)_n\cdots(a_m)_n}{(b_2)_n\cdots(b_m)_n}\,\frac{z^n}{n!} $$

satisfying linear differential equations

$$ \left(\theta\prod_{j=2}^{m}(\theta+b_j-1) - z\prod_{j=1}^{m}(\theta+a_j)\right)y = 0, \qquad \text{where} \quad \theta = z\frac{\mathrm{d}}{\mathrm{d}z}, $$

of order $m = 2$, 3, 4 and 5. In the 'classical' situation (when $m = 2$ or 3), most of the corresponding identities come from modular parametrizations of the series; on this way we can give, for example, algebraic expressions for the generating series of the Apéry numbers, Domb's numbers and many others, in hypergeometric forms. Different methods (analytic transformations of the differential equations and study of their monodromy) allow us to prove all other identities we have discovered. A particular example of our findings for hypergeometric differential equations of order 4 and 5 may be interpreted as a higher analogue of Clausen's formula

$$ {}_2F_1\left(\begin{matrix} a,\, b \\ a+b+\frac{1}{2} \end{matrix} \,\middle|\, z\right)^2 = {}_3F_2\left(\begin{matrix} 2a,\, 2b,\, a+b \\ a+b+\frac{1}{2},\, 2a+2b \end{matrix} \,\middle|\, z\right). $$