Report on "Number Theory and Physics at the Crossroads" (08w5077)
September 21–26, 2008

The fourth of this series of workshops in the interface of number theory and physics met at BIRS in the week of September 21 for five days. The workshop was a huge success. Altogether thirty-nine mathematicians and physicists converged at the BIRS for the five day's scientific endeavor. There were 22 one hour talks, and lots of time was allotted for informal discussions.

Lectures by mathematicians were designed to familarize physicists on modular forms, quasimodular forms, modularity of Galois representations, zeta-functions and L-series, among others. Vice versa, lectures by physicists were intended toward educating mathematicians about some aspects of mirror symmetry, conformal field theory, quantum field theory, black holes in connection with number theory.

Topics of lectures ranged from various aspects of modular forms, differential equations, conformal field theory, black holes, wall-crossings, topological strings and Gromov–Witten invariants, holomorphic anomaly equations, mirror symmetry, among others. More detailed descriptions of scientific activities will be reported on in Section 4.

Though number theorists and string theorists have been working on modular forms, quasimodular forms, Jacobi forms and more generally automorphic forms in their respective fields, there have been very little interactions between the two sets of researchers, although with some exceptions. In other words, both camps have been living in parallel universes. This workshop brought together researchers in number theory, algebraic geometry, and physics (string theory) whose common interests are centered around modular forms. We witnessed very active and intensive interactions of both camps from early mornings to late nights. We all felt that all things modular have came together at BIRS from both sides: number theory and physics (in particular, string theory). At the end of the workshop, all participants felt that both camps have finally crossed boundaries and established relatively comfortable rapport.

There was a strong desire to have this kind of workshops more frequently at BIRS. However, the deadline to submit a follow-up workshop in 2010 was September 29, 2008! and the organizers felt that we needed more time to access the impact of the 2008 workshop and then plan for the next one. Consequently, we are planning to submit a follow-up proposal for the year 2011. The organizers will be Victor Batyrev, Chuck Doran, Sergei Gukov, Noriko Yui and Don Zagier.

Contents

1. Organizers
2. Press Release: Number Theory and String Theory at the Crossroads
3. Summary of scientific and other objectives
4. Summary of scientific activities
5. References
6. Participants
7. Titles and abstracts at the workshop

1. Organizers:

Charles F. Doran
Department of Mathematics
University of Alberta
Edmonton, AB, Canada T6G 2G1
doran@math.ualberta.ca
Sergei Gukov
Department of Mathematics
University of California
Santa Barbara, CA 93106 USA
gukov@math.ucsb.edu
Noriko Yui
Department of Mathematics and Statistics
Queen's University
Kingston, Ontario, Canada K7L 3N6
E-mail: yui@mast.queensu.ca
Don Zagier
Max-Plunck Institut fúr Mathematik
Vivatsgasse 7
53321 Bonn, Germany
Don.Zagier@mpim-bonn.mpg.de

Originally, Helena Verrill (Louisiana State University, USA) was listed as an organizer. However, she declined to serve as an organizer due to her maternity duties.

2. Press Release: Number Theory and Physics at the Crossroads

In the recent years, the world has seen explosive interactions between Number Theory, Arithmetic and Algebraic Geometry, and Theoretical Physics (in particular, String Theory). To name a few, the classical modular forms, quasi-modular forms, and Jacobi forms appear in many areas of Physics, e.g., in mirror symmetry, topological quantum field theory, Gromov-Witten invariants, Calabi-Yau manifolds and in block holes. Also modularity questions of Calabi-Yau varieties and other higher dimensional varieties in connection with Langlands Program are getting considerable attention and feedbacks from physics. Zeta-functions and L-series enter scenes at various places in physics. Via renormalization, Feynman integrals are related to multiple zeta-values, and purportedly to motives. Calculations of the energy and charge degeneracies of black holes lead surprisingly to Jacobi forms and Siegel modular forms.

There have been strong desires among mathematicians and physicists for more workshops directed to the areas of number theory and physics at the crossroads. This workshop responded to that demand and brought together many leading researchers working in the interface of number theory, arithmetic/algebraic geometry and theoretical physics to BIRS.

The newly launched international research journal, "Communications in Number Theory and Physics" (http://www.intlpress.com/CNTP) published by International Press, will provide a venue for dissemination of results at this crossroads well into the future. Participants are encouraged to submit their written up talks and further new results to the journal.

3. Summary of scientific and other objectives

Physical duality symmetries relate special limits of the various consistent string theories (Types I, II, Heterotic string and their cousins, including F-theory) one to another. By comparing the mathematical descriptions of these theories, one reveals often quite deep and unexpected mathematical conjectures. The best known string duality to mathematicians, Type IIA/IIB duality also called *mirror symmetry*, has inspired many new developments in algebraic and arithmetic geometry, number theory, toric geometry, Riemann surface theory, and infinite dimensional Lie algebras. Other string dualities such as Heterotic/Type II duality and F-Theory/Heterotic string duality have also, more recently, led to series of mathematical conjectures, many involving elliptic curves, K3 surfaces, and modular forms.

In recent years, we have witnessed that modular forms, quasi-modular forms and automorphic forms play central roles in many areas of physics, e.g., quantum field theory, conformal field theory, mirror symmetry, and 4D gauge theory. Most prominently, generating functions counting the number of curves on Calabi–Yau manifolds (e.g., Gromov–Witten invariants), elliptic genera/partition functions of conformal field theory, and generating functions in 4D gauge theory are all characterized by some kinds of modular forms (classical modular forms, quasi-modular forms, Jacobi forms, Siegel modular forms. etc.)

This has led to a realization that we ought to assess with vigor the role of number theory, in particular that of modular forms, in physics in general. Indeed, there have been at least three efforts along this line in Canada, in terms of a series of workshops devoted to this goal. One of the first in this series was the Fields workshop on "Calabi–Yau Varieties and Mirror Symmetry", 2001; its follow-up five-day workshop was held at BIRS in 2003. The most recent one was the five-day workshop on "Modular Forms and String Duality" at BIRS in 2006. The Proceedings of these three past workshops have been published (see references). This brings the workshop on "Number Theory and Physics at the Crossroads" as the fourth in this series.

It brought together mathematicians and physicists working on problems inspired by string theory. Many researchers in string theory and number theory, working on the same or related problems from different angles came together at BIRS. This synthesis proved powerful and beneficial to both parties involved; simply put, the workshop was a huge success. There was an overwhelming consensus from researchers working at the crossroads of number theory and physics to organize this kind of workshop more frequently. In particular, many researchers are extremely eager to have another five-day workshop at BIRS in two or three years time.

A new research journal "Communications in Number Theory and Physics" (published by International Press of Boston) has been launched, specifically devoted to subject areas at the crossroads of number theory and physics. The editors-in-chief of this new journal are Robert Dijkgraaf, David Kazdhan, Maxim Kontsevich, and Shing-Tung Yau. The journal has entered its second year, and has proved to be a spectacular success! Some members of the editorial board took part in the workshop.

One of the principal goals of this workshop is to look at various modular forms, zeta-functions, L-series, Galois representations, arising from Calabi–Yau manifolds, conformal field theory, quantum field theory, and 4D gauge theory. The subject area of interest might be classified into not clearly disjoint sets of the following subjects:

(a) Modular, quasimodular, Siegel, and Jacobi modular forms, and their applications.

(b) Topological string theory, mirror symmetry and modular forms.
(c) Modularity of Galois representations, and arithmetic questions.
(d) Conformal field theory and modular forms.
(e) Holomorphic anomaly equations.
(f) Differential equations.
(g) Wall-crossing formula.
(h) Other topics in the interface of number theory and physics.

4. Summary of scientific activities

The workshop's kick-off talk was delivered by Sergei Gukov giving scientific directions to the entire workshop.

S. Gukov illustrated by a number of examples analogies and cross currents between number theory and physics. For instance, counting certain invariants in number theory (.e.g., zeta-functions, L-series concocted by counting number of rational points), and some counting invariants in physics (e.g. Z-functions, partition functions in quantum field theory formed by counting Gromov-Witten invariants, or BPS states, elliptic genera in conformal field theory) share common modular properties.

(a) Modular, quasimodular, Siegel, and Jacobi modular forms, and their applications

D. Zagier gave an introduction of mock modular forms, a new type of modular object whose theory was developed by S. Zwegers and which has potential applications to the theory of black holes.

A. Dabholkar explained a recent application of Siegel modular forms in counting the microstates of black holes. For instance, the Igusa cusp Siegel form of genus 2 appreared in a description of the partition function of dyonic black holes in $N = 4$ compactifications.

A. Clingher discussed a family of lattice polarized K3 surfaces polarized by the rank 17 lattice $H_2 \oplus E_8 \oplus E_7$. He explained how to classify these K3 surfaces in terms of Siegel modular forms.

(b) Topological string theory, mirror symmetry and modular forms

V. Bouchard discussed modular properties of generating functions of open orbifold Gromov–Witten invariants. The solutions to loop equations in matrix model theory yield B-model amplitudes, and they are quasimodular forms of weight 0 for $SL_2(\mathbf{Z})$ in case of genus 1.

E. Scheideggar reported on his proof of the Yau–Zaslow conjecture for all curves on K3 surfaces.

(c) Modularity, and arithmetic questions

D. Ramakrishnan discussed the audacity of hope focusing on a specific question, *the geometric realization problem*: Let π be a $\mathbf{Q}$-rational. even self-dual regular cusp form o $GL_n(\mathbf{Q})$ of weight $m > 0$. Then does there exist a Calabi–Yau variety $X/\mathbf{Q}$ of dimension m and a motive M such that $\pi \leftrightarrow M \subset H^m(X)$ with $M^{m,0} \neq 0$?

R. Schimmrigk gave a physicist's understanding of modularity of Calabi–Yau threefolds over $\mathbf{Q}$ and D-branes.

(d) Conformal field theory and modualr forms

T. Gannon's talk was concerned with rational conformal field theory (RCFT) or rational vertex operator algebras (RVOA). The main point was that characters associated to RVOA are vector-valued modular forms for $SL_2(\mathbf{Z})$ or $\Gamma(2)$. Moonshine modules are explained as such examples.

C. Keller discussed a generalization of recursion relations between n-point functions to those of the $N = 2$ supersymmetry case. The recursion relations yielded generalized Eisenstein series, which were then used to construct modular covariant differential operators on the space of weak Jacobi forms. His talk was video-taped.

J. Manschot reported on the relation between supersymmetric black holes of $N = 2$ supergravity and modular forms. In particular, it was shown that microscopic counting function of some class of black holes is expressed in terms of Jacobi forms.

(e) Holomorphic anomaly equations

S. Hosono reported on his recent work on the BCOV rings over the moduli space of Calabi–Yau threefolds, which are regarded as generalizations of the ring of almost holomorphc modular forms and quasimodular forms for elliptic curves.

A. Klemm reported that modularity and the gap conditions make the holomorphic anomaly equation completely integrable for non-compact Calabi–Yau manifolds. As an application of this result, he laid out an algorithm for computing higher genus closed string amplitudes in terms of almost holomorphic modular forms.

(f) Differential equations

Candelas discussed special geometry of Calabi–Yau threefolds over $\mathbf{C}$ and $\mathbf{F}_p$, mostly focusing on one-parameter family of quintic mirror Calabi–Yau threefolds.

Lewis discussed an algorithm based on the Griffiths–Dwork for computing Picard–Fuchs differential equations for families of singular varieties, e.g., families of lattice polarized K3 surfaces.

Yamazaki reported on a new algorithm for constructing all rational solutions to the KdV hierarchy, which was based on the study of degenerate fibers of the Mumford systems in terms of Jacobian variety.

Zudilin used analytic method to study Picard–Fuchs differential equations of order m for periods of (1-parameter) families of Calabi–Yau manifolds ($m = 2, 3, 4, 5$). All differential equations were of hypergeometric type and found some new algebraic transformations among hypergeometric series satisfying certain linear differential equations of order $m \leq 5$.

(g) Wall-crossing formula

A. Neiztke and D. Gaiotto explained the wall-crossing formula due to Kontsevich and Soibelmann from geometric and physical points of view.

(h) Miscellaneous topics

P. Gunnells gave a lively talk on Weyl group Dirichlet series in several complex variables. He presented an idea of construction of Weyl group Dirichlet series.

D. Kazhdan discussed Satake isomorphism for Kac–Moody groups.

M. Kerr considered two different constructions of motivic cohomology classes on families of toric hypersurfaces and on Kuga varieties. As an application, he discussed how algebraic K-theory can be used in local mirror symmetry.

F. Rodriguez–Villegas described a tight connection between the geometry of character varieties of Riemann surfaces and the Macdonald polynomials arising from combinatorics.

5. References

Books and Lecture Notes

[B1] **The 1,2,3 of Modular Forms**, by J.H. Bruinier, G. van der Geer, G. Harder and D. Zagier, Lectures at a Summer School in Nordfjordeid, Norway, Universitext, Springer-Verlag, 2008.

[B2] **Calabi–Yau Varieties and Mirror Symmetry**, edited by N. Yui and J. Lewis, Fields Institute Communications Vol. **38**, Proceedings of the Fields Institute Workshop on Calabi–Yau Varieties and Mirror Symmetry 2001, AMS/FIC, 2003.

[B3] **Mirror Symmetry V**, edited by N. Yui, S.-T. Yau and J. Lewis, AMS/IP Stud. in Advanced Math. bf 38, Proceedings of the BIRS Workshop on Calabi–Yau Varieties and Mirror Symmetry 2003, AMS/IP, 2006.

[B4] **Modular Forms and String Duality**, edited by N. Yui, H. Verrill and C. Doran, Fields Institute Communications Vol. **54** Proceedings of the BIRS Workshop on Modular Forms and String Duality 2006, AMS/FIC, 2008.

[CNTP] **Communications in Number Theory and Physics**, International Press of Boston, launched in 2007. http://www.intlpress.com/CNTP

The recent book [B1] about elliptic modular forms, Hilbert modular forms and Siegel modular forms would become standard introduction to modular forms of one, two and more variables.

The most recent references on topics in the interface of number theory and physics are [B2,B3, B4]. All three books are proceedings of the workshops on number theory and physics held at the Fields Institute (2001), and subsequently att BIRS (2003) and (2006), edited by Noriko Yui et al. These references have served as the cornerstone for the rapidly developing topics discussed at the workshop.

The newly launched international research journal [CNTP] has been devoted to topics in the interface of number theory and physics.

Articles

The recent articles relevant to the talks presented at this workshop may be found on arXiv.

6. Participants

We had in total 39 participants for the workshop, out of which 14 were either graduate students or postdoctoral fellows. We had two last minutes cancellations (Xenia de la Ossa (Oxford, UK), and Kentaro Hori (Toronto, Canada)).

1. Batyrev, Victor (University of Tuebingen, Germany)
2. Bouchard, Vincent (Harvard, USA) (Postdoctoral fellow)

3. Candelas, Philip (University of Oxford, UK)
4. Clingher, Adrian (University of Missouri–St. Louis, USA)
5. Dabholkar, Atish (CNRS, Paris)
6. Dimofte, Tudor (Caltech, USA) (Graduate student)
7. Doran, Charles (University of Alberta, Canada)
8. Eager, Richard (University of California at Santa Barbara) (Graduate student)
9. Gaiotto, Davide (IAS Princeton, USA) (Postdoctoral fellow)
10. Gannon, Terry (University of Alberta, Canada)
11. Sergei Gukov (University of California at Santa Barbara, USA)
12. Gunnells, Paul (University of Massachusetts, USA)
13. Sinobu Hosono (University of Tokyo, Japan)
14. Kazhdan, David (Hebrew University, Israel)
15. Keller, Christoph (ETH Zürich, Swiztherland) (Postdoctoral fellow)
16. Kerr, Matt (University of Durham, UK)
17. Albrecht Klemm (University of Bonn, Germany)
18. Konishi, Yukiko (Kyoto University, Japan)
19. Lewis, Jacob (University of Washington, USA) (Graduate student)
20. Ron Livné (Hebrew University, Israel)
21. Manschot, Jan (University of Amsterdam, Holland) (Postdoctoral fellow)
22. Marion, Samantha (University of Alberta, Canada) (Graduate student)
23. McKay, John (Concordia University, Canada)
24. Miller, Robert (University of Washington, USA) (Graduate student)
25. Minabe, Satoshi (IHES, France) (Postdoctoral fellow)
26. Neitzke, Andy (IAS Princeton, USA) (Postdoctoral fellow)
27. Novoseltsev, Andrey (University of Alberta, Canada) (Graduate student)
28. Ramakrishnan, Dinakar (Caltech, USA)
29. Rodrigues Villegas, Fernando (University of Texas at Austin, USA)
30. Samol, Kira (University of Mainz, Germany) (Postdoctoral fellow)
31. Scheideggar, Emanuel (University of Augsburg, Germany)
32. Schimmrigk, Rolf (Indiana University South Bend, USA)
33. Walcher, Johannes (CERN, Swiztherland)
34. Yamazaki, Takao (Tohoku University, Japan)
35. Yang, Yifan (National Chiao Tung University, Taiwan)
36. Yeats, Karen (Boston University) (Postdoctoral fellow)
37. Yui, Noriko (Queen's University, Canada)
38. Zagier, Don (Max-Planck Institute for Mathematics Bonn, Germany/ College de France, France)
39. Zudilin, Wadim (Steklov Mathematical Institute, Russia/Max-Planck Institute for Mathematics Bonn, Germany)

7. Titles and Abstracts of Talks at the Workshop

SEPTEMBER 22, 2008

9:00–9:20 **S. Gukov**: *Overview of the workshop*

9:30–10:30 **F. Rodriguez-Villegas**: *Mixed Hodge polynomials of character varieties of Riemann surfaces*

Ever since Weil we know that counting points of varieties over finite fields yields topological information about them. In this talk I will describe such a calculation for the varieties of the title (parameterizing representations of the fundamental group of a Riemann surface into GL_n).

I will first discuss the main ingredients of the calculation, which involves an array of techniques from combinatorics and representation theory of finite groups of Lie type. In the process we discover an unexpected relation to certain quiver varieties. I will describe some conjectures that the outcome of the calculation naturally gave rise to. These predict the full mixed Hodge polynomials of the varieties and give a geometric backbone to the connection between the character and quiver varieties.

Besides their intrinsic interest the varieties in question are closely related to the moduli spaces of Higgs bundles on the surface. Somewhat surprisingly we discover a tight connection between the geometry of these character varieties and the Macdonald polynomials of combinatorics.

This is joint work with T. Hausel and E. Letellier

11:00–12:00 **A. Neitzke** : *Wall-Crossing and Hyperkahler Geometry, Part I*

We will describe recent work on the physical and geometric interpretation of the Kontsevich-Soibelman wall-crossing formula (WCF). We argue that the WCF (in the "non-gravitational" case) expresses the continuity of a certain hyperkahler metric, which arises physically as the moduli space of gauge theory on R^3xS^1, and can be constructed by solving a certain infinite-dimensional Riemann-Hilbert problem. In the first talk we describe our physical setup, the hyperkahler metric and its relation to the WCF. In the second talk we describe a close connection between this construction and the "$tt*$ geometry" of Cecotti and Vafa, and explain some specific examples which arise from D-brane constructions; in these examples the relevant hyperkahler spaces are moduli spaces of ramified Higgs bundles.

This talk has two parts. Part I will be given by A. Neitzke, and Part II by G. Daiotto.

14:30–15:30pm **D. Gaiotto**: *Wall-Crossing and Hyperkahler Geometry, Part II*

16:00–17:00 **T. Gannon**: *Vector-valued modular forms and Moonshine*

The bulk of my talk will review the theory of vector-valued modular forms for subgroups of the modular group, being developed by Peter Bantay and myself. I'll explain applications to conformal field theory and to moonshine. In particular, I'll explain how the 4-point conformal blocks on a sphere, and the 1- and 2-point conformal blocks on a torus, for any rational conformal field theory, fit into this framework, and how this therefore suggests a far-reaching extension of Monstrous Moonshine.

19:00–20:30 **D. Zagier** : *Part I: Mock modular forms*

19:00–20:30 **A. Dabholkar** : *Part II: Modular forms and black holes*

A recent application of Siegel modular forms for counting the microstates of black holes is discussed. The partition function of dyonic black holes in N=4 string compactifications is naturally given in terms of inverse of certain Siegel modular forms of Sp(2, Z) and its congruence subgroup. Fourier coefficients of these partition functions give the black hole degeneracies. In particular it is shown how the contour dependence in extracting the Fourier coefficients and its relation to the moduli dependence of the black hole degeneracies. Possible connections with mock modular forms are outlined.

SEPTEMBER 23, 2008

9:30–10:30 **V. Bouchard**: *Topological open strings on orbifolds*

Using the new recursive approach to the B-model inspired by matrix models, we study modular properties of topological open string amplitudes on mirrors of toric Calabi-Yau threefolds. As an application, we "modular transform" the large radius amplitudes to the orbifold point. Through mirror symmetry, the resulting amplitudes compute a new type of invariants: open string Gromov-Witten invariants of orbifolds.

11:00–12:00 **E. Scheideggar**: *Noether–Lefshetz Theory and the Yau–Zaslow Conjecture*

The Yau–Zaslow conjecture determines the reduced genus Gromov–Witten invariants of K3 surfaces in terms of the Dedekind η-function. Classical intersections of curves in the moduli space of K3 surfaces with Noether–Lefschetz divisors are related to 3–fold Gromov–Witten theory via the K3 invariants. The classical intersections of these curves and divisors are determined in terms of vector-valued modular forms. The 3–fold invariants are calculated using mirror symmetry. Via a detailed study of the STU model (determining special curves in the moduli space of K3 surfaces), we prove the Yau-Zaslow conjecture for all curve classes on K3 surfaces.

14:30–15:30 **C. Keller**: *Conformal field theory and modular differential operators for weak Jacobi forms*

For bosonic conformal field theories, there are recursion relations between n-point functions first introduced by Zhu. Applying these relations, one can obtain in a natural way modular covariant differential operators acting on modular forms.

These recursion relations can be generalized to the $N = 2$ supersymmetric case. In this case the object of interest is the elliptic genus, which is a weak Jacobi form of weight 0. The recursion relations motivate the introduction of generalized versions of the Eisenstein series. These generalized Eisenstein series can then be used to construct modular covariant differential operators on the space of weak Jacobi forms.

16:00–17:00 **T. Yamazaki**: *Degenerate fibers of the Mumford system and rational solutions to the KdV hierarchy*

This is a joint work with P. Vanhaecke and R. Inoue. We study the structure of a degenerate fiber of the Mumford system in term of the (compactified) Jacobian variety. As an application, we obtain a new algorithm to construct all rational solutions to the KdV hierarchy.

19:00–20:00 **D. Kazhdan**: *Satake isomorphism for Kac–Moody groups*

20:00–21:00 **M. Kerr**: *The Abel-Jacobi map on the Eisenstein symbol*

In this talk we consider two different constructions of motivic cohomology classes on families of toric hypersurfaces and on Kuga varieties. Under suitable modularity conditions on the former we say how the constructions "coincide", obtaining a complete explanation of a phenomenon observed by Villegas, Stienstra, and Bertin in the context of Mahler measure.

(This is where the AJ computation on the Kuga varieties, done using our formula with J. Lewis and S. Mueller-Stach, will be summarized.) We will use this to elducidate the consequences of a conjecture of Hosono and the role played by algebraic K-theory in local mirror symmetry. The material I will cover in my talk is mostly joint work with Charles Doran.

SEPTEMBER 24, 2008

9:30–10:30 **D. Ramakrishnan**: *Modular forms and Calabi-Yau varieties*

This talk will furnish an explanation, with a few key (positive) examples, of the following question which evolved in the speaker's joint work with Kapil Paranjape: Given a regular cusp form f on $GL(n)$ with rational coefficients and (motivic) weight w, is there a Calabi-Yau variety X over the rationals of dimension w, and equipped with an involution not fixing $H^{w,0}(X)$, such that the (rank n) motive $M(f)$ of f occurs in $H^w(X)$? Moreover, can one choose X to be a "bare-bone envelope" of $M(f)$, i.e., whose cohomology contains only Artin-Tate motives besides that of F? The simplest cases to consider are the classical holomorphic newforms for $SL(2, Z)$ of weight $2k$ and rational coefficients, such as the Delta function. Time permitting, the talk will also briefly explore the compatibility of our question with Langlands's principle of functoriality, especially the product structure. The focus of this lecture will be in the converse direction to the usual, deep association of modular forms to Calabi-Yau (and more general)

11:00–12:00 **S. Hosono**: *BCOV ring and anomaly equations*

I will introduce a certain differential ring, which I call BCOV ring, defined over the moduli space of Calabi-Yau hypersurfaces. Then, I will write the holomorphic anomaly equation due to Bershadsky-Cecocci-Ooguri-Vafa (BCOV) as an differential equation in this BCOV ring in general.

As an application, I will focus on the modular anomaly equation for $\frac{1}{2}$K3, which is written in the ring of quasi-modular forms. I will show that the BCOV holomorphic anomaly equation in this case is equivalent to the modular anomaly equation.

19:00–20:00 **P. Candelas**: *Special geometry for CY manifolds over* $\mathbf{C}$ *and* $\mathbf{F}_p$

20:00–21:00 **J. Manschot**: *Partitition functions for supersymmetric black holes*

In this talk, I will review the connection between supersymmetric black holes of $N = 2$ supergravity and modular forms. It is shown that the microscopic counting of a specific class of black holes is captured by a (generalized) Jacobi form, or equivalently a vector-valued modular form. This reproduces the entropy as suggested by supergravity. In the second part of the talk, I will discuss negative weight Poincare series, and the calculation of the dimension of the space of the relevant vector-valued modular forms.

SEPTEMBER 25, 2008

9:30–10:30 **A. Clingher** : *Lattice Polarized K3 Surfaces and Siegel Modular Form*

This talk will discuss a special family of complex algebraic K3 surfaces polarized by the rank-seventeen lattice $H + E_8 + E_7$. In terms of Hodge theory, these surfaces are naturally related to principally polarized abelian surfaces. I will outline the geometry of the correspondence as well as present an explicit classification of these special K3 surfaces in terms of Siegel modular forms. This is joint work with Charles Doran.

11:00–12:00 **A. Klemm**: *Integrability of the holomorphic anomaly equation*

We show that modularity and the gap condition makes the holomorphic anomaly equation completely integrable for non-compact Calabi-Yau manifolds. This leads to a very efficient formalism to solve the topological string on these geometries in terms of almost holomorphic modular forms. The formalism provides in particular holomorphic expansions everywhere in moduli space including large radius points, the conifold loci, Seiberg-Witten points and the orbifold points. It can be also viewed as a very efficient method to solve higher genus closed string amplitudes in the $\frac{1}{N^2}$ expansion of matrix models with more then one cut.

14:30–15:30 **R. Schimmrigk**: *Motivic L-functions in string theory and D-branes*

Motivic L-functions have been useful to understand the geometry of string compactifications in terms of the theory on the worldsheet. In this talk this application of L-functions is extended in two ways. The first extension relates L-functions to D-branes, providing a second physical interpretation of this object. The second extension uses L-function to establish relations between bosonic flat string theory and supersymmetric compactified strings.

16:00–17:00 **W. Zudilin**: *Algebraic transformations of Calabi–Yau differential equations*

My talk will be based on joint work with Heng Huat Chan, Gert Almkvist and Duco van Straten.

In our study of Picard–Fuchs differential operators of Calabi–Yau type we discover some curious relations of hypergeometric series

$$ {}_mF_{m-1}\left(\begin{matrix} a_1, & a_2, & \dots, & a_m \\ & b_2, & \dots, & b_m \end{matrix} \,\middle|\, z\right) = \sum_{n=0}^{\infty} \frac{(a_1)_n(a_2)_n\cdots(a_m)_n}{(b_2)_n\cdots(b_m)_n}\,\frac{z^n}{n!} $$

satisfying linear differential equations

$$ \left(\theta\prod_{j=2}^{m}(\theta+b_j-1) - z\prod_{j=1}^{m}(\theta+a_j)\right)y = 0, \qquad \text{where} \quad \theta = z\frac{\mathrm{d}}{\mathrm{d}z}, $$

of order $m = 2$, 3, 4 and 5. In the ‘classical’ situation (when $m = 2$ or 3), most of the corresponding identities come from modular parametrizations of the series; on this way we can give, for example, algebraic expressions for the generating series of the Apéry numbers, Domb’s numbers and many others, in hypergeometric forms. Different methods (analytic transformations of the differential equations and study of their monodromy) allow us to prove all other identities we have discovered. A particular example of our findings for hypergeometric differential equations of order 4 and 5 may be interpreted as a higher analogue of Clausen’s formula

$$ {}_2F_1\left(\begin{matrix} a,\, b \\ a+b+\frac12 \end{matrix} \,\middle|\, z\right)^2 = {}_3F_2\left(\begin{matrix} 2a,\, 2b,\, a+b \\ a+b+\frac12,\, 2a+2b \end{matrix} \,\middle|\, z\right). $$

19:00–20:00 **J. Lewis**:*Normal Forms and Picard-Fuchs Equations for Families of K3 Surfaces over Modular Varieties*

We will start with an overview of the Griffiths-Dwork algorithm for using residues to compute Picard-Fuchs equations for families of Calabi-Yau varieties. This algorithm applies to generically smooth families of hypersurfaces in projective space, and has been generalized to families of ample quasi-smooth varieties in simplicial projective toric varieties. We will discuss both difficulties and successes in applying the algorithm to families of singular varieties. The specific families of lattice-polarized K3 surfaces studied are those supported on modular curves, Humbert surfaces, and Shimura curves under the geometric correspondence described in the talk by A. Clingher. This is joint work with A. Clingher, C. Doran, and U. Whitcher.

20:00–21:00 **P. Gunnells**: *Weyl group multiple Dirichlet series*

Weyl group multiple Dirichlet series are Dirichlet series in several complex variables whose coefficients are constructed from n-th order Gauss sums, with groups of functional equations isomorphic to Weyl groups. Such series in more than one variable first appeared in the work of Siegel, who constructed a series attached to the A2 root system by taking the Mellin transform of a half-integral weight Eisenstein series. Ultimately all such series are expected to be Whitaker coefficients of Eisenstein series on metaplectic groups, although this has only been proved in certain cases.

Unlike the usual Dirichlet series, multiple Dirichlet series do not in general have an Euler product. Instead, they satisfy a "twisted multiplicativity": the coefficients of an n-th order series are multiplicative up to certain products of n-th order power residue symbols. Nevertheless, description of these series boils down to specification of their p-parts.

In this talk we describe a construction of Weyl group multiple Dirichlet series that is uniform for all root systems and for all n. We construct the p-parts using a deformation of the Weyl character formula. The resulting p-parts are fascinating combinatorial objects that resemble characters of representations of simple complex Lie algebras, but with each weight multiplied by a product of n-th order Gauss sums.

SEPTEMBER 26, 2008

9:00–12:00 *Informal discussions*