Random Schrödinger Operators: Universal Localization, Correlations, and Interactions
April 19 to 24, 2009

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
	Lecture rooms available after 16:00 (if desired)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall (if desired)
	Beverages and small assortment of snacks available on a cash honour-system.

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome to BIRS by BIRS Station Manager, Max Bell 159
9:00–9:50	F. Germinet: *Anderson localization for random Schodinger operators*
10:00–10:25	H. Schulz-Baldes: *Random Dirac operators with time reversal symmmetry*
10:30–11:00	Coffee Break, 2nd floor lounge, Corbett Hall
11:00–11:50	M. Disertori: *Supersymmetric non linear sigma models in Anderson localization*
12:00–13:00	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
15:00–15:30	Coffee, 2nd floor lounge, Corbett Hall
15:30–16:20	M. Goldstein: *Schrödinger equations with potentials generated by shifts and skew-shifts*
16:30–16:55	E. Hamza: *The unitary Anderson model*
17:00–17:25	H. Boumaza: *Localization for a matrix-valued Anderson-Bernoulli model*
17:30–17:55	R. Shterenberg: *Asymptotic expansion of the integrated density of states of a two-dimensional periodic Schrödinger operator*
18:00–19:30	Dinner

Tuesday

7:00–9:00	Breakfast
9:00–9:50	J. M. Combes: *Statistics of eigenvalues for the Anderson model*
10:00–10:25	M. Stoiciu: *Eigenvalue Statistics for Random CMV Matrices*
10:30–10:55	Coffee Break, 2nd floor lounge, Corbett Hall
11:00–11:50	V. Tchoulaevski: *Multi-particle disordered systems: New challenges*
12:00	Group Photo; meet on the front steps of Corbett Hall
12:00–13:30	Lunch
	Free Afternoon
17:30–19:30	Dinner

Wednesday

7:00–9:00	Breakfast
9:00–9:50	D. Damanik: *Pseudorandom Potentials: Open Problems and Some Recent Results*
10:00–10:25	H. Krüger: *The potential* $V(n) = f(n^\rho \pmod 1)$
10:30–10:55	Coffee Break, 2nd floor lounge, Corbett Hall
11:00–11:50	W.-M. Wang: *Resonant perturbations of Hamiltonian systems in infinite dimensions*
12:00–13:30	Lunch
15:00–15:30	Coffee, 2nd floor lounge, Corbett Hall
15:30–16:20	F. Klopp: *Non monotonous random operators*
16:30–16:55	C. Sadel: *Random phase hypothesis for a one dimensional N-orbital model*
17:00–17:25	A. Elgart: *Anderson localization for sign-indefinite single site potentials*
17:30–19:30	Dinner
20:00	Discussion: The Anderson transition

Thursday

7:00–9:00	Breakfast
9:00–9:50	L. Erdoes: *Wegner estimate and level repulsion for Wigner random matrices*
10:00–10:25	F. Hoecker-Escuti: *"Lifshitz" tail for low density Bernoulli-Anderson Model*
10:30–10:55	Coffee Break, 2nd floor lounge, Corbett Hall
11:00–11:25	N. Dombrowski: *Edge-conductance for a magnetic wall and Iwatsuka Hamiltonian*
11:30–11:55	A. Figotin: *Wave-corpuscle mechanics for elementary charges*
12:00–13:30	Lunch
14:00	F. Klopp: *Level spacing of eigenvalues* (informal lecture)
15:00–15:30	Coffee, 2nd floor lounge, Corbett Hall
15:30–15:55	Y. Kang: *Diffusion of wave packets in a Markov random potential*
16:00–16:25	D. Lenz: *Absolutely continuous spectrum for random operators on substitution trees*
16:30–16:55	O. Post: *Continuity of the IDS on random length metric graphs*
17:00–17:25	B. Metzger: *The discrete Gross-Pitaevskii model and condensation in the single particle ground state*
17:30	W. M. Wang: *Nonlinear Anderson localization* (informal lecture)
17:30–19:30	Dinner

Friday

7:00–9:00	Breakfast
9:00	Informal Lectures and Discussions
10:00	Coffee Break, 2nd floor lounge, Corbett Hall
11:30–13:30	Lunch
Checkout by 12 noon.	

** 5-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

Random Schrödinger Operators: Universal Localization, Correlations, and Interactions
April 19 to 24, 2009

ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Boumaza, Hakim** (Keio University)
Title: *Localization for a matrix-valued Anderson-Bernoulli model*
Abstract: We will present a localization result, both in the exponential and dynamical senses, for a random, matrix-valued, one-dimensional continuous Schrödinger operator acting on $L^2(\mathbb{R}) \otimes \mathbb{C}^N$, $N \geq 1$. For this, we combine results of Klein, Lacroix and Speis (1991) for matrix-valued discrete Anderson-Bernoulli models and results of Damanik, Sims and Stolz (2001) for scalar-valued continuous Anderson-Bernoulli models. Their approach, based upon multi-scale analysis, requires intermediate results such as the separability of Lyapunov exponents, the Hölder regularity of these exponents and of the IDS and a Wegner estimate. In our talk, we will focus on the first step, the positivity of the Lyapunov exponents. In particular, we present a criterion, coming from the theory of Lie groups, used to compute the closure of the group generated by the transfer matrices.

Speaker: **Combes, Jean Michele** (Univeristé de Toulon)
Title: *Statistics of eigenvalues for the Anderson model*
Abstract: We review recent results obtained in collaboration with F.Germinet, P.Hislop, A.Klein and F.Klopp on Wegner and Minami estimates for both discrete and continuum Anderson model. They provide the necessary tools to prove Poisson statistics of rescaled eigenvalues in the localized regime, thus extending to the continuum case the result obtained by Minami for the discrete Anderson model.

Speaker: **Damanik, David** (Rice University)
Title: *Pseudorandom Potentials: Open Problems and Some Recent Results*
Abstract: A bounded infinite sequence of real numbers is called a pseudorandom potential if the Schrödinger operator with this sequence as a potential has spectral properties akin to those of a typical Schrödinger operator with a random sequence as a potential. We present several sequences that are conjectured to be pseudorandom potentials based on heuristics and numerics and then explain some related rigorous results.

Speaker: **Disertori, Margherita** (Université de Rouen)
Title: *Supersymmetric non linear sigma models in Anderson localization*
Abstract: It has been known since the pioneering work of Wegner and later Efetov, that information about the spectral and transport properties of Anderson type models can be inferred from the correlation functions of statistical mechanical models of a certain kind. In the physics literature one usually assumes the sigma model approximation, which is believed to capture the essential features of the energy correlations and transport properties of the underlying quantum system. I'll give an introduction to the techniques involved and some results.

Speaker: **Dombrowski, Nicolas** (Université de Clergy-Pontoise)
Title: *Edge-conductance for a magnetic wall and Iwatsuka Hamiltonian* Abstract: In a joint work with F.Germinet and G.Raikov, we show that a magnetic wall modelized by a Iwatsuka Hamiltonian creates an edge-curent. We prove the quantization of the edge-conductance and its stability under suitable magnetic perturbation. This result has to goal to showing that the edge-current not depends of the nature of the wall but only of the breaking of the time reversal symmetry.

Speaker: **Elgart, Alexander** (Virginia Tech)
Title: *Anderson localization for sign-indefinite single site potentials*
Abstract: TBA

Speaker: **Erdoes, Laszlo** (Ludwig Maximilians Universität München)
Title: *Wegner estimate and level repulsion for Wigner random matrices*
Abstract: We consider N by N Hermitian random matrices with independent identically distributed entries (Wigner matrices). The matrices are normalized so that the average spacing between consecutive eigenvalues is of order 1/N. Under suitable assumptions on the distribution of the single matrix element, we first prove that, away from the spectral edges, the empirical density of eigenvalues concentrates around the Wigner semicircle law on energy scales bigger than 1/N. This result establishes the semicircle law on the optimal scale and it removes a logarithmic factor from our previous result. We then show a Wegner estimate, i.e. that the averaged density of states is bounded. Finally, we prove that the eigenvalues of a Wigner matrix repel each other, in agreement with the universality conjecture. This is a joint work with H.T. Yau and B. Schlein.

Speaker: **Figotin, Alexander** (University of California Irvine)
Title: *Wave-corpuscle mechanics for elementary charges*
Abstract: In 1920 Schrodinger inspired by ideas of de Broglie on the material wave introduced his wave mechanics in which a particle is modeled by a wave packet. As it was pointed out by M. Born the interpretation of a particle by a wave packet has problems: "the wave packets must in course of time become dissipated, and on the other hand the description of the interaction of two electrons as a collision of two wave packets in ordinary three-dimensional space lands us in grave difficulties". To address those problems we introduce a concept of wave-corpuscle to describe spinless elementary charges interacting with the classical EM field. Every charge interacts only with the EM field and is described by a complex valued wave function over 4-dimensional space time continuum. A system of many charges interacting with the EM field is defined by a local, gauge and Lorentz invariant Lagrangian with a key ingredient - a nonlinear self-interaction term providing for a cohesive force assigned to every charge. An ideal wave-corpuscle is a spatially localized solitary wave which is an exact solution to the Euler-Lagrange equations which are reduced to a certain nonlinear Schrodinger equation. We show that the wave-corpuscle remains spatially localized when it is free or even when it accelerates in a homogeneous electric field. Two or more interacting charges are well defined even when they collide.

Speaker: **Germinet, Francois** (Université de Clergy-Pontoise)
Title: *Anderson localization for random Schodinger operators*
Abstract: In this review talk, we shall mention recent results on localization. It features universal localization for continuum Anderson Hamiltonians and localization for Poisson Schrodinger operators. Spectral as well as dynamical localization are proven for these models.

Speaker: **Goldstein, Michael** (University of Toronto)
Title: *Schrödinger equations with potentials generated by shifts and skew-shifts*
Abstract: In this talk we give an exposition of the ideas, methods and technological tools developed in last ten years for the analysis of one-dimensional difference Schrödinger equations with potentials generated by shift and skew-shift

$$[H(x,\omega)\varphi](n) = -\varphi(n-1) - \varphi(n+1) + V(T_\omega^n(x))\varphi(n) = E\varphi(n), \;\; n \in \mathbb{Z}.$$

Here T_ω is a shift $T_\omega(x) = x + \omega, \omega \in \mathbb{T}^\nu$, or a skew-shift, $T_\omega(x,y) = (x+\omega, y+x)$, $(x,y) \in \mathbb{T}^2$, $\omega \in \mathbb{T}$, and T_ω^n, $n \in \mathbb{Z}$, stands for the n-th iteration of T. We state also the central open problems in this field.

Speaker: **Hamza, Eman** (Michigan State University)
Title: *The unitary Anderson model*
Abstract: TBA

Speaker: **Hoecker-Escuti** (Université Paris 13)
Title: *"Lifshitz" tail for low density Bernoulli-Anderson Model*
Abstract: We will discuss a Lifshitz tail type estimate for the integrated density of states of a discrete random Schrdinger Hamiltonian in a low density regime, the potential of which consists of independent identically distributed Bernoulli random variables.

Speaker: **Kang, Yang** (Michigan State University)
Title: *Diffusion of wave packets in a Markov random potential*
Abstract: In this talk we consider the evolution of a tight bonding wave packet propagating in a time dependent potential. If the potential evolves according to a stationary Markov process, then the square amplitude of the wave packet converges, after diffusive rescaling, to a solution of a heat equation. This is a joint work with Jeffrey H. Schenker.

Speaker: **Klopp, Frédéric** (Université Paris 13)
Title: *Non monotonous random operators*
Abstract: The talk is devoted to the study of the ground state energy and the existence of Lifshitz tails near this energy for non monotonous random Schrdinger operators. Here, non monotonous means that the random operator is not a monotonous function of the random variables. We will review recent results, try to explain some of the new techniques and list some open problems. The models under consideration include alloy type models, displacement models, random magnetic potential models, etc.

Speaker: **Krüger, Helge** (Rice University)
Title: *The potential* $V(n) = f(n^\rho \pmod 1)$
Abstract: Consider the potential $V(n) = f(n^\rho \pmod 1)$ for $\rho > 0$ not an integer, and the associated Schrödinger operator H. I will study the spectrum and Lyapunov exponent of this operator, in particular deriving explicit formulas for it in the case $0 < \rho < 1$.

Speaker: **Lenz, Daniel** (Universität Jena)
Title: *Absolutely continuous spectrum for random operators on substitution trees*
Abstract: TBA

Speaker: **Metzger, Bernd** (Université Paris 13)
Title: *The discrete Gross-Pitaevskii model and condensation in the single particle ground state* Abstract: In the context of a tight-binding approximation of the Gross-Pitaevskii energy functional with a random background potential we want to discuss in dependence on the interaction coupling constant a criteria when the Gross-Pitaevskii ground state and the single particle ground state coincide.

Speaker: **Olaf Post** (Humboldt Universität Berlin)
Title: *Continuity of the IDS on random length metric graphs*
Abstract: We establish several properties of the integrated density of states for random quantum graphs: Under appropriate ergodicity and amenability assumptions, the integrated density of states can be defined using an exhaustion procedure by compact subgraphs. A trace per unit volume formula holds, similarly as in the Euclidean case. Our setting includes periodic graphs. For a model where the edge lengths are random and vary independently in a smooth way we prove a Wegner estimate and related regularity results for the integrated density of states. These results are illustrated for an example based on the Kagome lattice. In the periodic case we characterise all compactly supported eigenfunctions and calculate the position and size of discontinuities of the integrated density of states. (joint work with 'Daniel Lenz, Norbert Peyerimhoff and Ivan Vecelic')

Speaker: **Sadel, Christian** (Universität Erlangen, Georgia-Tech)
Title: *Random phase hypothesis for a one dimensional N-orbital model*
Abstract: This is joint work with H. Schulz-Baldes. We consider a random Schrdinger operator on the

strip, acting as discrete Laplacian in the infinite direction, with random hopping between the orbitals, i.i.d. for each slice. This randomness is coupled with a small coupling constant λ. This system can be described by transfer matrices. In order to calculate the Lyapunov exponents associated to them, one has to consider the induced Markov process on a flag manifold which is diffeomorphic to a quotient of the unitary group U(N). Within the band the free Hamiltonian (without randomness) has only elliptic channles and all Lyapunov exponents are zero. The random phase hypothesis states that after an adequate basis change, the invariant measure for this Markov process is given by the Haar measure, at least for small λ. We show that for $\lambda \to 0$, all families of invariant measures converge to the Haar measure in a weak sense. To obtain this we actually consider a more general situation of random Lie group elements acting on a compact homogeneous space. As a consequence we get perturbative formulas for the Lyapunov exponents showing that they are equally spaced to lowest order.

Speaker: **Schulz-Baldes** (Universität Erlangen)
Title: *Random Dirac operators with time reversal symmetry*
Abstract: Quasi-one-dimensional stochastic Dirac operators with an odd number of channels, time reversal symmetry but otherwise efficiently coupled randomness are shown to have one conducting channel and absolutely continuous spectrum of multiplicity two. This follows by adapting the criteria of Guivac-Raughi and Goldsheid-Margulis to the analysis of random products of matrices in the group $\mathrm{SO}^*(2L)$, and then a version of Kotani theory for these operators. Absence of singular spectrum can be shown by adapting an argument of Jaksic-Last if the potential contains random Dirac peaks with absolutely continuous distribution. (Joint work with Christian Sadel)

Speaker: **Shterenberg, Roman** (University of Alabama at Birmingham)
Title: *Asymptotic expansion of the integrated density of states of a two- dimensional periodic Schrödinger operator*
Abstract: We prove the complete asymptotic expansion of the integrated density of states of a two-dimensional Schrödinger operator with a smooth periodic potential.

Speaker: **Stoiciu, Mihai** (Williams College, UC Irvine)
Title: *Eigenvalue Statistics for Random CMV Matrices*
Abstract: The CMV matrices are the unitary analogues of one-dimensional discrete Schrodinger operators. We consider CMV matrices with random coefficients and we study the statistical distribution of their eigenvalues. For slowly decreasing random coefficients, we show that the eigenvalues are distributed according to a Poisson process. For rapidly decreasing coefficients, the eigenvalues have rigid spacing (clock distribution). For a certain critical rate of decay we obtain the circular beta distribution. We will also explain the connection with random Schrodinger operators, discuss a few open questions and present several numerical results.

Speaker: **Tchoulaevski, Victor** (Université de Nantes)
Title: *Multi-particle disordered systems: New challenges*
Abstract: We give an overview of recent results and techniques in the multi-particle Anderson localization theory, on a lattice and in a Euclidean space of an arbitrary finite dimension, and discuss some open and challenging problems in this new direction of mathematical physics of random media. We argue that the spectral aspect of localization phenomena, often considered as the central one in the single-particle localization theory, may be less important for infinite systems of interacting particles, and that the insight from the nonlinear localization theory may prove instructive for further developments in the linear theory.

Speaker: **Wang, Wei-Min** (Université Paris Sud)
Title: *Resonant perturbations of Hamiltonian systems in infinite dimensions*
Abstract: We develop a resonant perturbation theory for linear and nonlinear Schroedinger equations. In this lecture, we focus on the linear theory, where we prove eigenfunction localization (in Fourier) for the 2D periodic Schroedinger operator on the square torus, solving a classic problem in spectral theory.

In the second part of the lecture, we present the proof of long time Anderson localization for the nonlinear random Schroedinger equation (joint work with Z. Zhang).