ABSTRACTS
(in alphabetic order by speaker surname)

Pavel Bleher

Random matrix model with external source and a constrained vector equilibrium problem

We consider the random matrix model with external source, in case where the potential $V(x)$ is an even polynomial and the external source has two eigenvalues $\pm a$ of equal multiplicity. We show that the limiting mean eigenvalue distribution of this model can be characterized as the first component of a pair of measures (μ_1, μ_2) that solve a constrained vector equilibrium problem. The proof is based on the steepest descent analysis of the associated Riemann-Hilbert problem for multiple orthogonal polynomials.

We illustrate our results in detail for the case of a quartic double well potential $V(x) = \frac{1}{4}x^4 - \frac{t}{2}x^2$. We are able to determine the precise location of the phase transitions in the ta-plane, where either the constraint becomes active, or the two intervals in the support come together (or both).

This is a joint project with Arno Kuijlaars and Steven Delvaux.

Björn Gustafsson

Non-univalent solutions of the Polubarinova-Galin equation

We discuss solutions of the Polubarinova-Galin (PG) equation

$$\mathrm{Re}\,[\dot{f}(\zeta,t)\overline{\zeta f'(\zeta,t)}] = q(t) \quad (|\zeta| = 1)$$

in the case that the "mapping" function $f(\zeta, t)$ from the unit disk $\mathbb{D}$ is no longer univalent.

If f' has zeros in $\mathbb{D}$ new phenomena appear, for example given the $q(t) > 0$ there are many solutions of PG. One of them satisfies the corresponding Löwner-Kufarev equation $\dot{f}(\zeta,t) = \zeta f'(\zeta,t)P(\zeta,t)$, and this one can be lifted to a univalent map $\tilde{f}(\cdot,t) : \mathbb{D} \to \mathbb{C}$, where R is a certain Riemann surface over $\mathbb{C}$. This Riemann surface is not given in advance, but has to be created along with the solution. A proposed main result says that there exists a global (in time) weak solution of this kind. It could be called "universal weak solution" because it is simply connected and the classical weak solution, which may be multiply connected for some period of time, can be obtained from it by balayage.

The talk is based on work in progress in collaboration with Yu-Lin Lin, Taipei.

J. Harnad

Fermionic construction of tau functions and random processes

Tau functions expressed as fermionic expectation values [E. Date, M. Jimbo, M. Kashiwara, T. Miwa, Transformation groups for soliton equations, in: M. Jimbo, T. Miwa (Eds.), Nonlinear Integrable Systems Classical Theory and Quantum Theory, World Scientific, 1983, pp. 39 120] are shown to provide a natural and straightforward description of a number of random processes and statistical models involving hard core configurations of identical particles on the integer lattice, like a discrete version simple exclusion processes (ASEP), nonintersecting random walkers, lattice Coulomb gas models and others, as well as providing a powerful tool for combinatorial calculations involving paths between pairs of partitions. We study the decay of the initial step function within the discrete ASEP (d-ASEP) model as an example. For constant hopping rates we obtain VershikKerov type of asymptotic configuration of particles. Joint work with A.Yu. Orlov.

Jens Hoppe

Integrable structures in the relativistic theory of extended objects

A recently discovered dynamical symmetry (related to higher dimensional analogues of the Virasoro algebra), as well as older signs of integrability in the dynamics of strings and membranes, ..., will be presented.

Dmitry Khavinson

“Fingerprints” of the Two Dimensional Shapes and Lemniscates

The newly emerging field of vision and pattern recognition often focuses on the study of two dimensional “shapes”, i.e. simple, closed smooth curves. A common approach to describing shapes consists in defining a “natural” embedding of the space of curves into a metric space and studying the mathematical structure of the latter. Another idea that has been pioneered by Kirillov and developed recently among others by Mumford and Sharon consists of representing each shape by its “fingerprint”, a diffeomorphism of the unit circle. Kirillov’s theorem states that the correspondence between shapes and fingerprints is a bijection modulo conformal automorphisms of the disk. In this talk we discuss the recent joint work with P. Ebenfelt and Harold S. Shapiro outlining an alternative interpretation of the problem of shapes and Kirillov’s theorem based on finding a set of natural and simple fingerprints that is dense in the space of all diffeomorphisms of the unit circle. This approach is inspired by the celebrated theorem of Hilbert regarding approximation of smooth curves by lemniscates. We shall outline proofs of the main results and discuss some interesting function-theoretic ramifications and open questions regarding possibilities of numerical applications of this idea. Joint with Peter Ebenfelt and Harold S. Shapiro.

Seung-Yeop Lee

Random normal matrix by Riemann-Hilbert problem

We present a simple case where the orthogonal polynomials defined by area-integral can be replaced by the orthogonal polynomials on contours. Then we apply the standard steepest descent analysis of the relevant Riemann-Hilbert problem. We obtain the strong asymptotics of the orthogonal polynomials, and provide an example where merging (or break-off) transition is described by the Painleve II equation. This is the joint work with Ferenc Balogh, Marco Bertola, and Ken McLaughlin.

Yu-Lin Lin

Large-time behavior of multi-cut solutions to Hele-Shaw flows

In the case of zero surface tension Hele-Shw flows driven by injection, it is found that there is a general set of solutions, called multi-cut solutions which are the combination of rational functions and logarithm functions. In this talk, we show large-time behavior of multi-cut solutions $f(\zeta, t)$ in terms of some conserved quantities. These conserved quantities are called Richradson’ moments . One of the important step of the proof is to formulate ODEs for singularity of $f(\zeta, t)$. Part of this talk is joint-work with B. Gustafsson.

Erik Lundberg

Laplacian growth, elliptic growth, and singularities of the Schwarz potential

Solutions to the two-dimensional Laplacian growth problem can be described elegantly in terms of the singularities in the oil domain of the Schwarz function of the moving boundary. Namely, the non-physical singularities are stationary, and the physical singularities (corresponding to sources/sinks) obey simple dynamics. The same principle holds in higher dimensions in terms of the singularities of the Schwarz potential, introduced by D. Khavinson and H. S. Shapiro (1989). By further generalizing the Schwarz potential, this can also be extended to the elliptic growth problem. We discuss some exact solutions and questions arising in this connection.

Irina Markina, Alexander Vasil'ev

Parametrization of the Loewner-Kufarev evolution in Sato's Grassmannian

We discuss complex and Cauchy-Riemann structures of the Virasoro algebra and of the Virasoro-Bott group in relation with the Loewner-Kufarev evolution. Based on the Hamiltonian formulation of this evolution we obtain an infinite number of conserved quantities and provide embedding of the Loewner-Kufarev evolution into Sato's Grassmannian.

Robb McDonald

Two problems in Hele-Shaw free boundary flows

Two problems in Hele-Shaw free boundary flows are considered. First, the time-dependent evolution of source-driven Hele-Shaw free boundary flows in the presence of an obstacle are computed numerically. The Baiocchi transformation is used to convert the Hele-Shaw problem into a free boundary problem for a streamfunction-like variable $u(x, y, t)$ governed by Poisson's equation. Interpreting u as a streamfunction, at a given time the problem becomes that of finding a steady patch of uniform vorticity enclosing a point vortex of given strength such that the velocity vanishes on the free boundary and the tangential velocity vanishes on the obstacle. A combination of contour dynamics and Newton's method is used to compute such equilibria. By varying the strength of the point vortex these equilibria represent a sequence of source-driven growing blobs of fluid in a Hele-Shaw cell. Example computations are presented.

Second, an equation governing the evolution of a Hele-Shaw free boundary flow in the presence of an arbitrary external potential–generalized Hele-Shaw flow–is derived in terms of the Schwarz function of the free boundary. The new equation is used to re-derive some known explicit solutions for equilibrium and time-dependent free boundary flows in the presence of external potentials including uniform gravitational and centrifugal fields, and those with singularities in the potential field. Some new exact solutions for generalized Hele-Shaw flow are also constructed using the Schwarz function equation.

Alexander Orlov

Random Processes, Tau Functions and Growth of Young Diagrams

We consider a model suggested by M.Fisher in his presentation paper at Boltzmann medal rewarding, which may be called a random turn process (M. Fisher, "Walks, walls, wetting and melting", *J. Stat. Phys.*

34 667-729 (1984)). We will show that this model may be considered as a version of the Young diagram growth model and relate it to an integrable systems via a tau function approach. (It may be compared with relation between other growth domain problems like the Laplace growth where tau function also appeares.) In both cases tau functions play the role of the partition function of models. We will explain the fermionic construction of tau functions for two types of integrable hierarchies (A-type and B-type root systems) and present an explicit expression for the probability to achive a given Young diagram along the random turn process.

Makoto Sakai

Structure theorems on quadrature domains for subharmonic functions

For a finite or infinite positive measure, we assign a positive measure called partial balayage. By using the measure, we describe all quadrature domains of the given measure for subharmonic functions.

Leonard M. Sander

Numerical computation of the harmonic measure for percolation and DLA clusters

We have obtained the harmonic measure of diffusion-limited aggregate (DLA) clusters, percolation clusters, and FK clusters for the Potts model using a biased random-walk sampling technique which allows us to measure probabilities of random walkers hitting sections of clusters with unprecedented accuracy; our results include probabilities as small as 10^{-80}. We find the multifractal D(q) spectrum including regions of small and negative q. For percolation the results agree with results obtained from conformal field theory, and for DLA we settle several controversies in the literature. We have new results on DLA and percolation clusters in three dimensions, for which there is no theory of any kind, and which show interesting features. With David A. Adams and R. M. Ziff.

Tanya Savin

Schwarz symmetry principle and dynamical mother bodies

The talk consists of two separate parts which however have some connections from both mathematical and physical viewpoints.

First part is devoted to generalizations of the celebrated, point-to-point, Schwarz reflection principle. This point-to-point reflection, holding for harmonic functions subject to the Dirichlet or Neumann conditions on a real-analytic curve in the plane, almost always fails for solutions to more general elliptic equations. We will discuss non-local, point-to-compact set, reflection operators for different elliptic equations subject to different boundary conditions.

The second part of the talk is a joint project with Alexander Nepomnyashchy (Technion). We will introduce dynamical mother bodies arising in an attempt to answer the question: what distribution of sinks allows the complete removal of a droplet with an algebraic boundary from a Hele-Shaw cell.

Ahmed Sebbar

Level lines of Green's functions and third order ODE

To multiply connected domains, we associate a third order ordinary differential equation. We study the geometry of this equation as initiated by Cartan, Chern, concentrating on the Wünschmann invariant. The doubly connected case will be considered in details.

Saleh Tanveer

Time evolution and stability results in Hele-Shaw and Kinetic Regularization Problems

Razvan Teodorescu

Generating functions for Laplacian growth

Laplacian growth may be understood as an inverse moment problem in two dimensions, with special constraints satisfied by the moments. It can also be regarded as an Abel-Jacobi inverse problem, in the sense of integrable systems.

Both descriptions of the process can be formulated by using generating functions. In this talk, some of the relationships between the different generating functions are investigated.

Giovani L. Vasconcelos

Interface growth in two dimensions: A Loewner-equation approach

The problem of Laplacian growth in two dimensions is considered within the Loewner-equation framework. Initially, the problem of slit-like fingers is revisited and some known exact solutions are presented. Then a general class of growth models for an interface growing in the upper-half plane and the channel geometry is introduced and the corresponding Loewner equations are derived. Several examples are given including interfaces with one or more tips as well as multiple growing interfaces. A generalization of our interface growth model in terms of "Loewner domains," where the growth rule is specified by a time evolving measure, is also presented. The possibility of generating fractal surfaces within the context of Loewner domains will be briefly discussed.

Oksana Yermolayeva

Quantum Algebraically Integrable Systems and Conformal Mappings

Theory of algebraically integrable systems is a proper non-trivial extension of the theory of harmonic functions to the case of operators with variable coefficients. It is natural to consider analogs of Laplacian growth for such systems. We study conformal mappings describing domain dynamics governed by algebraically integrable equations and show that important properties of the Laplacian growth are preserved in this generalization.