Look Elsewhere Effect

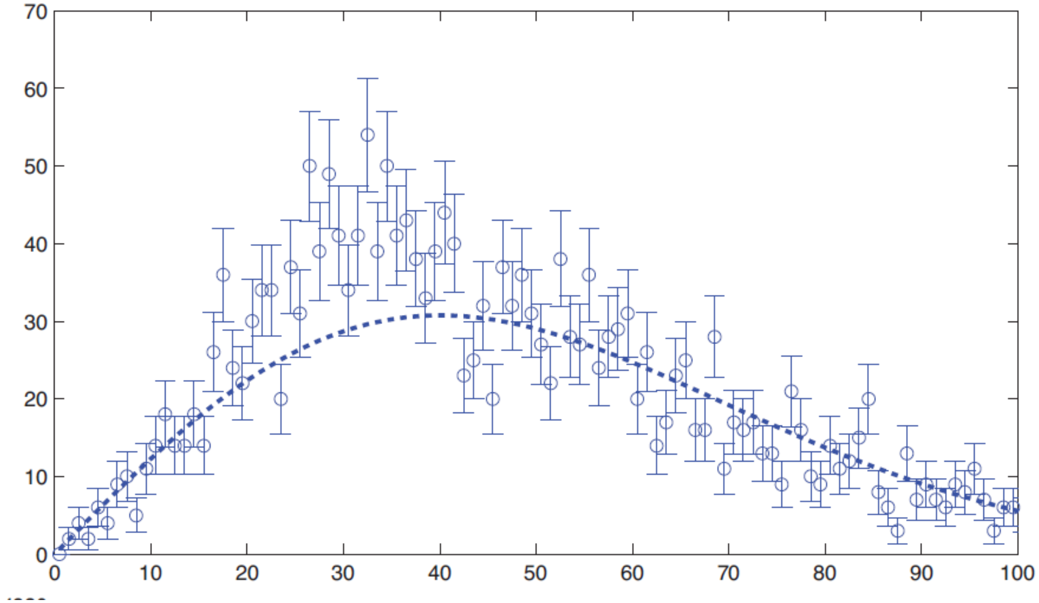
Eilam Gross and **Ofer Vitells**
Weizmann Institute of Science

A Plausible Thumb Rule for a Trial

Eilam Gross and **Ofer Vitells**
Weizmann Institute of Science

Look Elsewhere Effect

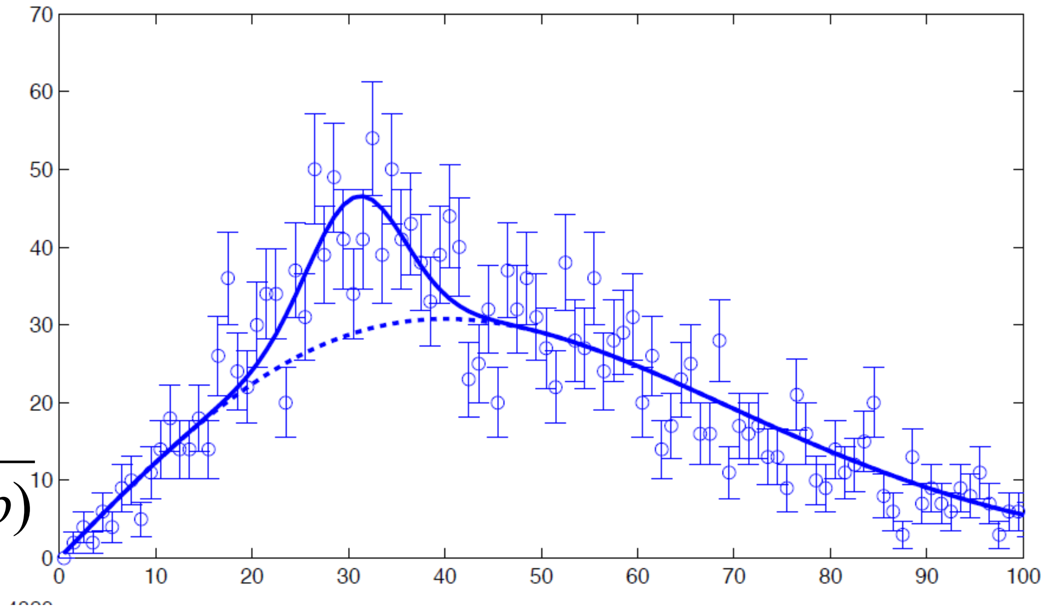
- Is there a signal here?



Look Elsewhere Effect

- Obviously @ m=30
- What is its significance?
- What is your test statistic?

$$t_{fix,obs} = -2\ln\frac{L(b)}{L(\hat{\mu}s(m=30)+b)}$$



Look Elsewhere Effect

- Test statistic

$$t_{fix\,obs} = -2\ln\frac{L(b)}{L(\hat{\mu}s(m=30)+b)}$$

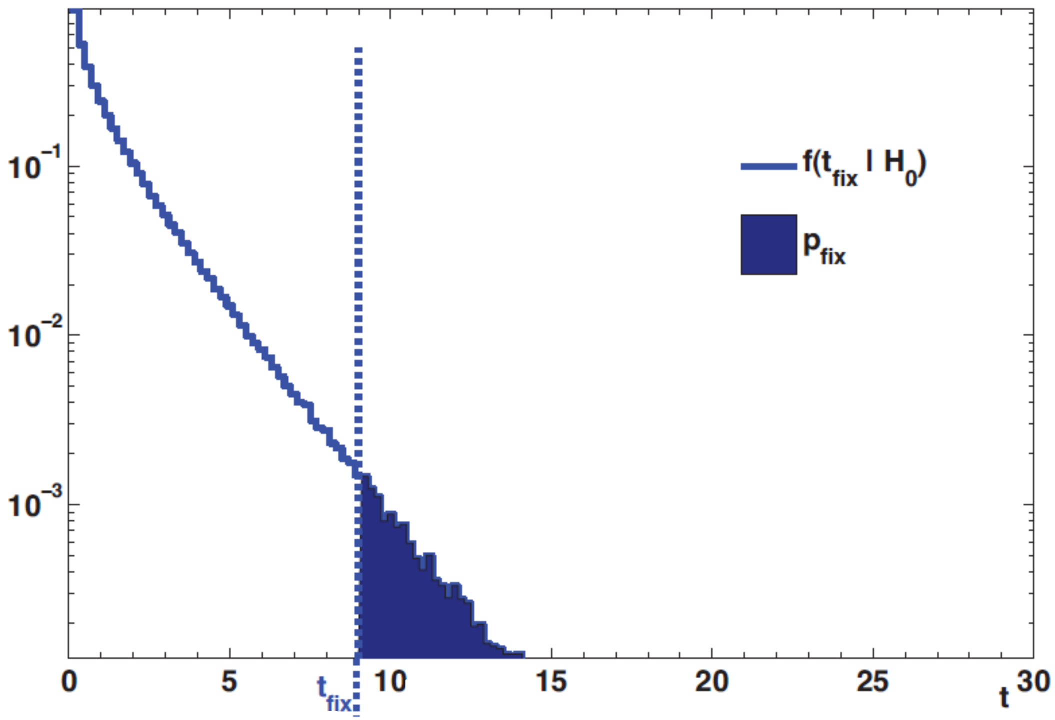
- What is the p-value?
- generate the PDF

$$f(t_{fix} | H_0)$$

and find the **p-value**

Wilks theorem:

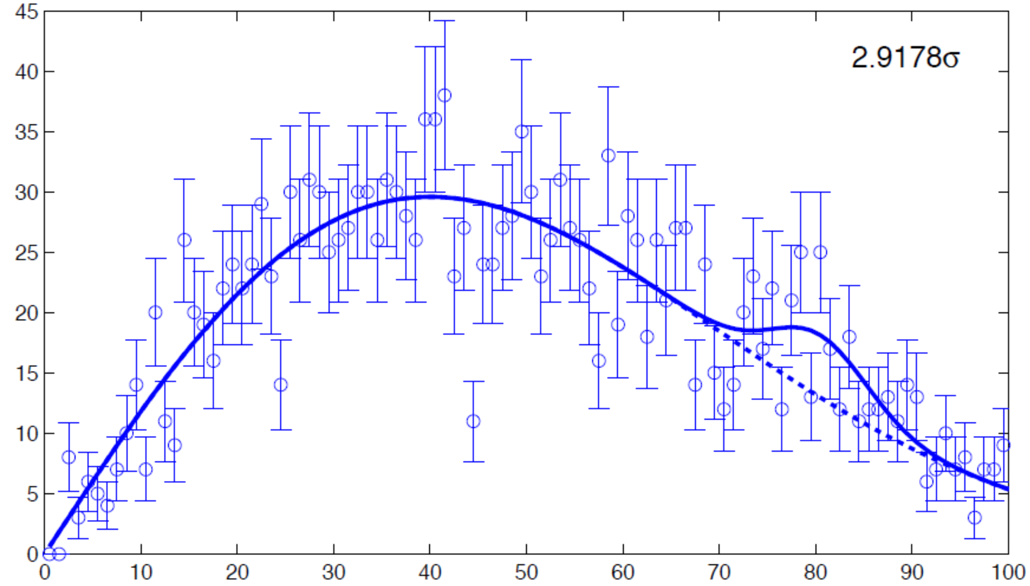
$$f(t_{fix} | H_0) \sim \chi^2_1$$



$$p_{fix} = \int_{t_{fix,obs}} f(t_{fix}|H_0)dt_{fix}$$

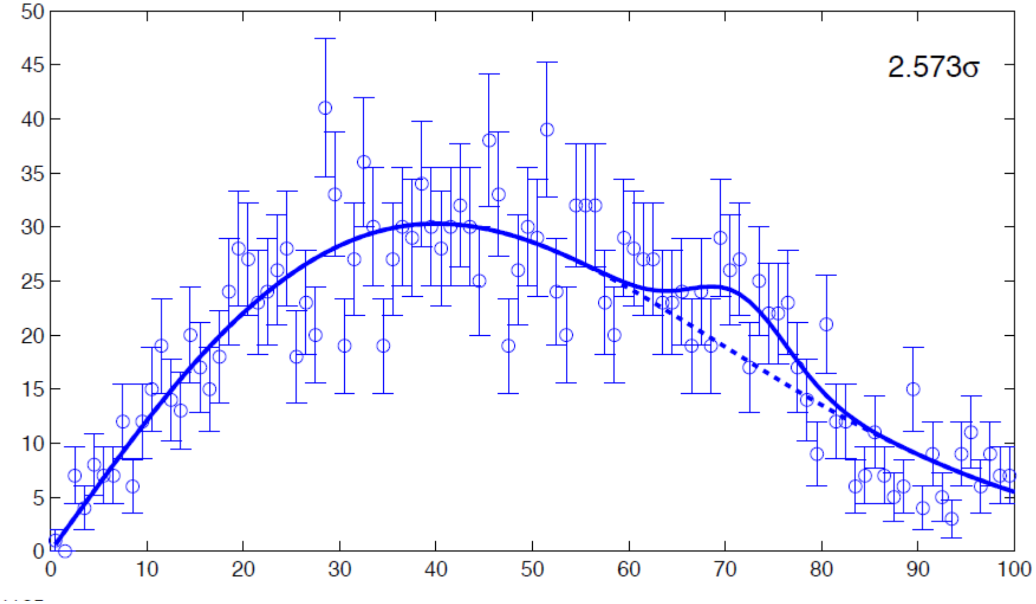
Look Elsewhere Effect

- Would you ignore this signal, had you seen it?



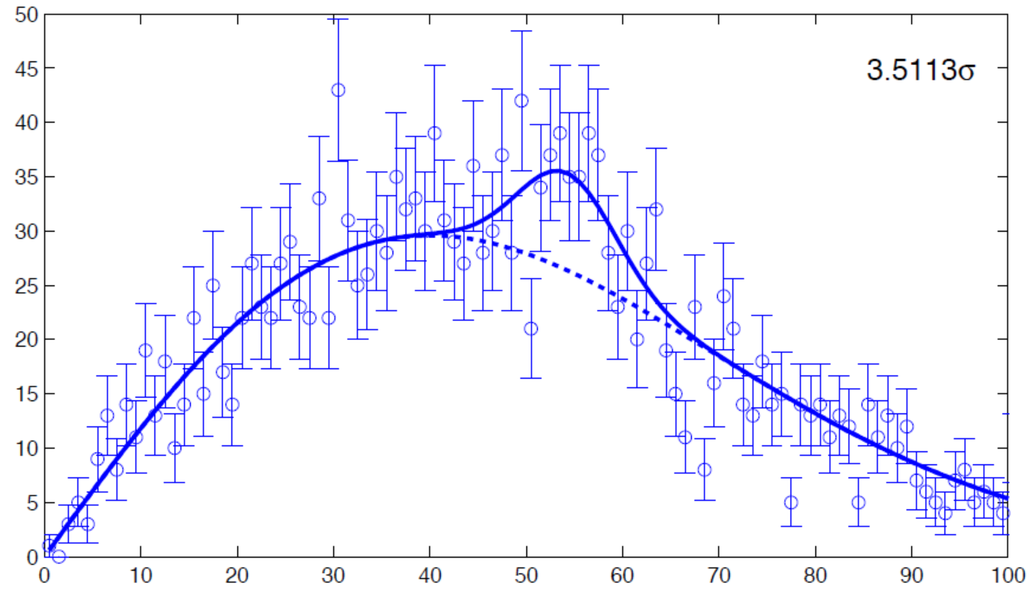
Look Elsewhere Effect

- Or this?



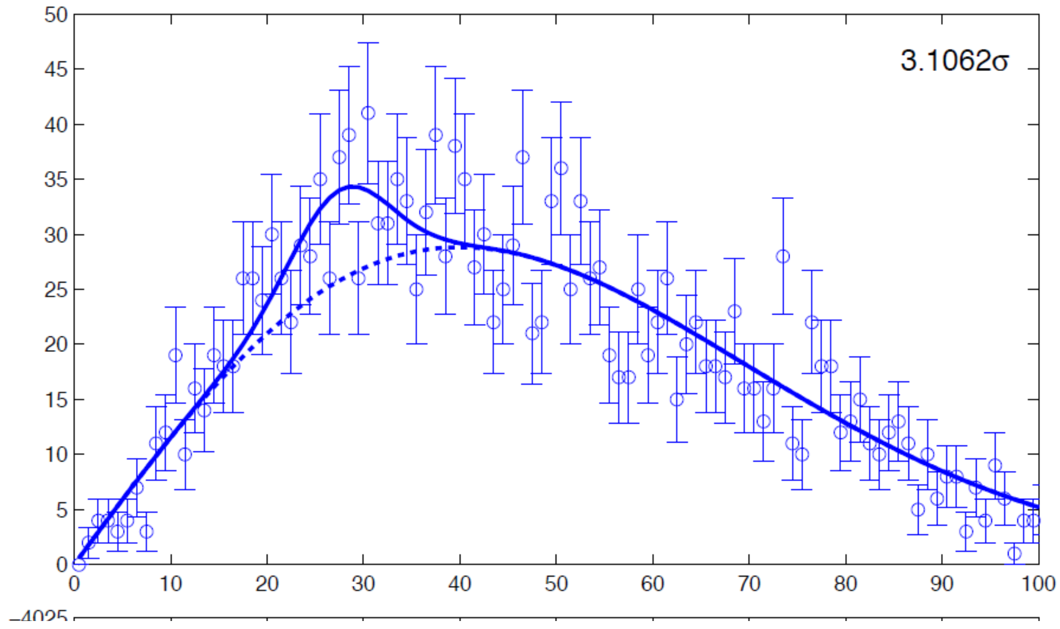
Look Elsewhere Effect

- Or this?



Look Elsewhere Effect

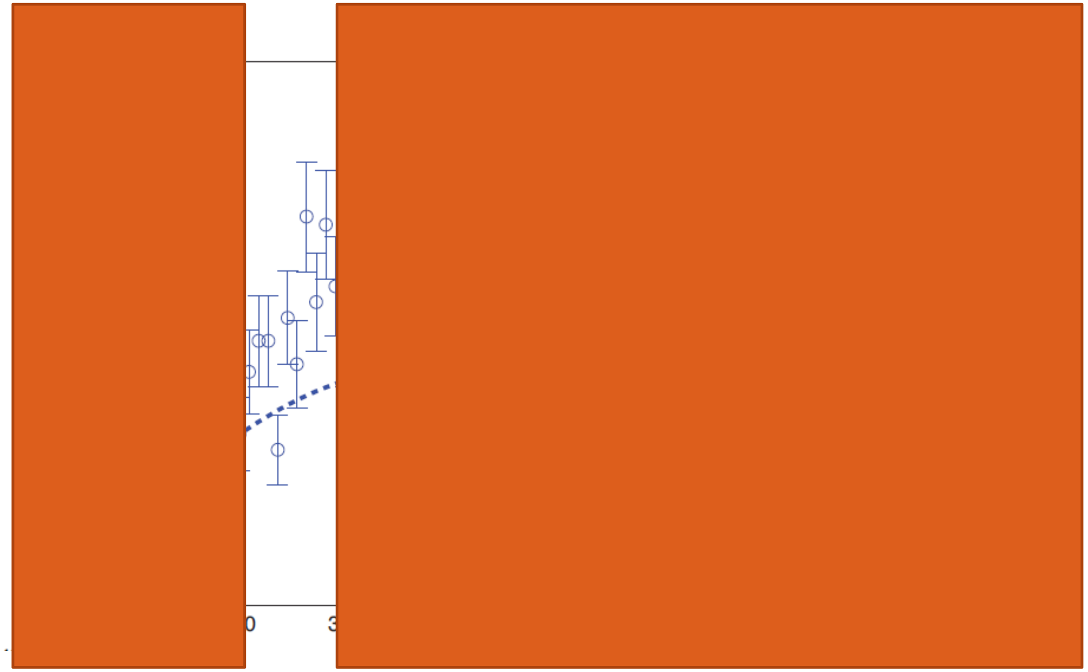
- Or this?
- Obviously NOT!
- ALL THESE "SIGNALS" ARE BG FLUCTUATIONS



Look Elsewhere Effect

- Having no idea where the signal might be there are two options
- **OPTION I:**

scan the mass range in pre-defined steps and test any disturbing fluctuations



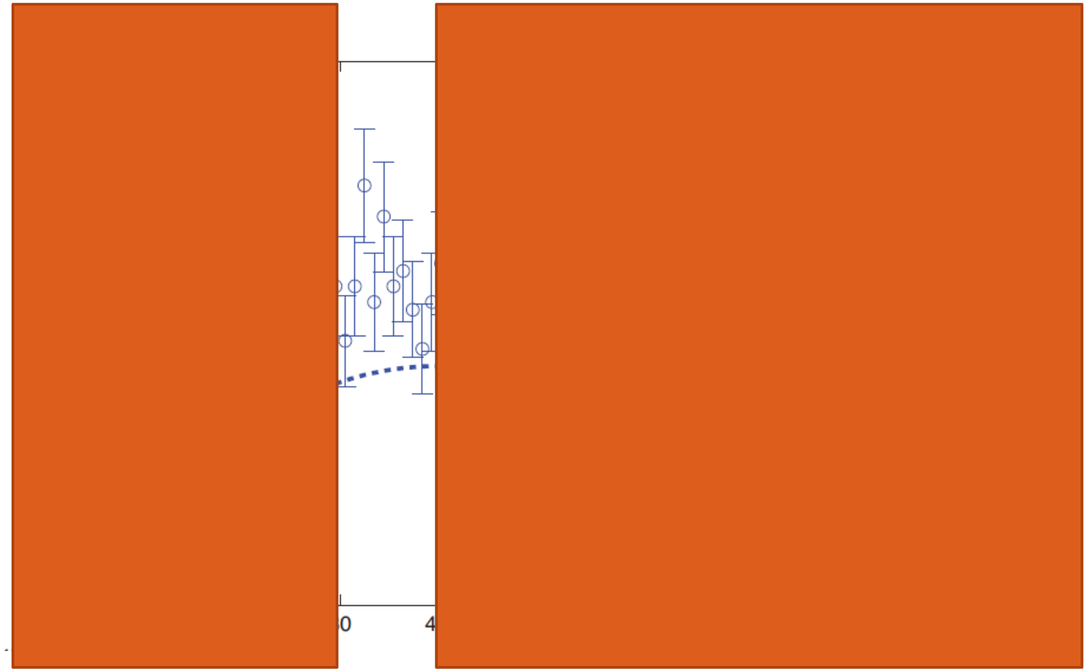
- Perform a fixed mass analysis at each point

$$t_{fix,\, obs}(\hat{\mu}) = -2\ln\frac{L(b)}{L(\hat{\mu}s(m)+b)}$$

Look Elsewhere Effect

- Having no idea where the signal might be there are two options
- **OPTION I:**

scan the mass range in pre-defined steps and test any disturbing fluctuations



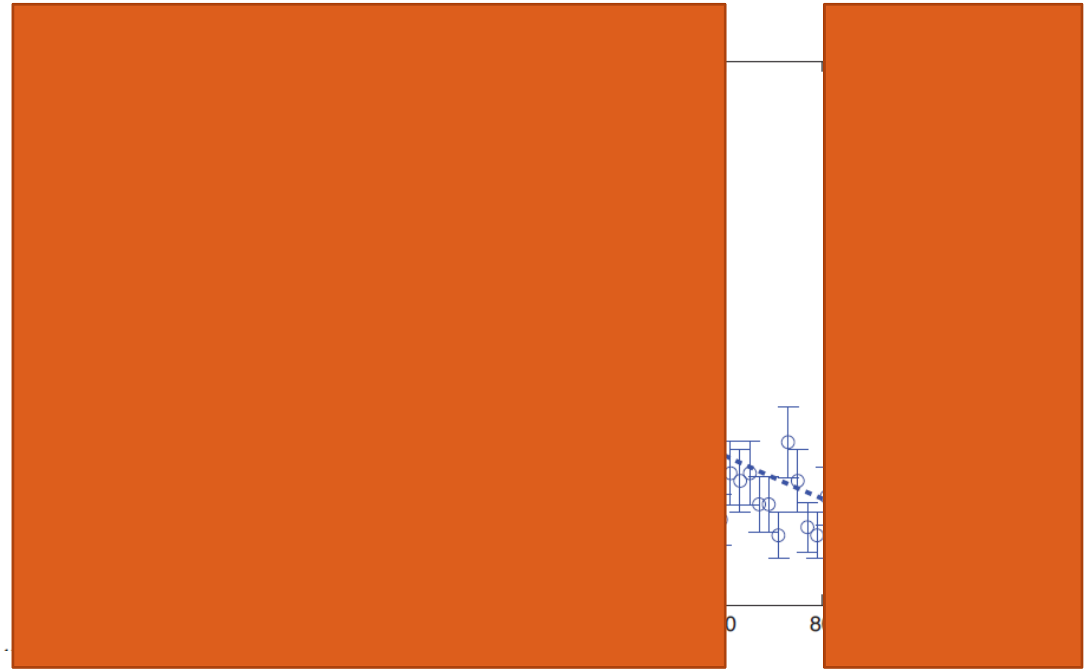
- Perform a fixed mass analysis at each point

$$t_{fix,\,obs}(\hat{\mu}) = -2\ln\frac{L(b)}{L(\hat{\mu}s(m)+b)}$$

Look Elsewhere Effect

- The scan resolution must be less than the signal mass resolution

- Assuming the signal can be only at one place, pick the one with the smallest p-value (maximum significance)

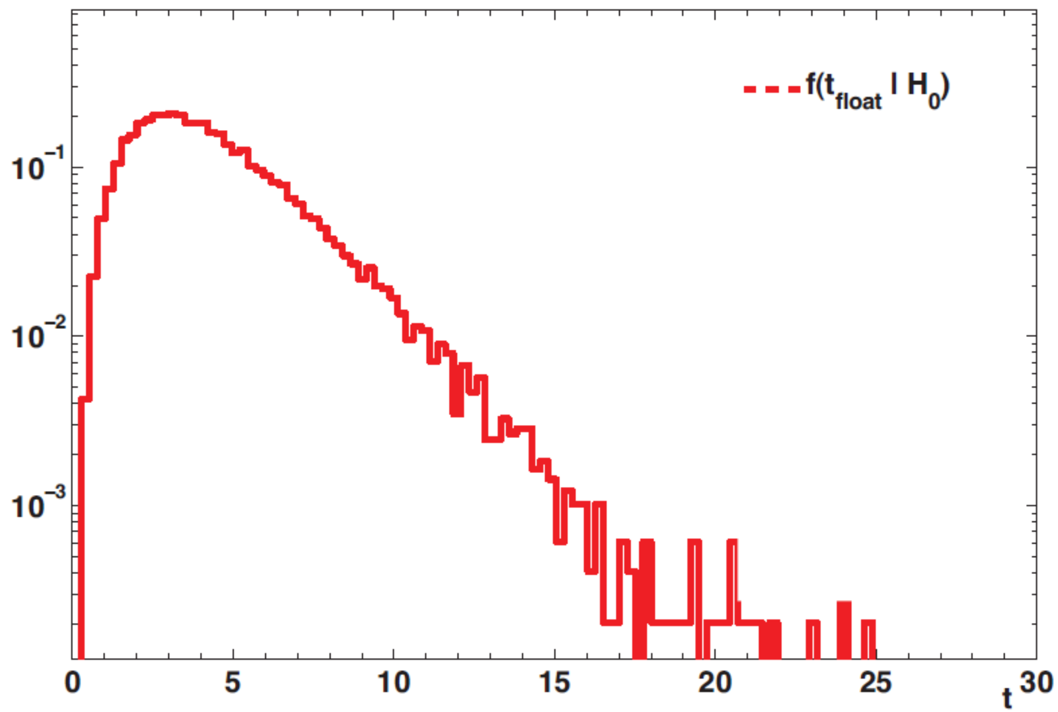


$$t_{fix,\,obs}(\hat{\mu}) = -2\ln\frac{L(b)}{L(\hat{\mu}s(m)+b)}$$

Look Elsewhere Effect

- The scan resolution must be less than the signal mass resolution

- Assuming the signal can be only at one place, pick the one with the smallest p-value (maximum significance)



- This is equivalent to **OPTION II**: leave the mass floating

$$t_{fix,\,obs}(\hat{\mu}) = -2\ln\frac{L(b)}{L(\hat{\mu}s(m)+b)}$$

This was shown by Cervero, Fayard, Kado and Polci, ATL-COM-PHYS-2009-382

The Thumb Rule

$$trial\ factor = \frac{p_{float}}{p_{fix}}$$

$$trial\ factor \stackrel{?}{=} \frac{range}{resolution} = \frac{\Gamma_m}{\sigma_m}$$

In an ATLAS note by

Tatiana Cervero, Louis Fayard, Marumi Kado, Francesco Polci

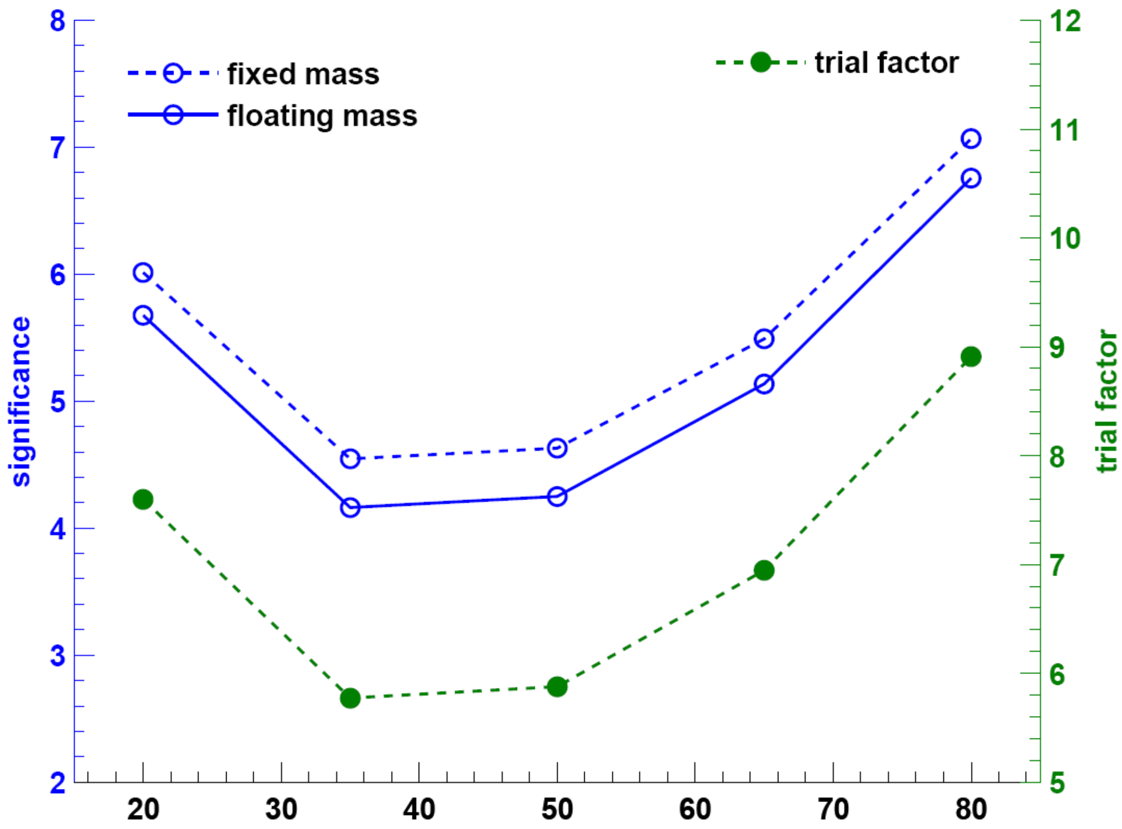
they tested the LEE with H→ $\gamma\gamma$ signal on top of a steep falling BG with a fixed mass significance of Z=3 σ , and found an agreement with the thumb rule (trial#~28).

The Worry

$$trial\ factor = \frac{p_{float}}{p_{fix}}$$

Back of the envelope:

$$trial\ factor \stackrel{?}{=} \frac{range}{resolution} = \frac{\Gamma_m}{\sigma_m}$$

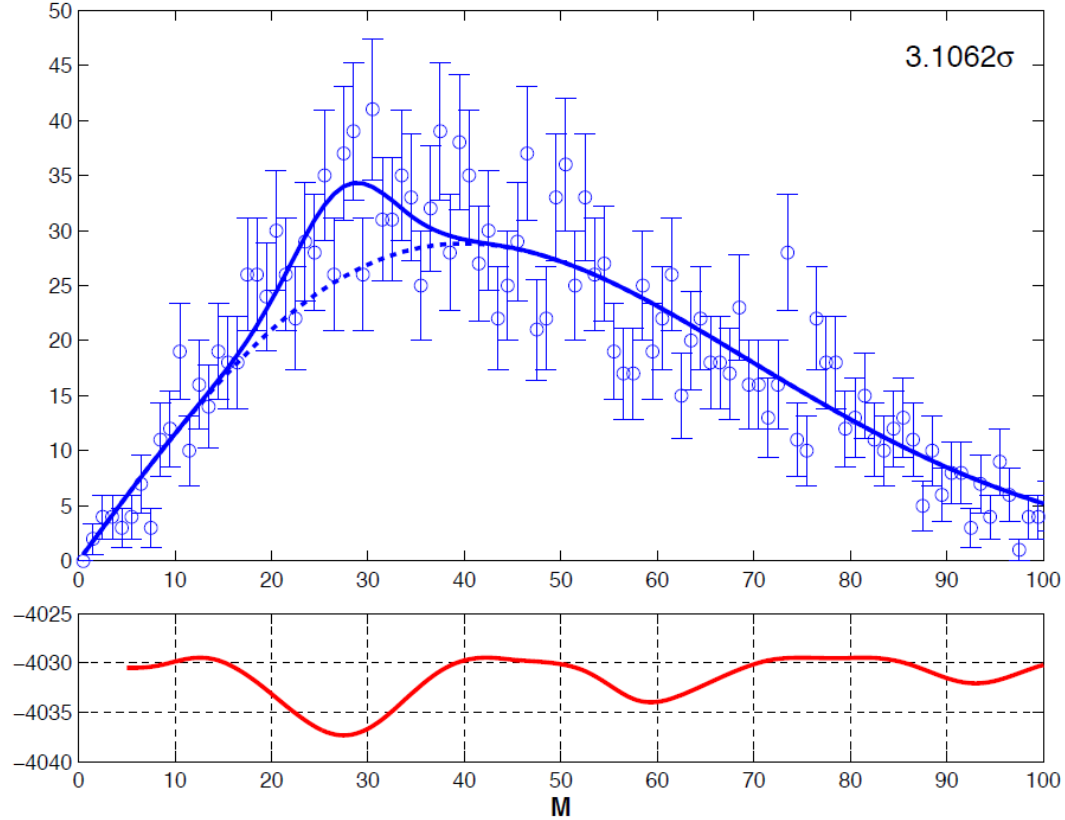


Look Elsewhere Effect: Floating Mass

- Having no idea where the signal might be you would allow the signal to be anywhere in the **search range** and use a modified test statistic

$$t_{float,obs}(\hat{\mu},\hat{m}) = -2\ln\frac{L(b)}{L(\hat{\mu}s(\hat{m})+b)}$$

- The p-value increases because more possibilities are opened



Look Elsewhere Effect

- the test statistis

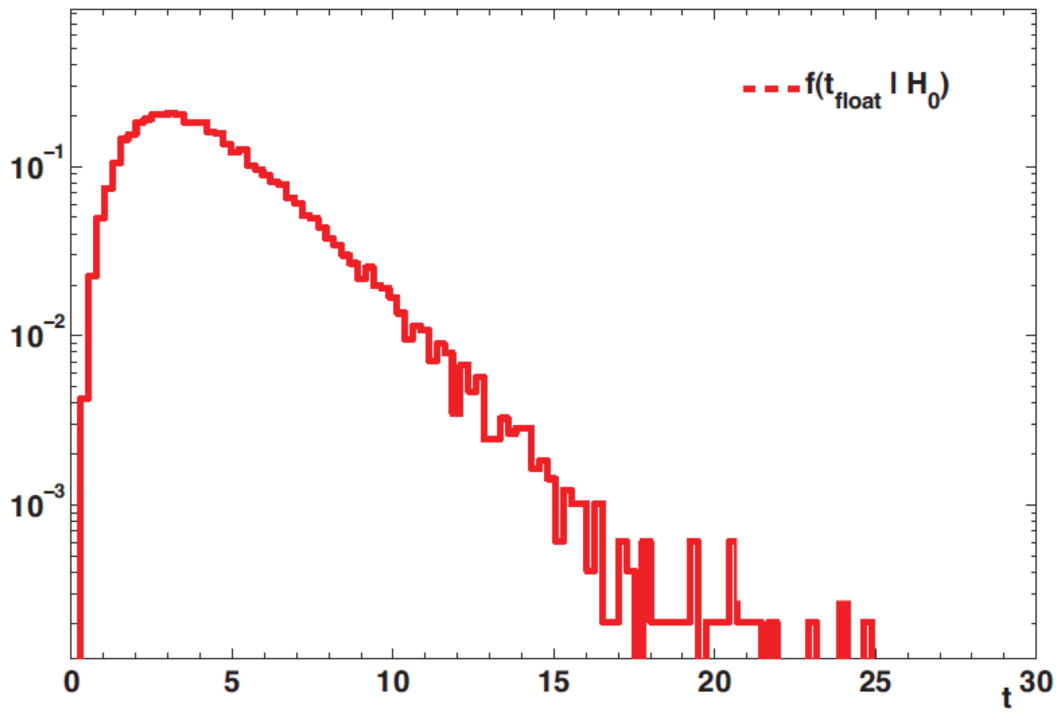
$$t_{float,\,obs}(\hat{\mu},\hat{m}) = -2\ln\frac{L(b)}{L(\hat{\mu}s(\hat{m})+b)}$$

- The null hypothesis PDF

$$f(t_{float} \mid H_0)$$

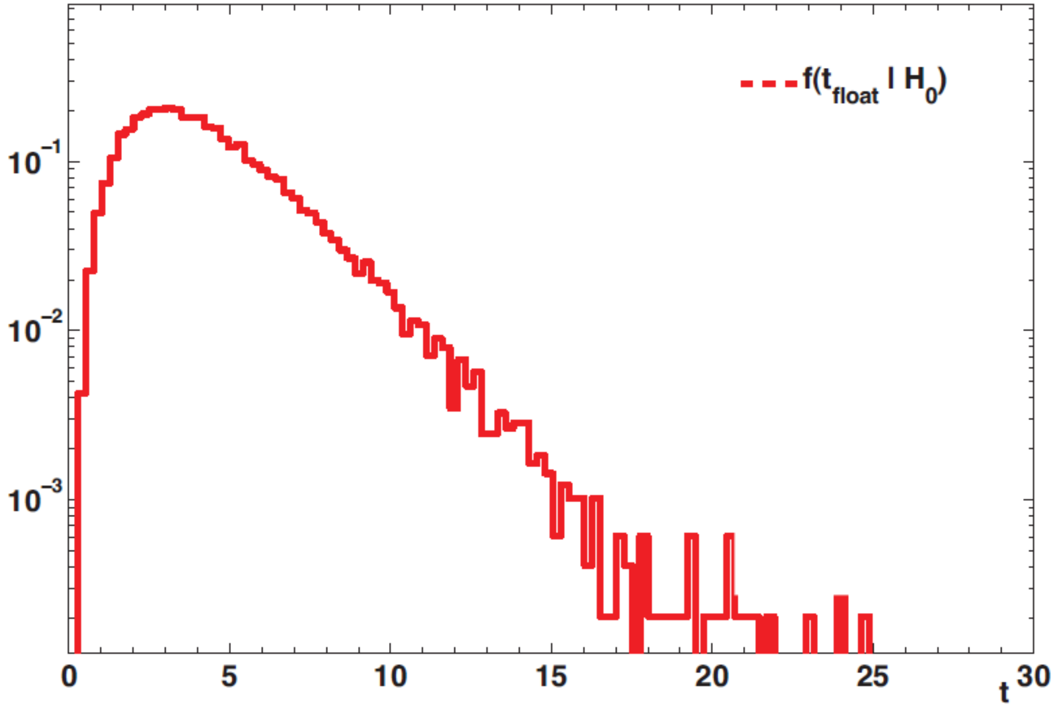
$$t_{float} = \max\left\{t_{float}^{(i)}\right\}$$

does **not** follow χ^2_2 because there are multiple minima depending on the size of the search range and resolution



Look Elsewhere Effect

- We rediscovered that by tossing χ^2_2 n times around some average <n>~# local minima, we can reproduce the distribution



Bill Quayle showed in PHYSTAT-LHC 2007 that by tossing χ^2_2 n times (with average <n>) and taking the maximum one, one can adjust <n> and reproduce the above PDF

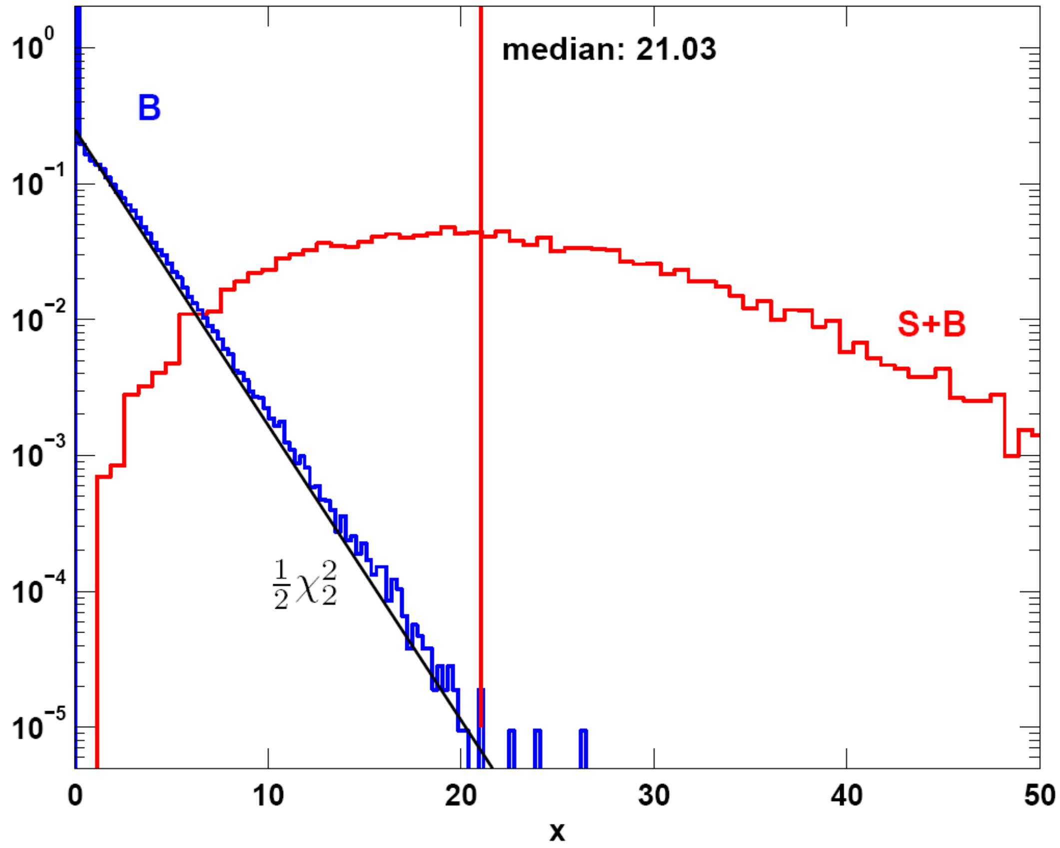
The Weakest Link or The Winning Link?

- If we define the values of the test statisitic at the local minima

$$t_{float}^{(i)},\ i = 1...N$$

$$\forall i \quad f(t_{float}^{(i)} \mid \mu = 0) \sim \chi_2^2$$

- The mass parameter is NOT defined under the null hypothesis, Wilks's theorem should not apply
- YET it works!



The Number of Local Minima

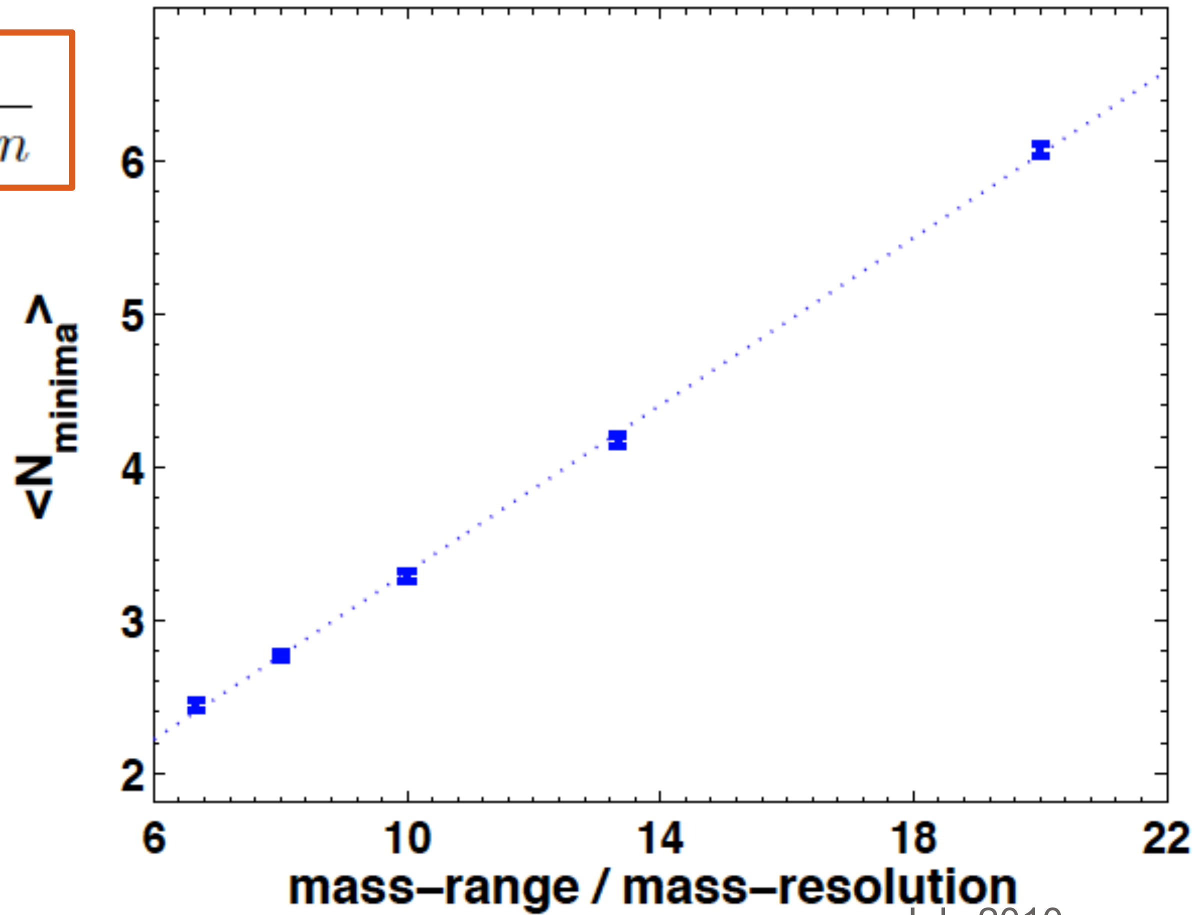
- The number of local minima depends on the famous thumb rule number

$$trial\#_{thumb} = \frac{mass\ range}{mass\ resolution}$$

- We find

$$\langle N \rangle \sim \kappa \frac{mass\ range}{\sigma_{mass}}$$

$$\kappa \approx \frac{1}{3}$$



Some Math

- If we define the values of the test statitic at the local mimima

$t^{(i)}_{float},\ i = 1 \ldots N$

and define

$$t_{float} = \max_i [t^{(i)}_{float}] \qquad P(t^{(i)}_{float} > t) = p_{\chi^2_2}$$

the p-value of the floating test statistics can be approximated by

$$p_{\chi^2_2} << 1 \Rightarrow P(t_{float} > t) \simeq p_{\chi^2_2} \langle N \rangle$$

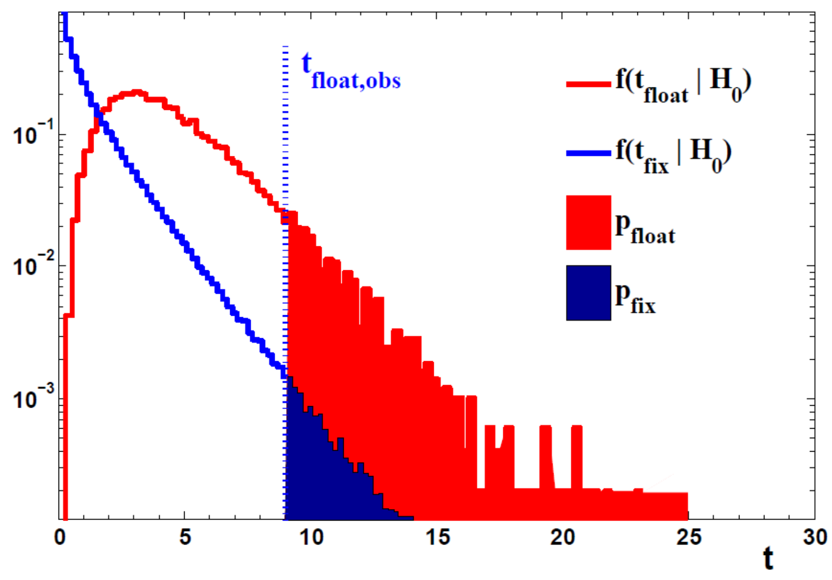
- Details of the calculation in

arXiv:1005.1891 [physics.data-an] 11 May, submitted for publication

Look Elsewhere Effect

- We can now ask the question: Assume the Higgs is observed at some mass $\hat{m}$ what is the probability for the background to fluctuate locally at the observed level (or more) $@m_H = \hat{m}$

$$t_{fix,obs} = t_{float,obs} = -2\ln\frac{L(b)}{L(\hat{\mu}s(\hat{m}=m=30)+b)}$$



- We can calculate the following p-value

$$p_{fix} = \int_{t_{obs}} f(t_{fix}|H_0)dt_{fix} < p_{float} = \int_{t_{obs}} f(t_{float}|H_0)dt_{f}$$

$$trial\,\# = \frac{\int_{t_{obs}} f(t_{float}|H_0)dt_{float}}{\int_{t_{obs}} f(t_{fix}|H_0)dt_{fix}} = \frac{p_{float}}{p_{fix}}$$

The Observed Trial Factor

$$trial\,\# = \frac{\int_{t_{obs}} f(t_{float} \mid H_0)\,dt_{float}}{\int_{t_{obs}} f(t_{fix} \mid H_0)\,dt_{fix}} = \frac{p_{float}}{p_{fix}}$$

- Using

$$p_{float} \approx p_{\chi_2^2} < N >$$

$$p_{fix} = p_{\chi_1^2}$$

$$trial\,\# \approx \frac{p_{\chi_2^2} < N >}{p_{\chi_1^2}} \approx \frac{e^{-\frac{t_{obs}}{2}} < N >}{\frac{1}{\sqrt{t_{obs}}}\sqrt{\frac{2}{\pi}}e^{-\frac{t_{obs}}{2}}} =< N > \sqrt{\frac{\pi}{2}}\sqrt{t_{obs}} \sim < N > Z_{fix}$$

- Note that

$$< N > \sim \kappa \frac{mass\ range}{mass\ resolution}$$

- The proportionality is indeed found empirically to be

$$trial\,\#_{observed} \sim \kappa \frac{mass\ range}{mass\ resolution} Z_{fix}; \quad \kappa = \frac{1}{3}$$

The Thumb Rule

$$trial\ factor = \frac{p_{float}}{p_{fix}}$$

$$trial\ factor \stackrel{?}{=} \frac{range}{resolution} = \frac{\Gamma_m}{\sigma_m}$$

In an ATLAS note by

Tatiana Cervero, Louis Fayard, Marumi Kado, Francesco Polci

they tested the LEE with $H \rightarrow \gamma\ \gamma$ signal on top of a steep falling BG with a fixed mass significance of $Z_{fix}=3$, and found an agreement with the thumb rule (trial#~28).

$$trial\#_{observed} \sim \kappa \frac{mass\ range}{mass\ resolution} Z_{fix}; \quad \kappa = \frac{1}{3}$$

Look Elsewhere Effect

$$trial\,\# = \frac{\int_{t_{obs}} f(t_{float}|H_0)dt_{float}}{\int_{t_{obs}} f(t_{fix}|H_0)dt_{fix}} = \frac{p_{float}}{p_{fix}}$$

- We find a thumb rule:

$$\Delta_m = mass-search-range$$

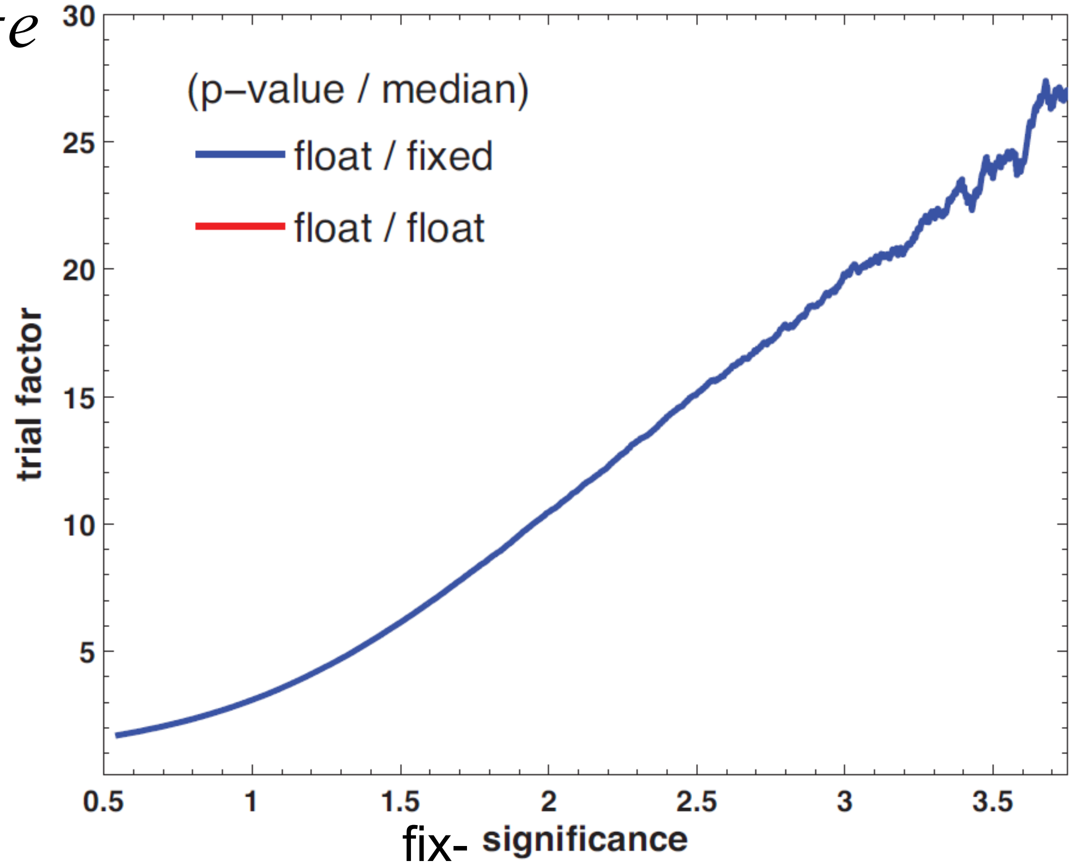
$$\sigma_m = mass-resolution$$

$$Z_{fix} = significance$$

$$trial\,\# \approx \kappa \frac{\Delta_m}{\sigma_m} Z_{fix}$$

E. Gross and O. Vitells

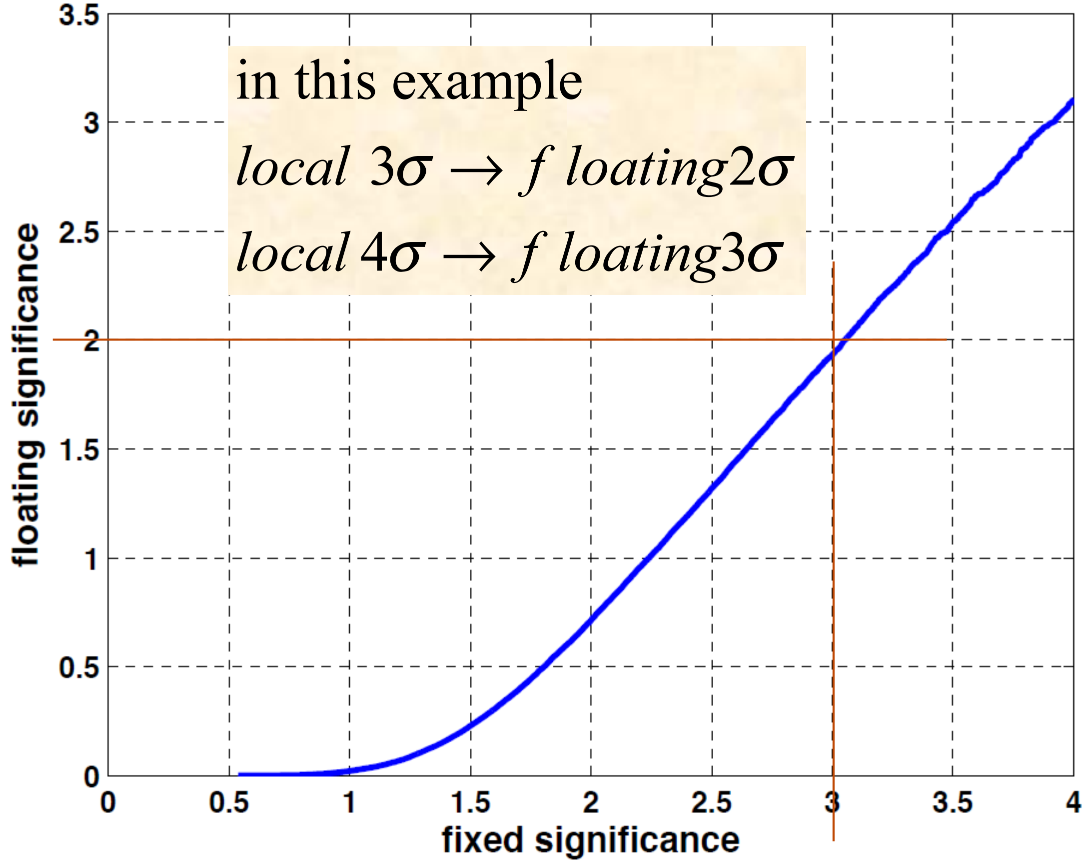
$$\kappa \approx \frac{1}{3}$$



Look Elsewhere Effect

- The Look Elsewhere Effect reduces the apparent significance

$$p_{float} = \int_{t_{float}} f(t_{float} | H_0) dt_{float}$$



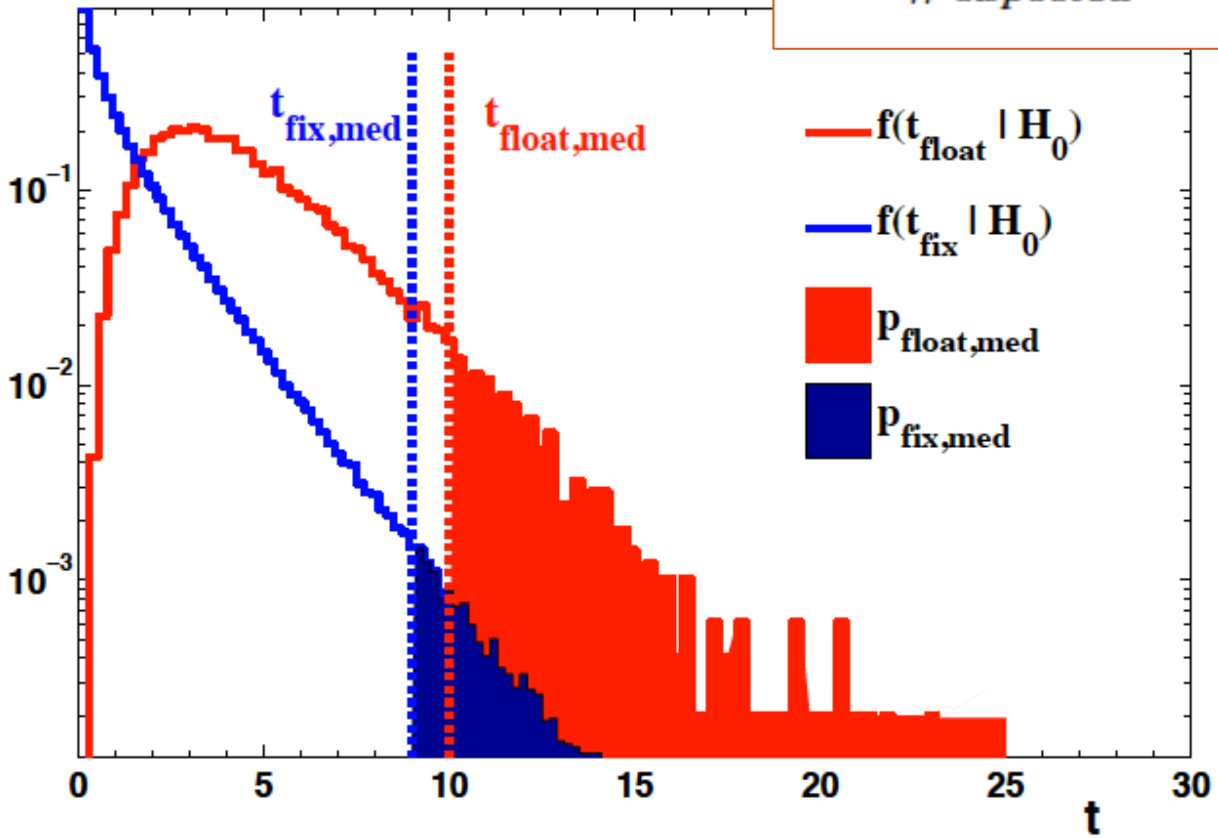
- It addresses the alternate hypothesis:

  A Higgs at some mass in the search-range

$$p_{fix} = \int_{t_{fix}} f(t_{fix} | H_0) dt_{fix}$$

The Expected Trial Factor

$$trial\#_{expected} = \frac{p_{float,med}}{p_{fix,med}}$$



We find $$trial\#_{expected} = \frac{1}{\sqrt{e}} trial\#_{observed}$$

Trial Factor Observed vs Expected
The New Thumb Rules

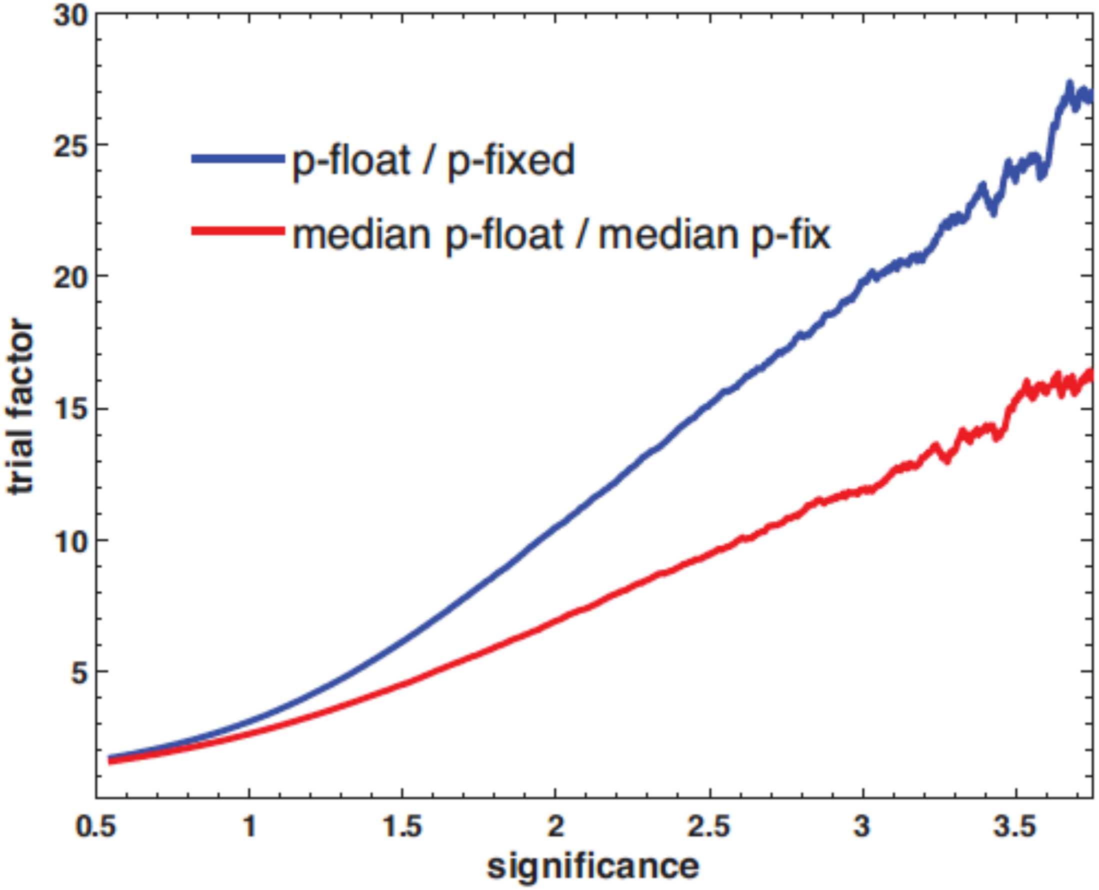
Forget the old thumb rule
Use this:

$$trial\#_{observed} \approx \kappa \frac{\Delta_m}{\sigma_m} Z_{fix}$$

$$trial\#_{expected} \approx \kappa \frac{1}{\sqrt{e}} \frac{\Delta_m}{\sigma_m} Z_{fix}$$

$$\kappa \approx \frac{1}{3}$$

- We have tested it on a CMS paper as well..



Conclusions

Based on the observation that the local t(i) distributes as a chi squared with 2 dof we have derived the following rule

$$trial\,\# \equiv \frac{p_{float}}{p_{fix}} \approx < N > Z_{fix}$$

This lead to a new thumb rule for calculating the trial#

$$trial\,\# \approx \frac{mass\ range}{mass\ resolution} Z_{fix} = \kappa \frac{\Delta_m}{\sigma_m} Z_{fix}$$

Kappa can easily be obtained via $< N >= \kappa \frac{mass\ range}{mass\ resolution}$

arXiv:1005.1891 [physics.data-an]

11 May, submitted for publication

Ms. Ref. No.: NIMA-D-10-00385

Title: Trial factors or the look elsewhere effect in high energy physics; Nuclear Inst. and Methods in Physics Research,

On thorough examination, your submission does not appear to be suitable for publication in our Journal. Although we have a section on statistical analysis, this is intended to be strictly connected to instrumentation, while your manuscript refers essentially to data analysis. It is suggested that a a more appropriate Journal for the subject covered is the Physical Review, Physics Letters or similar. Thank you for giving us the opportunity to consider your manuscript

Yours sincerely, Fabio Sauli

ditorNuclear Inst. and Methods in Physics Research, A

arXiv:1005.1891v1 [physics.data-an] 11 May 2010