Multivariable Operator Theory
August 15- August 20, 2010

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations. Note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
	Lecture rooms available after 16:00 (if desired)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall
	Beverages and a small assortment of snacks are available on a cash honor system.

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome by BIRS Station Manager, Max Bell 159
9:00-9:45	Douglas
9:45-10:15	Coffee Break, 2nd floor lounge, Corbett Hall
10:15-11:00	McCarthy
11:00-11:45	Davidson
11:30–13:00	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00	Group Photo; meet on the front steps of Corbett Hall
14:15-15:00	Ball
15:00-15:30	Coffee Break, 2nd floor lounge, Corbett Hall
15:30-16:15	Raghupati
16:15-17:00	Witt
17:00-17:45	W.-S. Li
17:45–19:30	Dinner

Tuesday

7:00–9:00	Breakfast
9:00-9:45	Popescu
9:45-10:15	Coffee Break, 2nd floor lounge, Corbett Hall
10:15-11:00	Muhly
11:00-11:45	Vinnikov
11:30–13:00	Lunch
14:15-15:00	Sundberg
15:00-15:30	Coffee Break, 2nd floor lounge, Corbett Hall
15:30-16:15	Vasilescu
16:15-17:00	Ambrozie
17:00-17:45	Dritschel
17:45–19:30	Dinner

Wednesday

7:00–9:00	Breakfast
9:00-9:45	Müller
9:45-10:15	Coffee Break, 2nd floor lounge, Corbett Hall
10:15-11:00	Shalit
11:00-11:45	Katsoulis
11:30–13:00	Lunch
Free afternoon	
17:30–19:30	Dinner

Thursday

7:00–9:00	Breakfast
9:00-9:45	Upmeier
9:45-10:15	Coffee Break, 2nd floor lounge, Corbett Hall
10:15-11:00	Englis
11:00-11:45	Yang
11:30–13:00	Lunch
14:15-15:00	Agler
15:00-15:30	Coffee Break, 2nd floor lounge, Corbett Hall
15:30-16:15	Paulsen
16:15-17:00	McCullough
17:00-17:45	Eschmeier
17:45–19:30	Dinner

Friday

7:00–9:00	Breakfast
9:00-9:45	Biswas
9:45-10:15	Coffee break
10:15-11:00	Richter
11:30–13:30	Lunch
Checkout by 12 noon.	

** 5-day workshops are welcome to use BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

ABSTRACTS
(in alphabetic order by speaker surname)

Calin Ambrozie

Remarks on truncated moments problems

Abstract: We use techniques of infinite dimensional Fenchel duality to derive conclusions on the existence of positive representing densities on the n-dimensional Euclidian space for certain finite sets of given moments.

Joseph Ball

Test functions, admissible kernels, Schur-Agler class an transfer functions: matrix-valued holomorphic functions on finitely-connected planar domains

Abstract: Dritschel-McCullough have shown how to obtain a transfer-function realization for the holomorphic mappings of a finitely-connected planar domain R into the closed unit disk (Schur class over R). By making connections with the characterization of the extreme points of the set of positive-real-part holomorphic functions on R (normalized to be 1 at some point p_0 in R), Pickering identified the Schur class over R as the Schur-Agler class associated with an explicitly identified class of test functions on R. We show how these results extend to contractive matrix-valued holomorphic functions on R. In particular, the set of extreme points for the positive-real-part matrix-valued holomorphic functions on R (normalized to be I at p_0) is much richer than in the scalar case. This picture sheds additional insight into the negative solution of the spectral-set questions for R (for the case where R has at least two holes) due to Dritschel-McCullough. This talk reports on preliminary joint work with Moisés Guerra-Huaman, also of Virginia Tech.

Ken Davidson

Nevanlinna-Pick interpolation and factorization of functionals

Abstract: I will discuss a very general framework for Nevanlinna-Pick type interpolation in subalgebras of the multiplier algebra of a reproducing kernel Hilbert space. We show that if the algebra has property $A_1(1)$, then it always has a family of kernel Hilbert spaces arising from cyclic invariant subspaces which are an NP family, meaning that they yield a family of Pick matrices which determine whether interpolation is possible. Many classical examples fit into this framework, as well as many other situations such as Bergman space.

Ronald G. Douglas

Resolutions and Quotients of Hilbert Modules

Abstract: Two complementary approaches to multivariate operator theory are, first, the study of classical concrete Hilbert modules such as those of Hardy, Bergman and Drury-Arveson, over various domains such as the unit ball and the polydisk and, second, the effort to determine the relationship between classical results for concrete Hilbert modules and various properties in the context of more general Hilbert modules. We will review some results for both approaches relating to resolutions and quotients, starting with the case of the disk and considering their possible extension to the ball and polydisk.

Miroslav Engliš

The Peter-Weyl decomposition for Toeplitz quantizations

Abstract: We describe the Peter-Weyl decomposition of the Toeplitz calculus (star product) on a bounded symmetric domain in $\mathbf{C}^\mathbf{n}$. Both the Berezin and the Berezin-Toeplitz cases are covered; the former recovers a result of Arazy and Ørsted, while the latter has not hitherto been known even in the simplest case of the unit disc. Finally, analogous decomposition is likewise obtained for the Toeplitz calculus on real symmetric domains. This is a joint work with Harald Upmeier (Marburg).

Jörg Eschmeier

Essential normality of homogeneous submodules

Abstract: Let $M \subset H(\mathbb{B})$ be a homogeneous submodule of the n-shift Hilbert module on the unit ball in $\mathbb{C}^n$. We show that a modification of an operator inequality used by Guo and Wang in the case of principal submodules is equivalent to the existence of factorizations of the form $[M_{z_j}^*, P_M] = (N+1)^{-1}A_j$, where N is the number operator on $H(\mathbb{B})$. Thus a proof of the inequality would yield positive answers to conjectures of Arveson and Douglas concerning the essential normality of homogeneous submodules of $H(\mathbb{B})$. We show that in all cases in which the conjectures have been established the inequality holds and leads to a unified proof of stronger results.

Elias Katsoulis

Semicrossed products of operator algebras and their C-envelopes*

Abstract: We generalize a recent result of Davidson and Katsoulis and we show that the c*-envelpe of the semicrossed product of the tensor algebra of a C"-correspondence X by a completely isometric automorphism α that fixes the diagonal is isomorphic to the crossed product of the Cuntz-Pimsner C*-algebra O_X by α. (co author: E. Kakariadis)

Wing Suet Li

Results and open problems related to the Horn Conjecture

Abstract: The proof of the Horn Conjecture, a characterization of self-adjoint matrices A, B, and C satisfied A+B+C=0 by the relations of their eigenvalues, achieved by Klyachko, Knutson and Tao, using sophisticated machinery from algebraic geometry and intricate combinatorics, has generated many new results in algebraic geometry and representation theory. With H. Bercovici and others, we were able to generalize the Horn Conjecture to some infinite dimensional settings, most noticeable to type II_1 factors. In this talk I will show some of our results, tools that we used, and, perhaps more interestingly many other questions to which we do not know the answer yet.

John McCarthy

Operator monotone functions of several variables

Abstract: In 1934 K. Loewner characterized functions f that are operator monotone (in the sense that if $A \leq B$ then $f(A) \leq f(B)$). I shall discuss extending Loewner's result to functions of several variables. The talk is based on joint work with Jim Agler and Nicholas Young.

P. Muhly - B. Solel

Contributions to Fully Matricial Function Theory: When a matrix of functions is a function of matrices

Fully matricial sets and functions arise quite naturally when one tries to build a complex function theory based on free algebras of various sorts. This was recognized first by J. Taylor (Adv. Math. **3** (1972). See section 6.) and has arisen anew in the work of Voiculescu on free analysis questions. He coined the terms "fully matricial sets and functions". But they are implicit in a lot of other work that has been evolving in recent years and they are becoming more and more explicitly studied. (See, in particular, the recent work by Helton, McCullough, Klepp, Putinar, Vinnikov and others on dimension free linear matrix inequalities and the work of Ball, Davidson, Katsoulis, Pitts, Popescu and others on free holomorphic functions.) In this talk we will discuss the sets and functions that arise as follows: Let E be a W^*-correspondence over a W^*-algebra M and let $H^\infty(E)$ be the Hardy algebra we build from (E, M) as described in Math. Ann. **330** (2004). Then for each normal representation σ of M on a Hilbert space H_σ there is a natural W^*-correspondence over $\sigma(M)'$, called the *σ-dual correspondence of E*, and denoted E^σ. This is a space of intertwining operators between σ and the representation induced by E in the sense of Rieffel, $\sigma^E \circ \varphi$, where φ gives the left action of M on E. The unit ball in the space of adjoints of E^σ, $\mathbf{D}(E^\sigma)^*$, is part of a fully matricial family of sets. This is because $E^{n\sigma}$ is in a very natural way $M_n(E^\sigma)$ for each positive integer n.

The importance of $\mathbf{D}(E^\sigma)^*$ lies in the fact that as σ runs over the space of normal representations of M, the points in $\mathbf{D}(E^\sigma)^*$ label (most of) the ultra-weakly continuous, completely contractive representations of $H^\infty(E)$ in $B(H_\sigma)$. For $\eta^* \in \mathbf{D}(E^\sigma)^*$, $\eta^* \times \sigma$ denotes the representation of $H^\infty(E)$ determined by η^*. Each each element $F \in H^\infty(E)$ gives rise to a function $\widehat{F}_\sigma : \mathbf{D}(E^\sigma)^* \to B(H_\sigma)$ defined by $\widehat{F}_\sigma(\eta^*) := (\eta^* \times \sigma)(F)$. It is an easy calculation to see that for each σ, the collection $\{\widehat{F}_{n\sigma}\}_{n\geq 1}$ forms a fully matricial function on the fully matricial set $\{\mathbf{D}(E^{n\sigma})^*\}_{n\geq 1}$.

When $M = \mathbf{C}$, $E = \mathbf{C}^d$ and σ is the one dimensional representation of $\mathbf{C}$, then $H^\infty(E)$ is the free semigroup algebra $\mathcal{L}_d$ and $\mathbf{D}(E^{n\sigma})^*$ is the space of row contractions of d $n \times n$ matrices. An $F \in H^\infty(E)$ has a representation in terms of a series indexed by the free semigroup on d generators, and the function $\widehat{F}_{n\sigma}$ is obtained by replacing the d generators by the d components of a row contraction. Thus $\widehat{F}_{n\sigma}$ lies in a certain completion of the algebra of d generic $n \times n$ matrices.

We address the inverse question: Given a family of functions $\{\Phi_\sigma\}$, where σ runs over all the normal representations of M and where $\Phi_\sigma : \mathbf{D}(E^\sigma)^* \to B(H_\sigma)$, when does there exist an element $F \in H^\infty(E)$ so that $\widehat{F}_\sigma = \Phi_\sigma$ for all σ. We describe two solutions. One is in terms of certain intertwining spaces and is connected to Taylor's original analysis, as well as to recent work by D. Kalyuzhnyĭ-Verbovetzkiĭ and V. Vinnikov. The second is based on our generalization of the Nevannlina-Pick Theorem and Lyapunov analysis.

We emphasize that although our results are formulated in terms of general W^*-algebras, W^*-correspondences and normal representations, they contain the situations when M and E are finite dimensional and σ is finite dimenionsional as special cases. These cases yield very interesting finite dimensional matrix balls expressed as $\mathbf{D}(E^\sigma)^*$. The functions we study lie in completions of spaces of polynomial maps studied in invariant theory.

Vladimir Müller

Joint essential numerical range

Abstract. Let T be a bounded linear operator on a complex Hilbert space H.

The numerical range $W(T) = \{\langle Tx, x\rangle : x \in H, \|x\| = 1\}$ is a useful tool in operator theory.

The essential numerical range $W_e(T)$ is defined as the set of all $\mathbf{C}$ such that there exist an orthonormal sequence $(x_n) \subset H$ such that $\langle Tx_n, x_n\rangle \to \lambda$.

Similar definitions can be also used for n-tuples of operators.

We will discuss properties of the joint essential numerical range. The following theorem will be proved.

Theorem. Let $T_1, \ldots, T_n \in B(H)$. Then there are compact operators $K_1, \ldots, K_n \in B(H)$ such that

$$W(T_1 + K_1, \ldots T_n + K_n)^- = W_e(T_1, \ldots, T_n).$$

The essential numerical range is closely related with the following problem of Olsen, which is still open:

Problem. Let $T \in B(H)$ and let p be a polynomial. Does there exists a compact operator K such that $\|p(T + K)\| = \|p(T)\|_e$?

We show that the Olsen problem has a positive answer for the polynomials $p(z) = z^k$.

Gelu Popescu

Free biholomorphic classification of noncommutative domains

Abstract: We present recent results concerning the theory of free holomorphic functions on noncommutative Reinhardt domains generated by positive regular free holomorphic functions in n noncommuting

variables. We show that the free biholomorphic classification of these domains is the same as the classification, up to unital completely isometric isomorphisms, of the corresponding noncommutative domain algebras. We also provide characterizations for the unitarily implemented isomorphisms of noncommutative Hardy algebras in terms of free biholomorphic functions between the corresponding noncommutative domains.

The most prominent feature of our presentation is the interaction between the theory of free holomorphic functions on noncommutative Reinhardt domains, the operator algebras generated by weighted creation operators on the full Fock space with n generators, and the classical complex function theory in several variables.

Mrinal Raghupati

Function theory for subalgebras of H^∞

Abstract: In the last two years there has been some progress on understanding the Pick interpolation problem for subalgebras of H^∞. The work was originally motivated by Agler and McCarthy's study of embedded disks and one-dimensional varieties.

In this talk I would like to discuss some of the results that have been obtained in the past couple of years in Nevanlinna-Pick interpolation, Toeplitz corona problems, interpolating sequences, corona problems and stable ranks for subalgebras of H^∞.

Orr Shalit

Operator algebraic geometry - classifying universal operator algebras with varieties

Abstract: Let I be homogeneous ideal of polynomials in d variables. To the ideal I one classically associates a variety V(I), consisting of all d-tuples of complex numbers that annihilate I. Since I is homogeneous, we may consider instead the "variety" Z(I), which is defined as the intersection of V(I) and the closed unit ball. Now consider the collection of all d-tuples of commuting operators that annihilate I. This "noncommutative" variety is encompassed by a certain concretely defined operator algebra A(I), which is the universal operator algebra generated by a row contraction subject to the relations in I.

In this talk I will report on our work on the classification of the algebras A(I) up to isomorphism, and up to (completely) isometric isomorphism. We find intriguing analogues to the classical correpondence between algebra and geometry. For example, if I and J are ideals determining smooth projective varieties, then A(I) and A(J) are isomorphic if and only if Z(I) and Z(J) are biholomorphically equivalent; A(I) and A(J) are isometrically isomorphic if and only if Z(I) and Z(J) are unitarilly equivalent.

Such results immediately raise the problem of when Z(I) and Z(J) are biholomorphically equivalent. We find that the complex geometry of these sets is very rigid. For example, when the corresponding projective varieties are smooth, then every biholomorphism between Z(I) and Z(J) is a linear map. When Z(I) and Z(J) are also irreducible, then this linear map must in fact be unitary. (Reporting on joint work in progress with Kenneth R. Davidson and Christopher Ramsey).

Harald Upmeier

Asymptotic Expansion of Invariant Operators on Compact Symmetric Spaces

Abstract: The geometric quantization of complex symmetric domains on Bergman spaces of holomorphic functions leads to invariant operators such as the Berezin transform linking the active and passive symbol of Toeplitz operators, and the Moyal type product representing the composition of two Bergman Toeplitz operators. In collaboration with Miroslav Englis, we have found explicit asymptotic expansions for these operators, generalizing work of Arazy-Orsted, also in the case of real symmetric spaces. This work was mainly devoted to the non-compact setting. In my talk, I will report on more recent work, also in collaboration with M. Englis, concerning invariant operators and their asymptotic expansion in the case of compact symmetric spaces such as the Riemann sphere, the projective spaces and the Grassmann manifolds. Here the Bergman type spaces have only finite dimension, and one has to consider the whole sequence of Toeplitz matrices. Also, the topology of the underlying spaces is more involved.

F. -H. Vasilescu

A Stability Equation for Truncated Moment Problems

Abstract: An abstract quadric equation can be used to characterize locally the flatness of the truncated moment problems, both in commutative and non-commutative cases.

Rongwei Yang

Operator model theory revisited

Abstract: In the vector-valued Hardy space $H^2(E)$, a subspace M is said to be invariant if it is invariant under T_z — multiplication by z. Compression of T_z to the space $H^2(E)/M$, up to unitary equivalence and a scalar multiple, represents every bounded linear operator on a separable comple Hilbert space. When $E = H^2(D)$, $H^2(E)$ is the Hardy space over the bidisk $H^2(D^2)$. This setting give rises to more explicit descriptions of some key elements in operator model theory such as defect operator and characteristic function. More importantly, the two variable nature of this setting adds new ingredients to this theory.