Schedule – Workshop on Branching Random Walk and Searching in Trees

Jan 31-Feb 5, 2009

	Monday	**Tuesday**	**Wednesday**
9:15-9:30	Welcome from BIRS		
9:30-10:15	Andreas Kyprianou	Serguei Popov	Michael Drmota
10:15-10:45	**Coffee break**	**Coffee break**	**Coffee break**
10:45-11:30	Elie Aidekon	Gerold Alsmeyer	Matthias Meiners
11:30-12:15	Sebastien Müller	Bénédicte Haas	John Biggins
12:15-2:00	**Lunch Break**	**Lunch Break**	**Lunch Break**
2:00-2:15	Open problem session	Group Photo	**Free afternoon**
2:15-3:00	Open problem session contd.	Yueyun Hu	
3:00-3:30	**Coffee break**	**Coffee break**	
3:30-4:15	Open problem/Working session	Working session	
4:15-5:00	Working session	Working session	

	Thursday	**Friday**
9:30-10:15	Matthias Winkel	Working Session
10:15-10:45	**Coffee break**	**Coffee break**
10:45-11:30	Nina Gantert	Working Session
11:30-12:15	Ralph Neininger	Working Session
12:15-2:00	**Lunch Break**	**Lunch Break**
2:00-3:00	Working Session	
3:00-3:30	**Coffee break**	Departure
3:30-5:00	Working session	

Abstracts

Serguei Popov
Shape and local growth for multidimensional branching random walks in random environment

We study branching random walks in random environment on the d-dimensional square lattice, $d \geq 1$. In this model, the environment has finite range dependence, and the population size cannot decrease. We prove limit theorems (laws of large numbers) for the set of lattice sites which are visited up to a large time as well as for the local size of the population. The limiting shape of this set is compact and convex, and the local size is given by a concave growth exponent. Also, we obtain the law of large numbers for the logarithm of the total number of particles in the process.

Matthias Meiners Fixed points of min- and smoothing transforms

In this talk, we study the fixed-point equation of the smoothing transform ($X \sim \sum T_i X_i$) where the X_i are iid with $X_i \sim X$ and $(X_i)_i$ and $(T_i)_i$ are independent) and a related min-type equation with applications in the probabilistic analysis of algorithms. We provide a full description of the sets of solutions to both equations under fairly mild assumptions on the underlying weight sequence $(T_i)_i$.

Joint work with Gerold Alsmeyer.

Sebastien Müller
On the trace of branching random walk

We prove that the trace of a transient branching random walk on a unimodular random graph is a.s. transient for simple random walk, has a.s. critical percolation probability less than one, and exponential volume growth. Examples for unimodular random graphs are in particular Cayley graphs and (quasi-) transitive graphs. (The trace is the subgraph that consists of all vertices and edges that were visited by a branching random walk).

The proof relies on the fact that the trace induces an invariant percolation on the family tree of the branching random walk. Furthermore, we prove that the trace is a.s. strongly recurrent for any branching random walk. This follows from the observation that the trace after appropriate biasing of the root is a unimodular random network. (joint work with I. Benjamini)

Andreas Kyprianou
Travelling waves for fragmentation processes

We formulate the analogue of the classical KPP equation for homogeneous fragmentation processes (a parabolic integro-differential equation) and study its bounded travelling waves. This involves looking at stopping lines for fragmentation processes together with some new results on strong laws of large numbers which are closely related to renewal theory of subordinators. This is based on joint work with Julien Berestycki (Paris VI) and Simon Harris (Bath).

Gerold Alsmeyer
Perpetuities and branching random walk

Perpetuities naturally arise when describing the stationary limit of iterations of random affine maps, but also in other contexts. An interesting connection with branching random walks emerges when studying the latter model under a size-biased measure. Based on recent work in collaboration with Alexander Iksanov (Kiev) und Uwe RŽsler (Kiel), the major purpose of this talk is to present a number of moment results for perpetuities and to explain how these and the afore-mentioned connection may be exploited to derive moment results for the branching random walk.

Bénédicte Haas
Scaling limits of Markov branching trees

A Markov branching family with a prescribed number of leaves is a sequence $(T_n, n \geq 1)$ of discrete rooted trees with n leaves such that, conditional on the fact that the root of T_n branches in k subtrees with, respectively, $n_1, ..., n_k$ leaves, these subtrees are independently distributed, with respective laws that of $T_{n_1}, ..., T_{n_k}$. We consider similarly Markov branching trees with a prescribed number of vertices. Our main result is that under a natural hypothesis, the Markov branching family $(T_n, n \geq 1)$ converges, correctly rescaled, to a continuous limiting tree. Among other examples, this allows us to describe the asymptotic behavior of Pólya trees (that are uniformly distributed among the set of un-ordered, un-labelled rooted trees with n vertices) and to recover the classical results on the continuous limits of Galton-Watson trees.

This is joint work with Grégory Miermont.

Matthias Winkel
Growth of Galton-Watson forests

We study certain consistent families $(F_\lambda)_{\lambda\geq 0}$ of Galton-Watson forests with lifetimes as edge lengths and/or immigrants as progenitors of the trees in F_λ. Various schemes for such consistent growth have been studied. We will focus here on consistency under Bernoulli leaf percolation, where F_μ has the same distribution as the subforest of F_λ spanned by blue leaves, where each leaf of F_λ is coloured in blue independently with probability μ/λ. Applications include limit theorems, descriptions of the genealogy of continuous-state branching processes with and without immigration, decompositions of supercritical trees. By including a spatial motion, the approach generalises to branching random walks and superprocesses. This is joint work, partly with Thomas Duquesne and partly with my student John Cao.

Nina Gantert
Nearly optimal paths in a branching random walk

Consider a discrete-time one-dimensional supercritical branching random walk. We study the probability that there exists an infinite ray in the branching random walk that always lies above the line of slope $\gamma-\varepsilon$, where γ denotes the asymptotic speed of the right-most position in the branching random walk. Under mild general assumptions upon the distribution of the branching random walk, we prove that when $\varepsilon\to 0$, the probability above decays like $\exp\{-\frac{\beta+o(1)}{\varepsilon^{1/2}}\}$, where β is a positive constant depending on the distribution of the branching random walk. In the special case of i.i.d. Bernoulli(p) random variables (with $0<p<\frac{1}{2}$) assigned on a rooted binary tree, this proves a conjecture of Robin Pemantle.

Based on joint work with Yueyun Hu and Zhan Shi

Michael Drmota
The binary-tree search equation

The purpose of this talk is to discuss the differential equation $\Phi'(u)=-1/\alpha^2\Phi(u/\alpha)^2$ which is fundamental in the asymptotic study of binary search trees. It's solution is closely related to the height and the profile distribution of BST's.

Let $\Psi(x)$ denote the inverse Laplace transform of $\Phi(u)$. Then (for a certain value α) this is the limiting tail distribution function of the properly shifted and sampled height distribution (or equalently the limiting tail distribution function of the smallest element of the related branching random walk). Furthermore, if $M(z)$ is the solution of the stochastic fixed point equation $M(z)=zU^{2z-1}M^{(1)}(z)+z(1-U)^{2z-1}M^{(2)}(z)$ and also the limiting prozess of the normalized profile $X_{n,2z\log n}/EX_{n,2z\log n}$. then $\Psi(x)$ equals (up to scaling) the Laplace transform of $M(z)$, where $\alpha=z^{1/(2z-1)}$.

Elie Aidekon
Survival probabilities for the killed branching random walk

We study the branching random walk in dimension 1 where particles which go below zero are killed and do not reproduce. In the subcritical and critical cases, the system dies out almost surely and we look at asymptotics of the survival probability at time n when n goes to infinity.

Joint work with B. Jaffuel

John Biggins

Anomalous spreading in the branching random walk.

Weinberger et al (2007) noticed that in certain multitype deterministic population models it is possible for types to 'work cooperatively' to produce a speed of spread that is faster than any single type could attain on its own even, when the type set is reducible. For the branching random walk it is possible to pin down this super-speed (which turns out to be an upper bound identified for the deterministic models). In the talk I will aim to set the scene, give an impression of how the branching random walk result arises and perhaps mention its connections with aspects of the deterministic theory (including certain coupled reaction-diffusion equations).

Yueyun Hu
Random walk in random environment on trees

This talk is based on a joint work with Gabriel Faraud (University Paris 13) and Zhan Shi (University Paris 6). We consider a class of recurrent random walks in random environments on trees. It turns out that the asymptotic behaviors of the walks are closely related to some path properties of a class of branching random walks.

Andreas Kyprianou
Travelling waves for fragmentation processes

We formulate the analogue of the classical KPP equation for homogeneous fragmentation processes (a parabolic integro-differential equation) and study its bounded travelling waves. This involves looking at stopping lines for fragmentation processes together with some new results on strong laws of large numbers which are closely related to renewal theory of subordinators. This is based on joint work with Julien Berestycki (Paris VI) and Simon Harris (Bath).

Ralph Neininger
Search tree methods applied to Polya urns.

Abstract: In the analysis of quantities of search trees such as size, path length, depth, etc., probability metrics have been used to exploit distributional recursive decompositions of search trees. In this approach, the so called "Contraction Method", a contraction on an underlying space of probability measures is crucial for the analysis. In the talk I will try to explain how these ideas can be used in the analysis of Polya urns.