Banff International Research Station

for Mathematical Innovation and Discovery

Number Theory and Physics at the Crossroads
May 8–13, 2011

ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **M. Ballard** (UPenn, Math.)
Title: **Matrix factorization categories for complete intersections with applications to Orlov spectra of triangulated categories**
Abstract: We provide a description of the category of singularities of a (graded) complete intersection in terms of a category that is a natural extension of category of matrix factorizations. This description utilizes work of L. Positselski on derived categories of curved differential graded algebras. In the style of T. Dyckerhoff, we give an analogous description of the dg-category of functors between the category of singularities of two (graded) complete intersection singularities. This allows us to compute the Hochschild cohomology of such categories. We also use these descriptions to prove that the Rouquier dimensions of the derived categories of a coherent sheaves on the self-product of the Fermat elliptic curve and on a closely associated K3 surface are two. This verifies a conjecture of Orlov in these cases. This is joint work with D. Favero and L. Katzarkov.

Speaker:**F. Brown** (Jussieu, Math.)
Title: **Modular forms in quantum field theory**
Abstract: In perturbative Quantum Field theory, physical predictions are obtained by computing the Feynman integrals associated to the graphs in the theory. These integrals are periods of the complement of a certain hypersurface associated to each graph, and, at least in low orders, are expressible in terms of the Riemann zeta function.

In the first part of the talk I will give an overview of recent work relating Feynman amplitudes to the theory of motives, and in the second part I will report on joint work with Oliver Schnetz in which we find a range of examples of graph hypersurfaces which are modular. This unexpected discovery disproves a certain number of conjectures about the arithmetic nature of graph hypersurfaces.

Speaker: **M. Bogner** (Mainz, Math.)
Title : **Symplectically rigid monodromy tuples induced by fourth order differential Calabi-Yau operators**
Abstract: We classify all $Sp_4(\mathbb{C})$-rigid, quasi-unipotent monodromy tuples having a maximally unipotent element and show that all of them can be constructed via tensor- and Hadamard-products of rank one tuples. Furthermore, we translate those constructions to the level of differential operators and investigate whether such a monodromy tuple is induced by a fourth order differential Calabi-Yau operator. We also obtain closed formulae for special solutions of those operators. This is joint work with Stefan Reiter.

Speaker: **V. Bouchard** (Alberta, Math.)
Title: **The geometry of mirror curves**
Abstract: According to the "remodeling conjecture", the generating functions of Gromov-Witten invariants of toric Calabi-Yau threefolds are fully determined in terms of a topological recursion. At the root of the

recursion is the geometry of the corresponding mirror curves. In this talk I will describe the geometry of mirror curves and the remodeling conjecture, focusing on the fate of "constant terms". In particular, I will explain how the "pair of pants" decomposition of mirror curves plays a role in the topological recursion, in mirror analogy to the topological vertex formalism on the Gromov-Witten side. This is joint work with Piotr Sulkowski.

Speaker: **A. Clingher** (Missouri–St. Louis, Math.)
Title: **On a Family of K3 Surfaces of Picard Rank 16**
Abstract: I will report on a classification for the K3 surfaces polarized by the lattice $H+E7+E7$. In terms of periods, the moduli space of these objects is a quotient of a four-dimensional bounded symmetric domain of type IV. Explicit normal forms will be presented, as well as a discussion of modular forms associated to this family.

Speaker: **S. Galkin** (IMPU, Math.)
Title: **Fano and Mathieu**
Abstract: There is a correspondence between G-Fano threefolds and conjugacy classes in Mathieu group M_{24}. Construction of cusp-forms from conjugacy classes in Mathieu group is well-known. It is less known that A-model on G-Fano threefolds also naturally produce modular forms. Why these two lists of modular forms are so similar is yet another moonshine.

Speaker: **T. Gannon** (Alberta, Math.)
Title : **Vector-valued automorphic forms and the Riemann-Hilbert problem**
Abstract: In this talk I'll sketch the basic theory of vector-valued automorphic forms for arbitrary finite-index subgroups of any genus-zero Fuchsian group of the first kind, and arbitrary representation and arbitrary weight. I'll describe the analogues here of Grothendieck's Theorem, Riemann-Roch, Serre duality, etc and show they can be sharpened into effective tools (e.g. for finding dimensions and basis vectors). A crucial role is played by Fuchsian differential equations. I'll focus on the most familiar case of $SL(2,Z)$, where there are plenty of direct applications to physics, geometry and algebra. This is joint work with Peter Bantay.

Speaker: **P. Gunnells** (UMass, Math.)
Title: **Metaplectic Whittaker functions and lattice models**
Abstract: Whittaker functions are special functions on algebraic groups that play an important role in number theory and representation theory. Just as the usual exponential function is the basic ingredient for Fourier expansions, Whittaker functions provide the special functions needed to do nonabelian harmonic analysis in the theory of automorphic forms.

In this talk we will discuss the structure of spherical Whittaker functions on finite covers of $GL(n)$ over p-adic fields (i.e. metaplectic groups). We will show how such functions are related to certain two-dimensional lattice models from statistical physics. In particular, we will show that metaplectic Whittaker functions can be described using partition functions attached to six-vertex lattice models. We will define and give examples of all the relevant objects.

This is joint work with Ben Brubaker, Dan Bump, Gautam Chinta, and Sol Friedberg.

Speaker: **S. Hosono** (Tokyo, Math.)
Title: **Mirror symmetry and projective geometry of Reye congruences**
Abstract: A line congruence is a congruence of lines given by a variety in Grassmannian $G(2,n+1)$. Reye congruence is a line congruence defined by a linear system of quadrics on $\mathbf{P}^n$, and for $n=3$ it's relation to Enriques surfaces is a well-studied subject in projective geometry.

In this talk, we will consider the Reye congruence for $n=4$, where we naturally come to a Calabi-Yau three fold X, (called generalized Reye congruence in [Oliva,1994]). We make a sutable mirror family to X by Batyrev-Borisov construction supplemented by a $\mathbf{Z}_2$ quotient. We will then observe that there appear

two different large complex structure limits in the complex structure moduli space. We identify one of them with the mirror to the Reye congruence X and for the other we find a new Calabi-Yau threefold Y, which we construct as the double cover of a determinantal quintic in $\mathbf{P}^4$ branched over a curve of genus 26 and degree 20. By using mirror symmetry, we will calculate the BPS numbers for both X and Y. It is observed that some of them have nice explanations as the numbers of curves on X and Y. It is conjectured that X and Y are derived equivalent although they are not biratinal. We announce a proof of this fact briefly, referring to a paper which will appear soon.

References:

[1] S. Hosono and H. Takagi, "Mirror symmetry and projective geometry of Reye congruences I", arXiv:1101.2746v1[mathAG].

[2] S.Hosono and H. Takagi, "Mirror symmetry and projective geometry of Reye congruences II – Derived equivalence —"

Speaker: **M. Kerr** (Washington University St. Louis, Math.)

Title: **On isotrivial families of $K3$ surfaces**

Abstract: We describe an explicit construction of K3-fibered Calabi-Yau threefolds, together with their period mappings into appropriate Mumford-Tate domains. This is joint work with A. Clingher and C. Doran, and based on their modular families of M-polarized K3 surfaces. Part of the talk will review their construction as well as the analogous story for elliptically-fibered K3's.

Speaker: **A. Klemm** (Bonn, Physics)

Title : **Omega backgrounds and generalized holomorphic anomaly equation**

Abstract: We derive an anomaly equation which incorporates the general Omega background in the B-model. We discuss applications to topological string thory on Calabi-Yau backgrounds and $N = 2$ gauge theory with (massive) flavors. Using geometric engineering on the Enriques Calabi-Yau we derive Seiberg-Witten curves for the conformal cases, which are comptabible with Nekrasovs partition function.

Speaker: **J. Manshot** (CEA, Physics)

Title: **The Betti numbers of moduli spaces of sheaves on the projective plane**

Abstract: Electric-magnetic duality of gauge theory implies modular properties for generating functions of invariants of moduli spaces of sheaves. I'll explain the computation of the generating functions of Betti numbers of moduli spaces of sheaves with rank $1, 2$ and 3 on the projective plane in terms of indefinite theta functions. The main ingredients are wall-crossing and the blow-up formula.

Speaker: **D. Morrison** (UC Santa Barbara, Math. & Physics)

Title: **K3 surfaces, modular forms, and non-geometric heterotic compactifications**

Abstract: Type IIB string theory has an $SL(2, Z)$ symmetry and a complex scalar field tau valued in the upper half plane, on which $SL(2, Z)$ acts by fractional linear transformations; this naturally suggests building models in which tau is allowed to vary. Although the $SL(2, Z)$-invariant function $j(\tau)$ can reveal some of the structures of these models, for their full construction and study we need $SL(2, Z)$ modular forms, particularly the Eisenstein series $E_4(\tau)$ and $E_6(\tau)$ and the corresponding Weierstrass equations. The Weierstrass equations can also be analyzed in algebraic geometry via the theory of elliptic curves. This approach leads to the "F-theory" compactifications of type IIB theory.

Similarly, the heterotic string compactified on T^2 has a large discrete symmetry group $SO(2, 18; Z)$, which acts on the scalars in the theory in a natural way; there have been a number of attempts to construct models in which these scalars are allowed to vary by using $SO(2, 18; Z)$-invariant functions. In our new work, we give (in principle) a more complete construction of these models, using $SO(2, 18; Z)$-modular forms analogous to the Eisenstein series. In practice, we restrict to special cases in which either there are no Wilson lines – and $SO(2, 2; Z)$ symmetry – or there is a single Wilson line – and $SO(2, 3; Z)$ symmetry. In those cases, the modular forms can be analyzed in detail and there turns out to be a precise theory of K3 surfaces with prescribed singularities which corresponds to the structure of the modular forms. Using

these two approaches – modular forms on the one hand, and the algebraic geometry of the K3 surfaces on the other hand – we can construct non-geometric compactifications of the heterotic theory.

This is a report on two joint projects: one with McOrist and Sethi and the other with Malmendier.

Speaker: **H. Movasati** (IMPA, Math.)
Title: **Mirror quintic Calabi-Yau modular forms**
Abstract: In this talk we first reintroduce the classical (quasi) modular forms using algebraic de Rham cohomology of elliptic curves and the corresponding Gauss-Manin connections. We then apply the same ideas to the one parameter family of mirror quintic Calabi-Yau threefolds and we get a new (quasi) modular form theory generated by seven series algebraically independent over the field of complex numbers. The modular group is the monodromy group of such a family and it is generated by two explicit matrices in the four dimensional symplectic group with integer coefficients. The automorphy factor in this case has image inside an algebraic group of dimension six which is generated by two multiplicative and four additive subgroups. We present the functional equation of such (quasi) modular forms, however, we emphasis that the characterization of such functions in the algebraic geometric context and through polynomial ordinary differential equations is much more convenient for calculations. At the end we present some conjectures following some similar statements in the case of elliptic curves and classical modular forms. The talk is based on the following articles which can be found in my homepage:

(1) Quasi-modular forms attached to elliptic curves, I, Lecture notes at Besse, France 2010
(2) Eisenstein type series for Calabi-Yau varieties , Nuclear Physics B, 847, 2011, 460-484.
(3) Quasi-modular forms attached to Hodge structures, Preprint.

Speaker: **M. Mulase** (UC Davis, Math.)
Title: **A topological recursion in B-model as the Laplace transform of a combinatorial equation**
Abstract: The topological recursion formula discovered by Eynard and Orantin in random matrix theory has been applied to Gromov-Witten theory by string theorists (Bouchard, Klemm, Marino, and Pasquetti), and has produced an effective conjectural formula that calculates both open and closed Gromov-Witten invariants of toric Calabi-Yau threefolds. Recently some special cases of this conjecture have been solved by mathematicians. In this talk, the key idea of these mathematical work, the Laplace transform playing the role of the mirror symmetry transformation, will be explained. This talk is based on my joint papers with Chapman, Eynard, Penkava, Safnuk, and Zhang.

Speaker: **B. Pioline** (LPTHE, Jussieu, Physics)
Title: **Automorphy in hypermultiplet moduli spaces**
Abstract: The hypermultiplet moduli space in type II string theories compactified on a Calabi-Yau threefold provides a unifying framework for Gromov-Witten invariants (worldsheet instantons), Donaldson-Thomas invariants (D-instantons) with a new type of invariants (NS5-instantons). String dualities require that this moduli space should be invariant under $SL(2,Z)$, or larger arithmetic groups obtained by combining $SL(2,Z)$ with monodromies and large gauge transformations. I will review recent progress in understanding quantum corrections to the perturbative moduli space metric consistently with these automorphic symmetries.

Speaker: **J. C. Rohde** (Hamburg, Math.)
Title: **Maximal automorphisms of Calabi-Yau manifolds versus maximally unipotent monodromy**
Abstract: Let X denote a Calabi-Yau 3-manifold. Moreover let p denote the period map of the F^2 bundle in the variation of Hodge structures of weight 3 of the local universal deformation of X. There are examples of Calabi-Yau 3-manifolds X satisfying that p is constant. In the case of these examples X cannot be a fiber of a maximal family of Calabi-Yau 3-manifolds with maximally unipotent monodromy. This contradicts the assumptions of a classical formulation of the mirror symmetry conjecture. Almost all known examples

of this kind arise from the observation that the F^2 bundle is an eigenspace of the non-trivial action of an automorphism of order 3 or 4 of the local universal deformation over its base space. Moreover the associated period domain is a complex ball containing a dense set of complex multiplication points in all known examples of this kind.

Speaker: **A. Sebbar** (Ottawa, Math.)
Title: **On the critical points of modular forms**
Abstract: In this talk, we study the critical points of modular forms. In particular, we prove that for each modular form f for a discrete group G, its derivative f' has infinitely many non-equivalent zeros, and all, but a finite number, are simple. Applications will also be provided.

Speaker: **R. Song** (Harvard, Math.)
Title: **The Picard-Fuchs systems of Calabi-Yau complete intersections in partial flag varieties**
Abstract: We introduce a system of differential equations associated to a smooth algebraic variety X with the action of a complex Lie group G and an ample G-linearized line bundle L on X. Assuming G acts on X with finitely many orbits, we show that this system is holonomic (in particular, its solutions form a locally constant sheaf of finite rank over a Zariski open dense subset). This construction recovers the GKZ systems when X is a toric variety. When $G = SL_n$, $X = G/P$ where P is a parabolic subgroup of G and $L = K_X^{-1}$, we get a holonomic system of differential equations to which period integrals on Calabi-Yau hypersurfaces in X are solutions. This can also be generalized to the case of Calabi-Yau complete intersections in X.

Speaker: **J. Stienstra** (Utrecht, Math.)
Title: **Dimer models and hypergeometric systems**
Abstract: This talk will be an updated review of the relation between dimer models and hypergeometric systems. First the definition of a dimer model will be given and illustrated with nice pictures. Then it will be shown how these pictures contain the equations for toric compactifications. Finally there will be comments on what this might tell about hypergeometric systems.

Speaker: **B. Szendroi** (Oxford, Math.)
Title: **Motivic DT theory of some local Calabi-Yau threefolds**
Abstract: Donaldson-Thomas theory is the enumerative theory of sheaves on Calabi-Yau threefolds, or more generally objects in CY3 categories. Work of Nekrasov and Hollowood-Iqbal-Kozcaz-Vafa suggested a q-refinement of this theory. This was realized mathematically using motivic invariants by Kontsevich-Soibelman and Behrend-Bryan-Szendroi, with closely related work done also by Dimofte-Gukov.

We aim to explain this theory, and a recent computation of the motivic invariants on the resolved conifold geometry, in all chambers of the space of stability conditions. This is joint work with Andrew Morrison, Sergey Mozgovoy and Kentaro Nagao.

Speaker: **R. Rodriguez-Villegas** (Texas–Austin, Math.)
Title: **The A-polynomial at $q = 1$, the dilogarithm and the asymptotics of q-series**
Abstract: I will discuss an approach to the value at $q = 1$ of the A-polynomial of a general quiver. (This polynomial counts the number of absolutely irreducible representations of the quiver over F_q.) The truncations of a formula of Hua for these polynomials yield q-series of the form

$$\sum_m q^{Q(m)} x^m / (q)_m$$

where Q is a quadratic form and m runs over a lattice. The asymptotics of this series as q approaches 1 is then related to a truncated form (conjecturally polynomial) of the A-polynomial at $q = 1$.

Speaker: **J. Walcher** (McGill, Math. & Physics)
Title: **New Normal Functions for Calabi-Yau Threefolds**

Abstract: The expansion of normal functions associated with families of algebraic cycles on Calabi-Yau threefolds around the large complex structure point has been given enumerative interpretation via mirror symmetry. This is an update on the most recent calculations, which exhibit several new interesting features.

Speaker: **U. Whitcher** (Harvey Mudd, Math.)
Title: **K3 Surfaces with S_4 Symmetry**
Abstract: Hypersurfaces in toric varieties offer a rich source of examples of K3 surfaces and Calabi-Yau varieties. We use a toric residue map to study variation of complex structure for families of K3 hypersurfaces with a high degree of symmetry. This talk describes joint work with Dagan Karp, Jacob Lewis, and two Harvey Mudd College undergraduates, Daniel Moore and Dmitri Skjorshammer.