Titles and abstracts

Joakim Arnlind: Poisson algebraic geometry of Kähler submanifolds

We show that the differential geometry of almost Kähler submanifolds can be formulated in terms of the Poisson structure induced by the inverse of the Kähler form. More precisely, the submanifold relations, such as Gauss' and Weingarten's equations, (as well as many other objects) can be expressed as Poisson brackets of the embedding coordinates. It is then natural to ask the following question: Are there abstract Poisson algebras for which such equations hold? We answer this question by introducing Kähler-Poisson algebras and show that an affine connection can be defined, fulfilling all the desired symmetries and relations, e.g. the Bianchi identities. Furthermore, as an illustration of the new concepts we derive algebraic versions of some well known theorems in differential geometry. In particular, we prove that Schur's lemma holds and that a lower bound on the Ricci curvature induces a bound on the eigenvalues of the Laplace operator.

Eric Bahuaud: Ricci flow of smooth asymptotically hyperbolic metrics

In this paper we prove that given a smoothly conformally compact metric there is a short-time solution to the Ricci flow that remains smoothly conformally compact. We apply recent results of Schn\"urer, Schulze and Simon to prove a stability result for conformally compact metrics sufficiently close to the hyperbolic metric.

Gérard Besson: Natural maps, differentiable rigidity, Ricci and scalar curvature

Maria Buzano: Homogeneous Ricci flow

We will consider the Ricci flow equation on compact homogeneous spaces G/K, with G a compact, connected Lie group and K a closed, connected Lie subgroup of G. We will consider the case in which there exists an intermediate Lie group H such that G>H>K. We will also assume that H/K is isotropy irreducible and all G-invariant Riemannian metrics on G/K are obtained from a fixed Riemannian submersion H/K to G/K to G/H, by rescaling the metrics on the fibre and on the base. We will show that the flow always develops a type I singularity in finite time and that, as we approach the singular time, the fibre H/K shrinks to a point and G/K converges in the Hausdorff-Gromov topology to G/H.

Gerhard Dziuk: Computation of geometric flows

Robert Haslhofer: A mass-decreasing flow in dimension three.

Abstract: After reviewing the relationship between Perelman's energy-functional, the stability of Ricci-flat spaces, and the ADM-mass from general relativity, I will introduce a mass-decreasing flow for asymptotically flat three-manifolds with nonnegative scalar curvature. This flow has a number of nice properties, in particular wormholes pinch off and non-trivial spherical space forms bubble off in finite time. The long time behaviour is delicate to analyze, but I conjecture that the flow squeezes out all the initial mass.

Fuquan Fang:

Brett Kotschwar: Ricci flow and the holonomy group

It is a well-known property of the Ricci flow equation that a solution with reduced holonomy restricted initially to some subgroup of SO(n) will have its holonomy likewise restricted for the entirety of its smooth lifetime. I will discuss the corresponding question of whether restricted holonomy at some non-initial time implies the same restriction at previous times, and prove that the reduced holonomy group of a smooth complete solution with uniformly bounded curvature cannot contract within the lifetime of the solution. This result has as geometric consequences, for example, that a solution is locally reducible at some time in its evolution if and only if it admits the same reduction at all previous times, and Kaehler at some time if and only it is Kaehler at all previous times. The problem is effectively one of unique continuation for a weakly-parabolic system; we reduce it to one of backwards uniqueness for a coupled system of partial-differential and ordinary-differential inequalities, amenable to an approach with Carleman-type inequalities. An analysis of a related (albeit simpler and fully-parabolic) system by the maximum principle also provides an elementary and self-contained proof of the forwards preservation of holonomy without reference to the classification and splitting theorems of Berger and DeRham.

Tobias Lamm: Geometric flows with rough initial data

We show the existence of a global unique and analytic solution for the mean curvature flow and the Willmore flow of entire graphs for Lipschitz initial data with small Lipschitz norm. We also show the existence of a global unique and analytic solution to the Ricci-DeTurck flow on euclidean space for bounded initial metrics which are close to the euclidean metric in L-infinity and to the harmonic map flow for initial maps whose image is contained in a small geodesic ball.

Phillippe LeFloch: Weakly regular T2 symmetric spacetimes. The global geometry of future developments

Under weak regularity assumptions, only, we develop a fully geometric theory of vacuum Einstein spacetimes with T2 symmetry, establish the global well-posedness of the initial value problem for Einstein's field equations, and investigate the global causal structure of the constructed spacetimes. Our weak regularity assumptions are the minimal ones allowing to give a meaning to the Einstein equations under the assumed symmetry and to solve the initial value problem. First of all, we introduce a frame adapted to the symmetry in which each Christoffel symbol can be checked to belong to some Lp space. We identify certain cancellation properties taking place in the expression of the Riemann and Ricci curvatures, and this leads us to a reformulation of the initial value problem for the Einstein field equations when the initial data set has weak regularity. Second, we investigate the future development of a weakly regular initial data set. We check that the area R of the orbits of symmetry must grow to infinity in the future timelike directions, and we establish the existence of a global foliation by the level sets of R. Our weak regularity assumptions only require that R is Lipschitz continuous while the metric coefficients describing the initial geometry of the orbits of symmetry are in the Sobolev space H1 and the remaining coefficients have even weaker regularity. We develop here the compactness arguments required to cover the natural level of regularity associated with the energy of the system of partial differential equations determined from Einstein's field equations.

Sylvain Maillot: Ricci flow with surgery on open 3-manifolds and applications

Applications of Ricci flow with surgery include Perelman's proof of Thurston's geometrization conjecture and Coda Marques's result on connectedness of spaces of metrics of positive scalar curvature. In this talk, I will discuss extensions of this work to noncompact manifolds. One goal is a Ricci flow proof of the geometrization conjecture in the noncompact, finite volume case, a result originally proved by Thurston using other methods. (This is joint with L. Bessières and G. Besson, partly in progress).

Reto Müller: Central blow-ups of Ricci flow singularities.

Scaling invariance properties of the heat equation motivated Hamilton to presume that singularities of the Ricci flow should be modeled by nontrivial gradient shrinking solitons. Perelman's W-entropy monotonicity strengthens this conjecture, showing that for any central blow-up sequence the shrinking soliton equation is approached in a weighted L^2-sense. Moreover, a similar (but more hidden) indication follows from the monotonicity of Perelman's reduced volume functional.

After explaining these motivations, we demonstrate how Hamilton's conjecture can be proved in the case of Type I Ricci flows, which is joint work with Joerg Enders and Peter Topping. Then, we will talk about possible extensions (in low dimensions) to the general case and present partial results obtained in collaborations with Robert Haslhofer and Carlo Mantegazza.

Artem Pulemotov: Parabolic equations and the Ricci flow on manifolds with boundary.

The first part of the paper discusses a second-order quasilinear parabolic equation in a vector bundle over a compact manifold M with boundary ∂M. We establish a short-time existence theorem for this equation. The second part of the paper is devoted to the investigation of the Ricci flow on M. We propose a new boundary condition for the flow and prove two short-time existence results.

Felix Schulze: On short time existence of the network flow.

I will report on joint work with T. Ilmanen and A. Neves on how to prove the existence of an embedded, regular network moving by curve shortening flow in the plane, starting from a non-regular initial network. Here a regular network consists of smooth, embedded line-segments such that at each endpoint, if not infinity, there are three arcs meeting under a 120 degree angle. In the non-regular case we allow that an arbitrary number of line segments meet at an endpoint, without an angle condition. The proof relies on gluing in appropriately scaled self-similarly expanding solutions and a new monotonicity formula, together with a local regularity result for such evolving networks. This short time existence result also has applications in extending such a flow of networks through singularities.

YuGuang Shi: Normalized Ricci flow on asymptotically hyperbolic manifold

This is my recent joint work with Jie Qing and my PhD student Jie Wu. In this talk we will investigate the behavior of normalize Ricci flow (NRF for short) whose initial metric is epsilon-Einstein on a complete,noncompact Riemannian manifold, and show that under non-degenerate condition on initial metric the solution to NRF exists for all time and converges exponentially fast to some Einstein metric. In addition, if the initial metric can be conformally compactified then the conformal structure of the infinity boundary of (M,g(t)) is preserved by NRF, and the limit is conformally compact Einstein metric.

Peter Topping:

Mu-Tao Wang: Mean curvature flows and isotopy problems

I shall discuss how mean curvature flows give canonical deformation of maps between Riemannian manifolds. Applications include estimations of null-homotopy constants of maps between spheres and smooth retractions of symplectomorphism groups of closed Riemann surfaces and complex projective spaces.

Jingyi Chen: Lagrangian mean curvature flow for entire Lipschitz graphs.

We prove longtime existence and estimates for solutions to a fully non-linear parabolic equation with C^{1,1} initial data u_0 satisfying either (1) â~H~R(1 + Î´)I_n â~I¤ D^2 u_0 â~I¤ (1+Î´)I_n for some positive Î´ or (2) u_0 is convex. We also show a supercritical condition on the Lagrangian phase is perserved under the equation. This is joint work with Albert Chau and Yu Yuan.

Miles Simon: Expanding solitons with non-negative curvature operator coming out of cones

We show that a Ricci flow of any complete Riemannian manifold without boundary with bounded non-negative curvature operator and non-zero asymptotic volume ratio exists for all time and has constant asymptotic volume ratio.

We show that there is a limit solution, obtained by scaling down this solution at a fixed point in space, which is an expanding soliton coming out of the asymptotic cone at spatial infinity. This is joint work with Felix Schulze.