Resent Researches on Quantum Systems Controls based on Lyapunov methods in USTC

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Concepts of Systems Controls

- To design an automatic control system by means of some system control theory and realize it

- Be belong to the area of Electric and Electronic Engineering (EEE)

- Department of Automation (or automatic control)

- All of the system control theories have the same principal, that is:

Negative feedback principle

- It is also called:

The automatic control principle

- The feedback control system is a closed-loop control system
- There are two kinds of control systems: open-loop control system and closed-loop control system
- Open-loop control system can be used in the simple controlled system with lower accuracy and robustness to the noise and disturbance or parameter uncertainty

- The variable of a feedback control system is always the error between the desired state and output state of the control system
- The control aim is always to make the error tend to a small value or zero which can be guaranteed if the control system is stable or convergent, and the control aim can be achieved automatically.

System Control Theories(1)

- Optimal control theory
- Adaptive control theory
- Lyapunov stability theorem (Lyapunov-based control method)
- Proportional, Integral and Derivative (P-I-D) Control
- Variable Structure Control (or Sliding Mode Control)

System Control Theories(2)

- Bang-Bang Control
- Robust Control
- Non-linear Control Theory
- Repetitive Control
- Recurrent Neural Network Control
- Fuzzy Logic Control
- …

- Optimal control methods are the most popular approaches that have been widely used specially in quantum chemistry fields.

- Since the mid 1980s, the quantum optimal control theory has attracted attentions from many researchers.

Lyapunov-based control method

- The Lyapunov indirect stability theorem which is used to judge whether a control system is stable or not without necessary of solving the (partial) differential equation
- People use this theorem to design a control law in the situation that the control system is surely stable

The Lyapunov indirect stability theorem

- For a given dynamical system

$$\dot{x}(t) = f(x)$$

- If one can find a scalar function V(x) has $V(x) \geq 0, \; for \; x \geq 0$

- At the same time there exists

$$\dot{V}(x) < 0 \quad or \quad \dot{V}(x) \leq 0$$

- Then this system is asymptotically stable (or stable)

The Procedure of Control law design

- For a control system

$$\dot{x}(t) = f(x) + Bu(x,t)$$

- If one can find a Lyapunov function V(x) such that exists $\dot{V}(u(x,t),x) \le 0$
- By means of the equation $\dot{V}(u(x,t),x) = 0$

one can obtain the control law

$$u(x,t) = F(x,t)$$

Similarity and difference between the optimal control and Lyapnuov-based control

The properties of these two control methods

- **Design procedure:** iterative algorithm to analytical function, or more complex to easier
- **Control strategy**: both are feedback control. The opitmal control system is designed by off-line, it is not suitable to the quantum physical control.

- **Control performance:** global optimal to local optimal (global optimal in the case the Lyapunov function is monotonic in the whole work area)
- **The convergence:** guarantee (control accuracy depends on the iterative number) to not guarantee (target state can not be achieved especially in quantum control if the Lyapunov function is not monotonic)

Applications

- The most of system control theories developed are based on the mathematical models of controlled systems
- They are universal in some sense.
- They have been applied successfully into the areas including the aeronautics and astronautics

Quantum Control

- eigenstate, superposition state, mixed state, entangled state etc.
- applications such as quantum information, quantum computation, and quantum communication, respectively.
- The transfer of different kinds of states in quantum systems has a important significance.

Some achievements in closed quantum system control

- The preparation and transfer control method design of the different kinds of states :

1) Eigenstate transfer
2) Superposition states preparation
3) Pure state control
4) Mixed state transfer
5) Purification of mixed state
6) Transfer from arbitrary pure state to target mixed state

Some research results on open quantum System control (1)

- Design of control sequence of pulses for the population transfer of high dimensional spin 1/2 quantum systems
- Purity and coherence compensation by using interactions
- Purity preservation of quantum systems by resonant field

Some research results on open quantum System control (2)

- Phase decoherence suppression in arbitrary *n*-level atom in Ξ-configuration with Bang-Bang controls

- Preparation of entanglement states in a two-spin system by Lyapunov-based method

The cooperation with the HNLPSM in real quantum device implementation

- Hefei National Laboratory for Physical Sciences at the Microscale in USTC

- Jiangfeng Du, Xing Rong, Nan Zhao, Ya Wang, Jiahui Yang & R. B. Liu, Preserving electron spin coherence in solids by optimal dynamical decoupling, Nature, 08470,29 Oct. 2009

1) Nuclear Magnetic Resonance spectrometer (NMR)
2) Electron Paramagnetic Resonance Spectrometer (EPR), also called Electron Spin Resonance (ESP)
3) Optical Detection of Magnetic Resonance (ODMR)

What we are doing is

1) Decoherence Suppression in a Cs (Cesium) *7/2* molecular with Bang-Bang Controls on NMR

2) Coherence Preservation on EPR by means of a new optimized dynamical decoupling strategy proposed

Conclusion

- Study the quantum system models existed
- Develop and propose the quantum control methods and technologies by means of combining them with the quantum system controlled properly
- Apply the proposed system control strategies into the actual experimental devices.

List of the publications

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Thanks