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Protecting Quantum Information with Optimal Control

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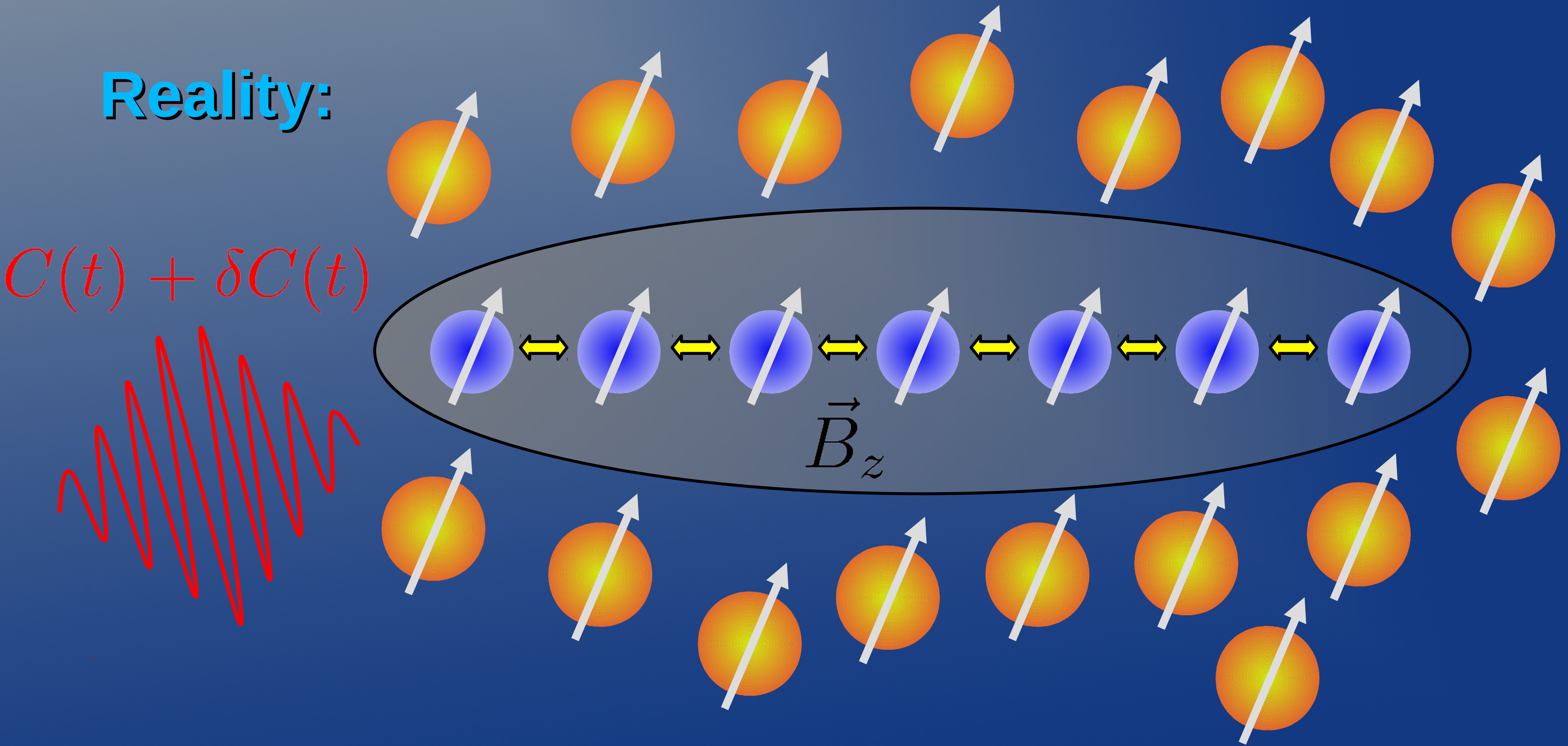
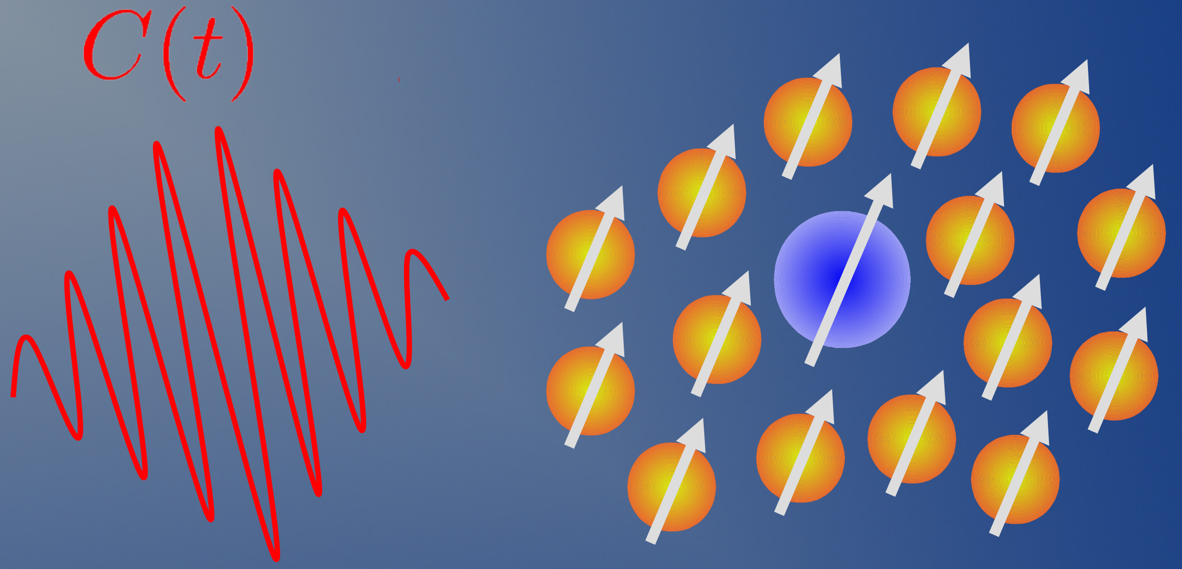
Quantum Information & Decoherence

Realistic physical systems are (noisy) open systems
→ they interact with the surrounding environment.

Reality:



Model Open System: Interacting Quantum Spins



$$H(t) = H_{\mathrm{s}}(t) + H_{\mathrm{int}} + H_{\mathrm{e}}$$

Schrödinger equation: $\dot{U}(t) = -\mathrm{i}H(t)U(t)$

Objective: Generate target system *time evolutions*

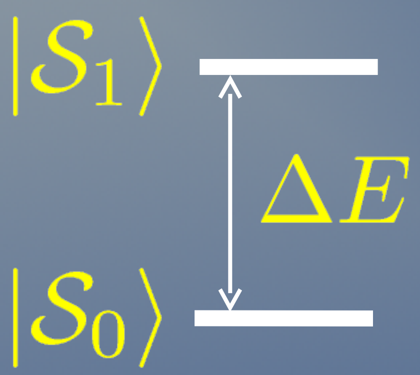
Model Open System: Interacting Quantum Spins

$$H = H_{\mathrm{s}} + H_{\mathrm{int}} + H_{\mathrm{e}}$$

$$H = H_0 - \vec{\sigma}\cdot\vec{C}(t) - \vec{\sigma}\cdot\vec{\Gamma} + H_{\mathrm{e}}$$

Control pulse area: $\theta(t) = \int_0^t \vec{C}(\tau)\mathrm{d}\tau$

Qubit Dynamics & the Bloch Sphere

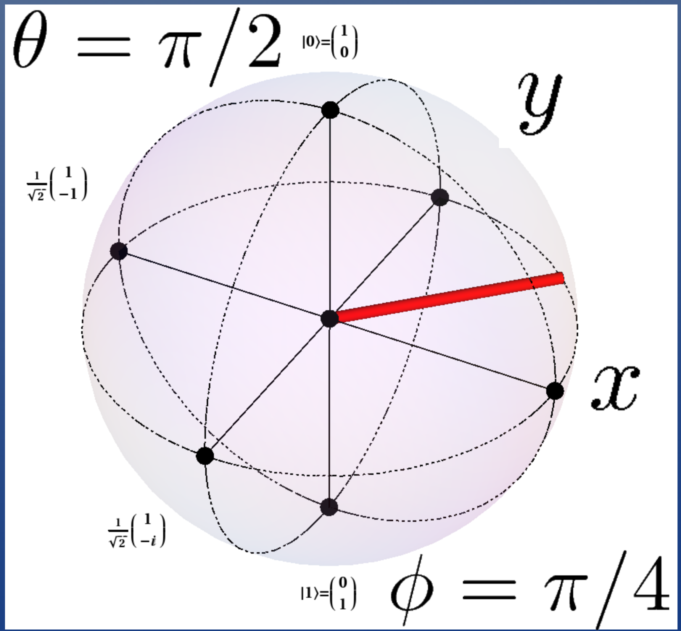


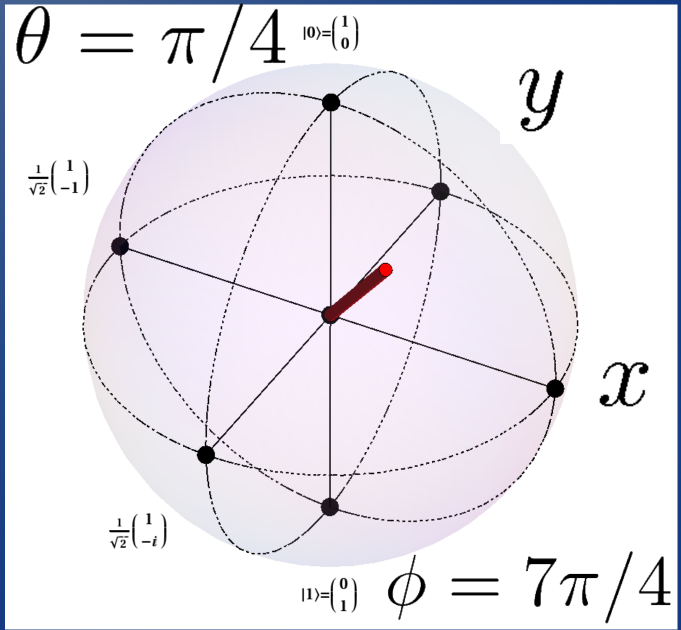
"Pure" spin states have the form:

$$|\psi\rangle = c_0|\mathcal{S}_0\rangle + c_1|\mathcal{S}_1\rangle,$$

where $|c_0|^2 + |c_1|^2 = 1.$

$$|\psi\rangle = \cos(\theta/2)|\mathcal{S}_0\rangle + \exp(\mathrm{i}\phi)\sin(\theta/2)|\mathcal{S}_1\rangle.$$

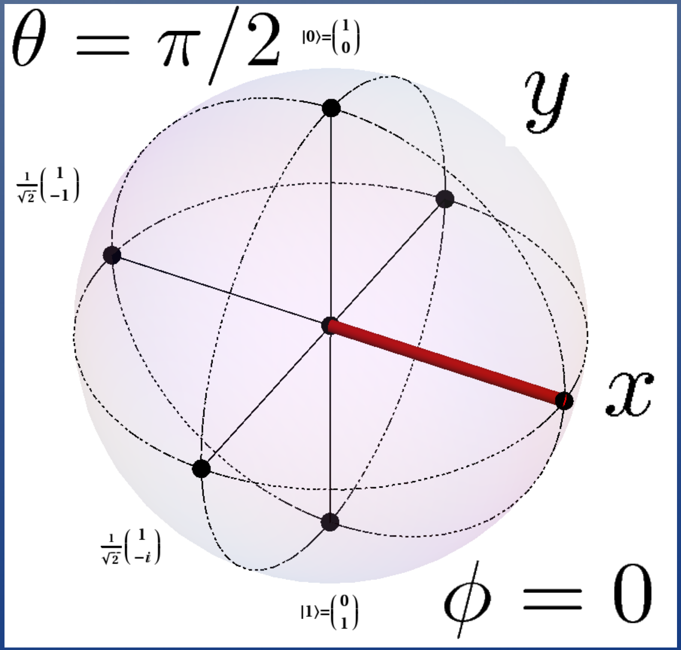




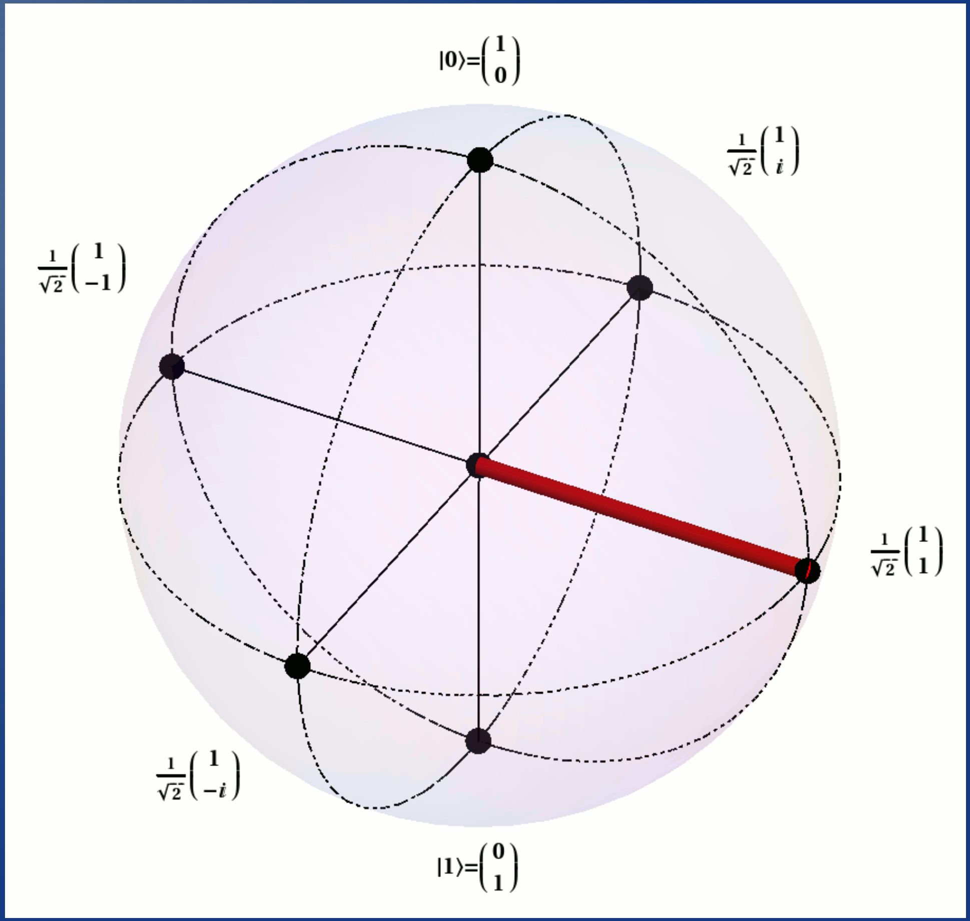
Qubit Dynamics & the Bloch Sphere: Memory Channels

Consider a decoherence process for states in the xy-plane, i.e.,

$$|\psi\rangle = \frac{|\mathcal{S}_0\rangle + \exp(\mathrm{i}\phi)|\mathcal{S}_1\rangle}{\sqrt{2}}.$$



Bloch vector dephasing → uncertainty in ϕ.



Qubit Dynamics & the Bloch Sphere: Memory Channels

With the "right" set of physical rotations, this error can be corrected → "Hahn-echo"

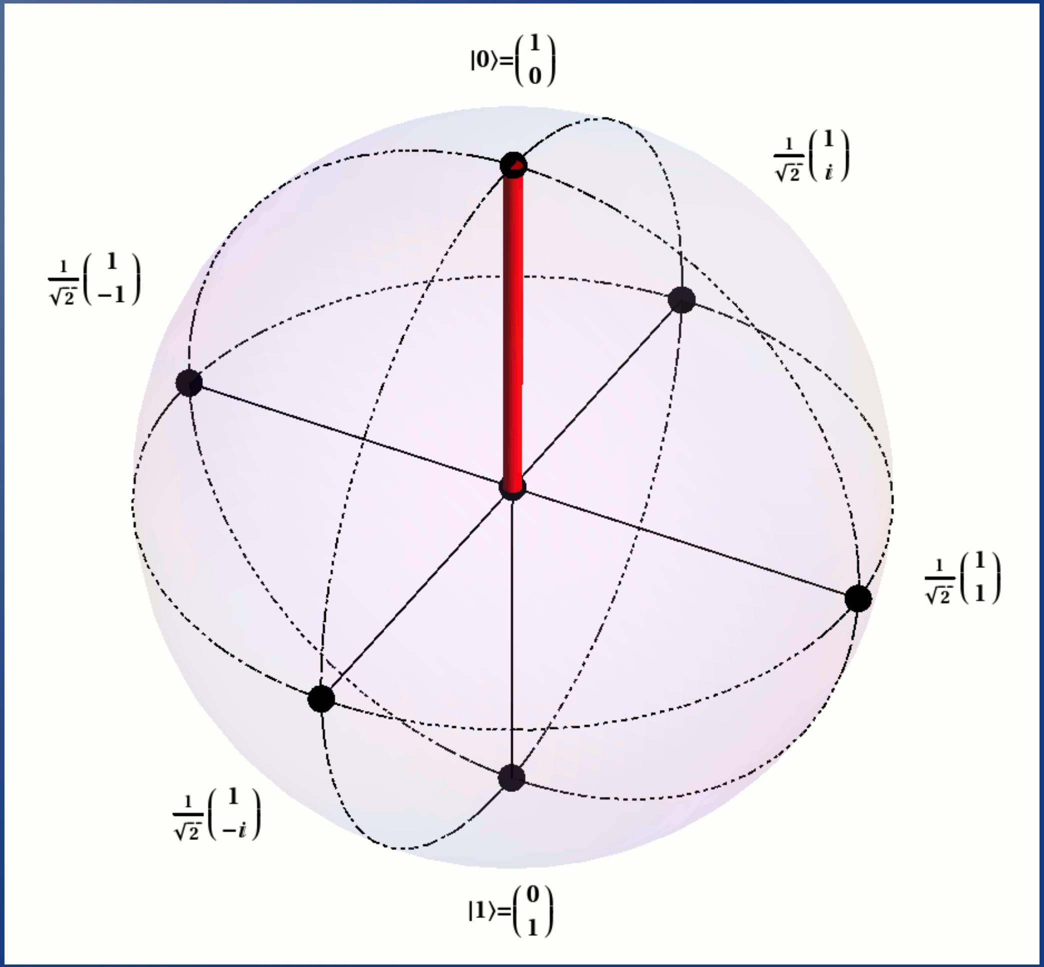
Hahn-echo pulse sequence:

$$\tau \longrightarrow Y_\pi \longrightarrow \tau \longrightarrow Y_\pi$$

Free evolution (↑ τ)

π-rotation about the y-axis (↑ Y_π)

π-pulses $\Rightarrow \theta(t_{\mathrm{f}}) = \pi$



Quantum Memory Channels: Dynamical-Decoupling Pulse Sequences

$$\prod_{i}^{N} U_i \approx \mathcal{I},$$ where U_i represent π and $\pi/2$ rotations and free evolutions.

This is an approximation to $\mathcal{I}$ because

- $\{U_i\}$ and N are finite
- Non-unitary evolution is corrected with unitary “time-reversal” operations

Dynamical-Decoupling Pulses

Aside from satisfying geometric and pulse area constraints, e.g.,

$$\theta(t_\mathrm{f}) = \int_0^{t_\mathrm{f}} \vec{C}(\tau)\,\mathrm{d}\tau,$$

What other control-field conditions can improve gate fidelity?

Dynamical-Decoupling Pulses

To remove 1st- and 2nd-order errors in π and $\pi/2$ z-axis rotations when $\vec{\Gamma} = \Gamma_x \Rightarrow \vec{\eta} = 0$:

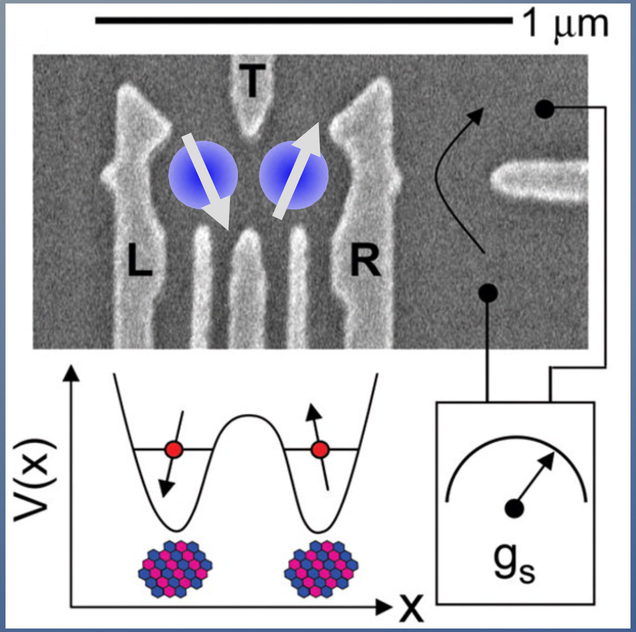
$$\eta_1 = \int_0^{t_\mathrm{f}} \sin[\theta(t)]\mathrm{d}t, \quad \eta_2 = \int_0^{t_\mathrm{f}} \cos[\theta(t)]\mathrm{d}t,$$

$$\eta_3 = \int_0^{t_\mathrm{f}} t\sin[\theta(t)]\mathrm{d}t, \quad \eta_4 = \int_0^{t_\mathrm{f}} t\cos[\theta(t)]\mathrm{d}t,$$

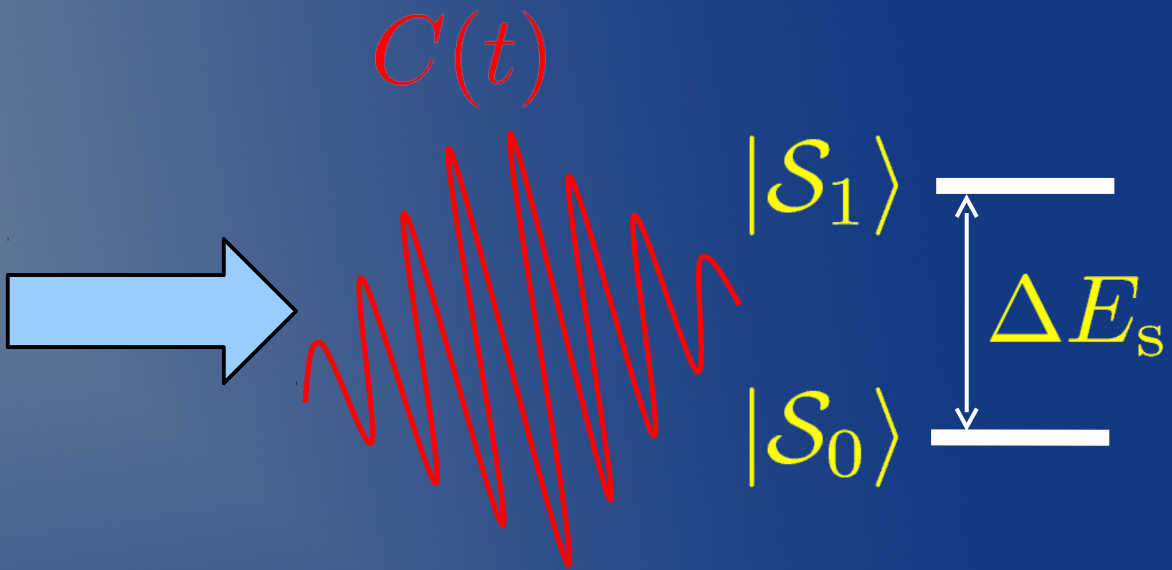
$$\eta_5 = \int_0^{t_\mathrm{f}}\int_0^{t_\mathrm{f}} \sin[\theta(t_1) - \theta(t_2)]\mathrm{sign}(t_1 - t_2)\mathrm{d}t_1\mathrm{d}t_2,$$

S. Pasini, *et al*., Phys. Rev. A, **80** (2009)

Double Quantum Dot: Effective One-Qubit Model



J. Petta, *et al*., Science, **309** (2005)



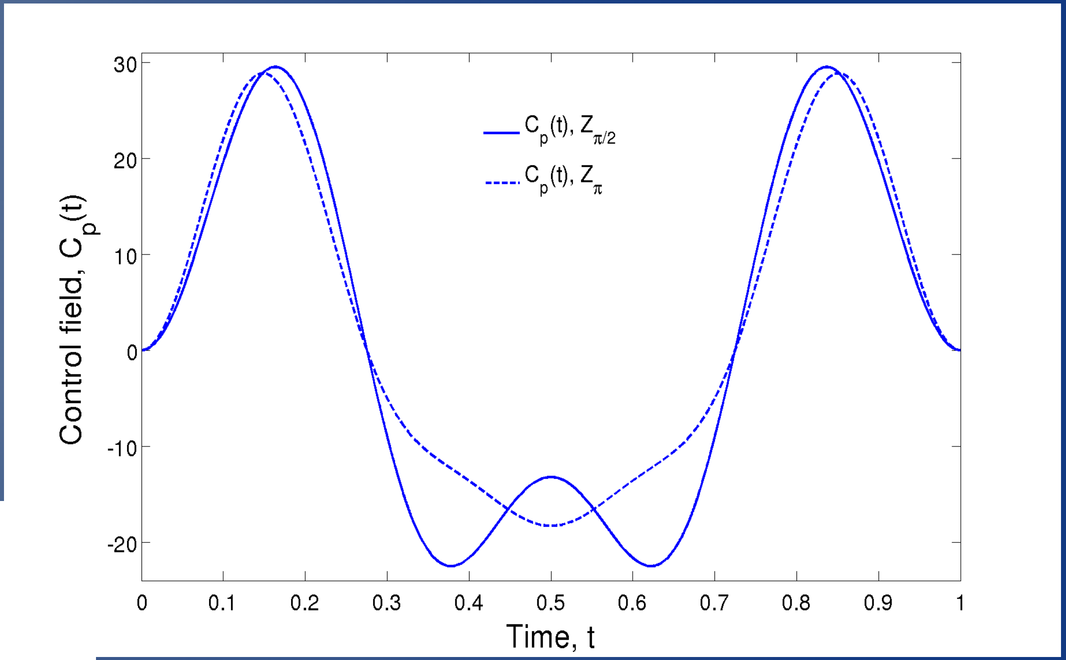
$$H = H_0 - \vec{\sigma}\cdot\vec{C}(t) - \vec{\sigma}\cdot\vec{\Gamma} + H_e$$

$$\rightarrow H = C_z(t)\sigma_z + \epsilon\sigma_x$$

Dynamical-Decoupling Pulses

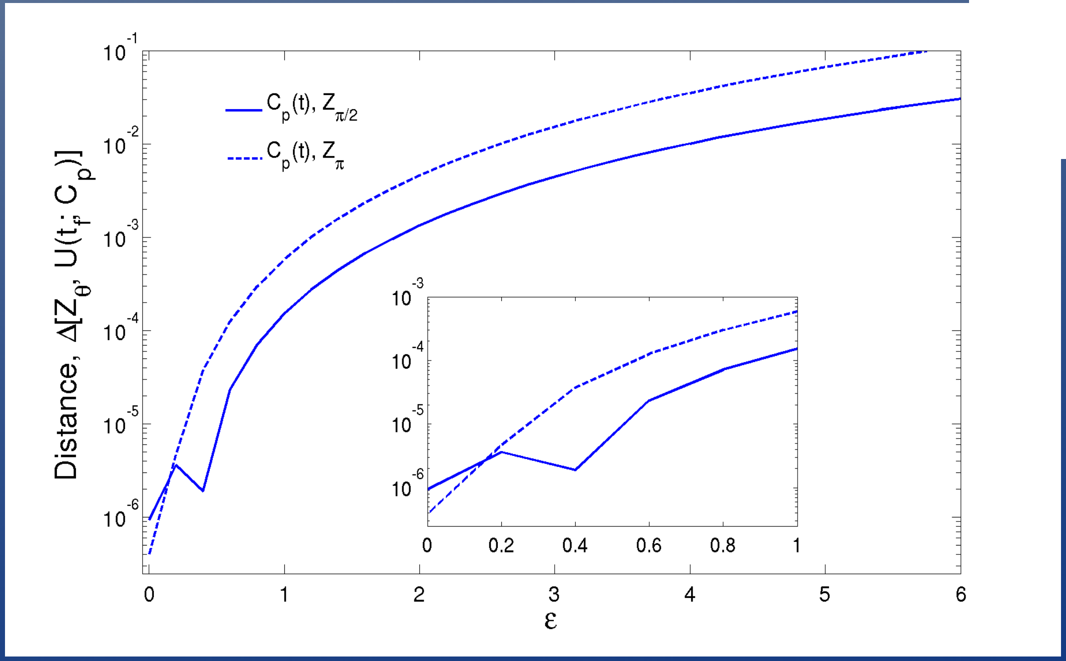
S. Pasini, *et al.*, Phys. Rev. A, **80** (2009)

Feasible controls satisfying $\vec{\eta} = 0$



Gate distance:

$$\Delta\left(Z_\theta, U_{t_\mathrm{f}}\right) = \sqrt{1 - \frac{1}{2}\left|\mathrm{Tr}\left(Z_\theta^\dagger U_{t_\mathrm{f}}\right)\right|}$$



Dynamical Decoupling + Optimal Control Pulses

By searching the space of controls satisfying

1. $\vec{\eta} \approx 0$

2. $\Delta\left(Z_\pi, U_{t_f}\right) = \sqrt{1 - \frac{1}{2}\left|\mathrm{Tr}\left(Z_\pi^\dagger U_{t_f}\right)\right|} \approx 0,$

and incorporating parameter estimates for ϵ , we improve control fidelities for Z_π .

Systematic searching $\longrightarrow$ Optimal control theory

Quantum Optimal Control Theory

- Define an objective:

$$\Delta\left(Z_\theta, U_{t_\mathrm{f}}\right) = \sqrt{1 - \frac{1}{2}\left|\mathrm{Tr}\left(Z_\theta^\dagger U_{t_\mathrm{f}}\right)\right|}$$

- Incorporate constraints:
  - Schrödinger’s equation
  - Experimental limitations of the control field

$$\mathcal{J} = \Delta \circ U_{t_\mathrm{f}} + \frac{\alpha}{2}\int_0^{t_\mathrm{f}} \left\|\vec{C}(t)\right\|^2 \mathrm{d}t$$

- Optimize iteratively
  - Evolutionary algorithms
  - Gradient-based methods $\nabla_c \mathcal{J} = 0$

DD+OC Optimization Procedure

After calculating $\nabla_c \mathcal{J}$, all gradient

directions $\nabla_c \eta_i$ are removed:

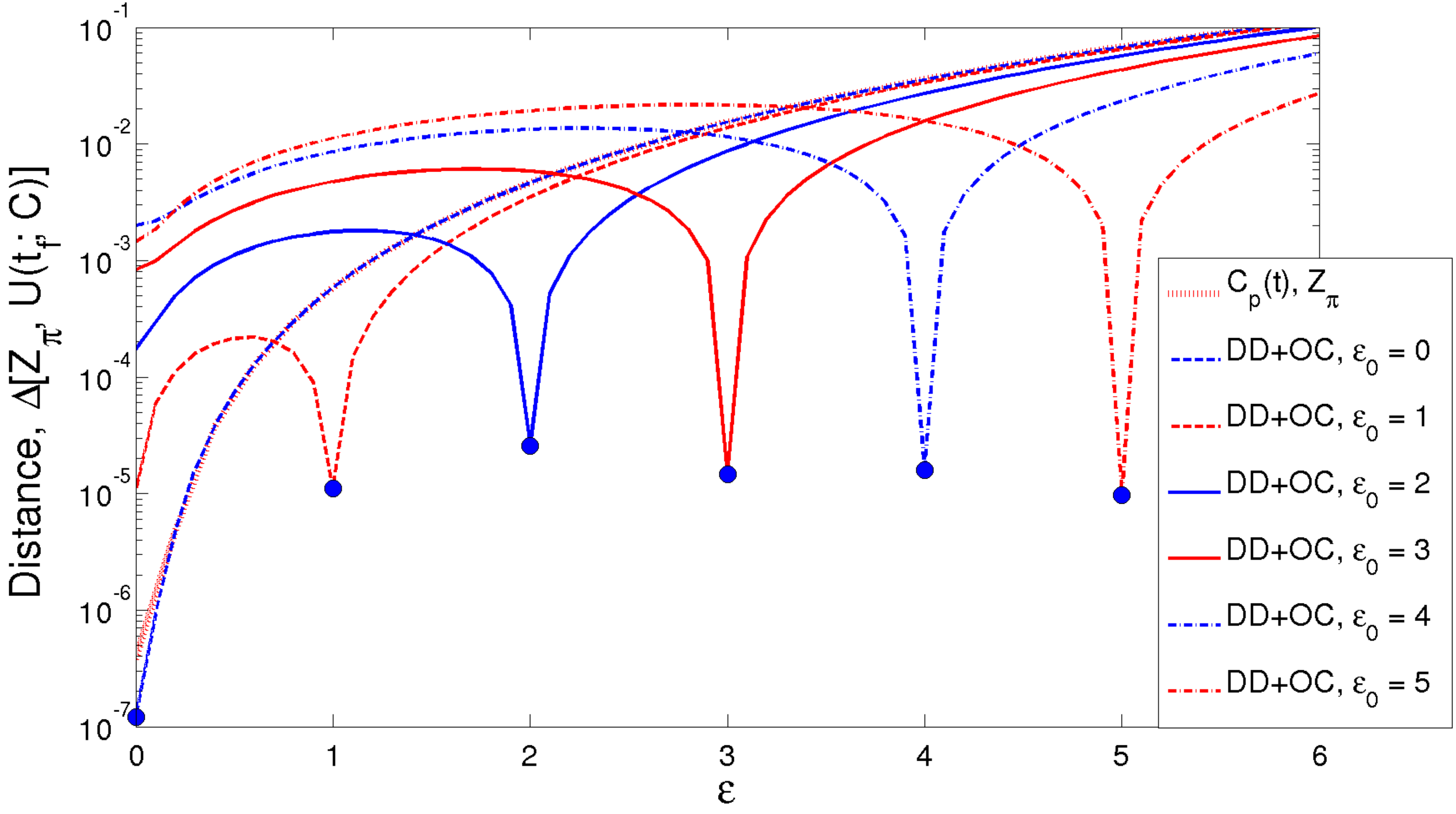
$$\nabla_c \mathcal{J} \longrightarrow \nabla_c \mathcal{J} - \sum_i p_i \nabla_c \eta_i \,,$$

so $\langle \nabla_c \mathcal{J}\,, \nabla_c \eta_i \rangle = 0$,

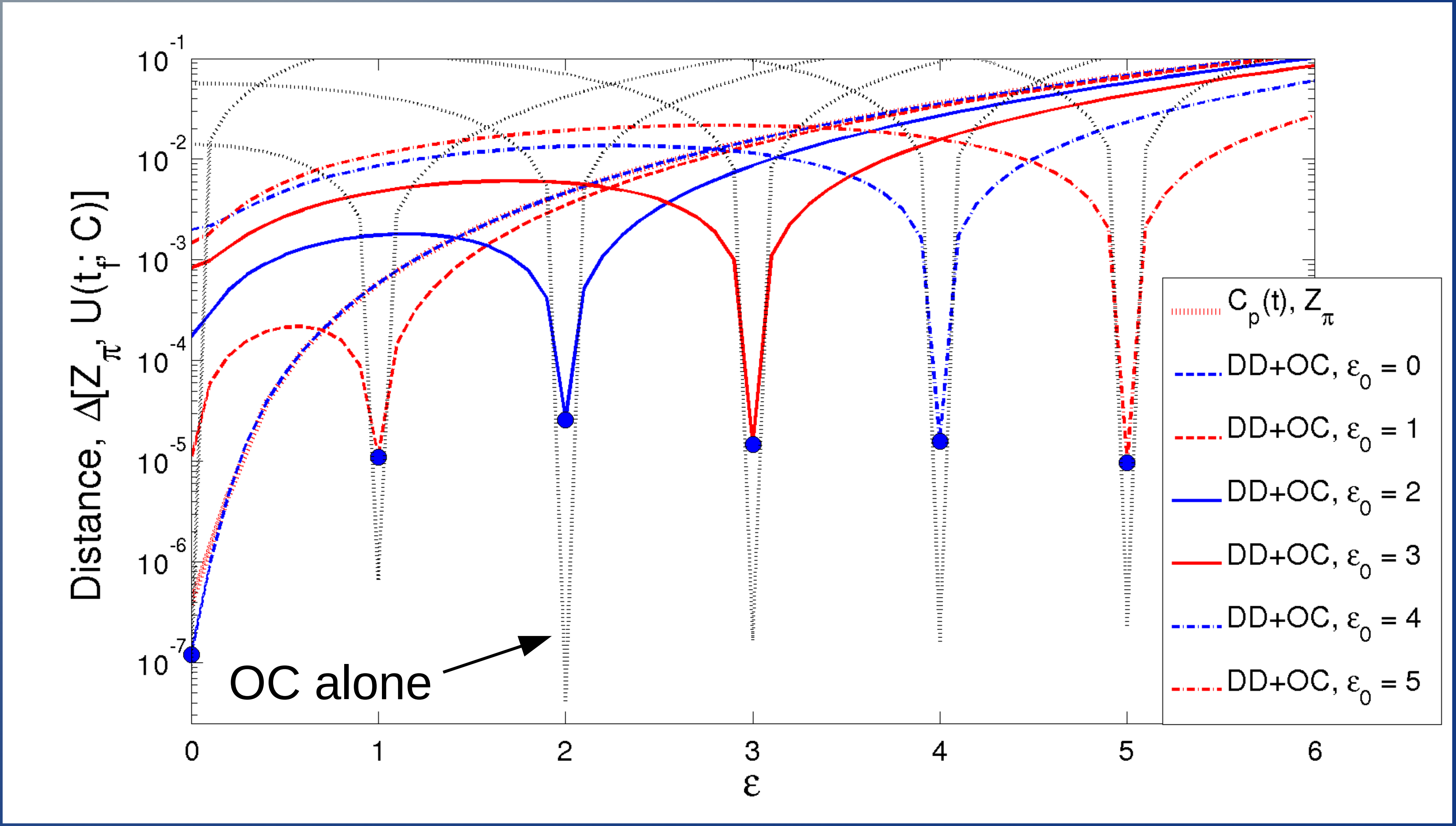
where $\vec{p}$ is constructed from

$G_{ij} = \langle \nabla_c \eta_i, \nabla_c \eta_j \rangle$ and $\langle \nabla_c \mathcal{J}\,, \nabla_c \eta_i \rangle$.

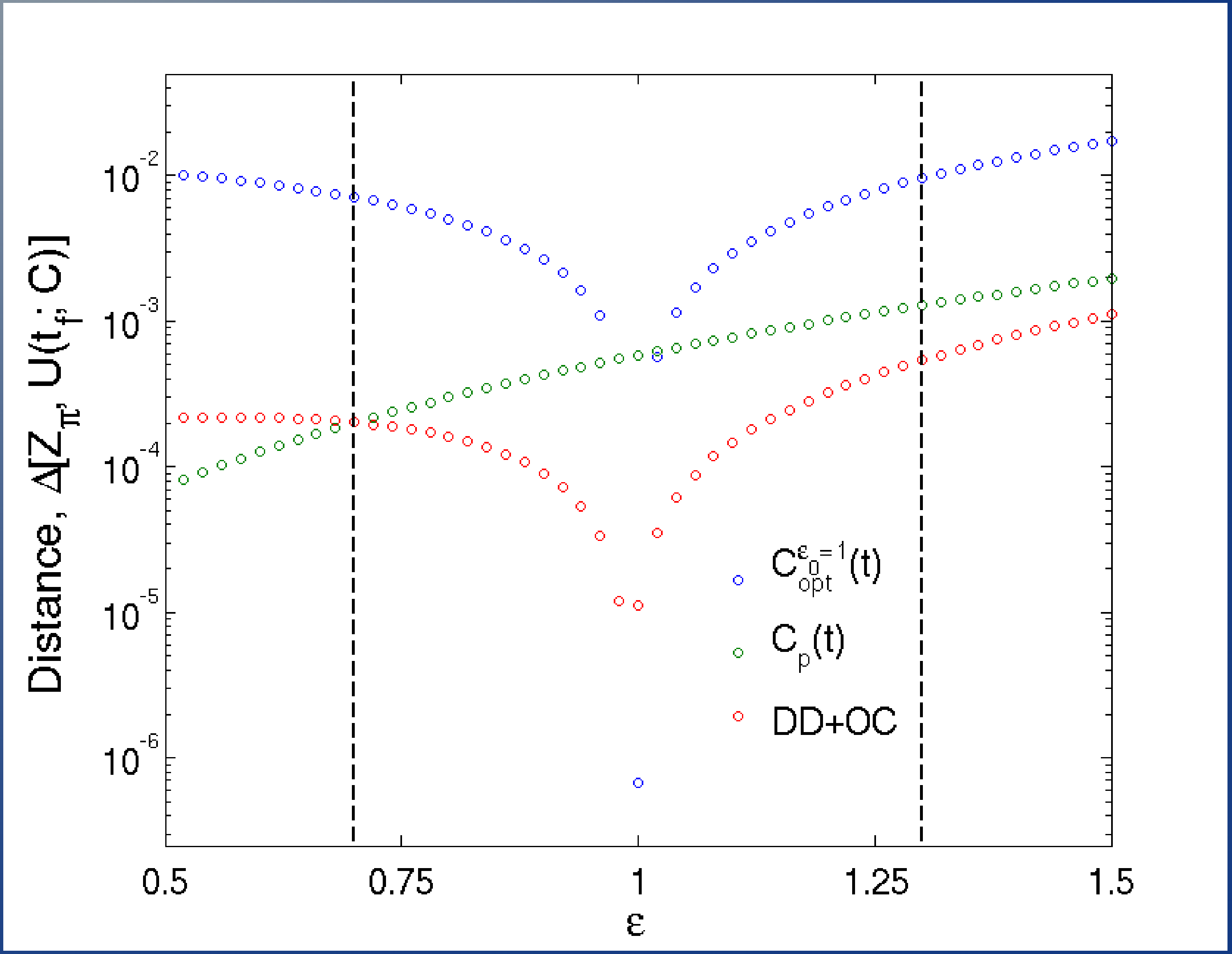
DD+OC Pulses: Gate Distances



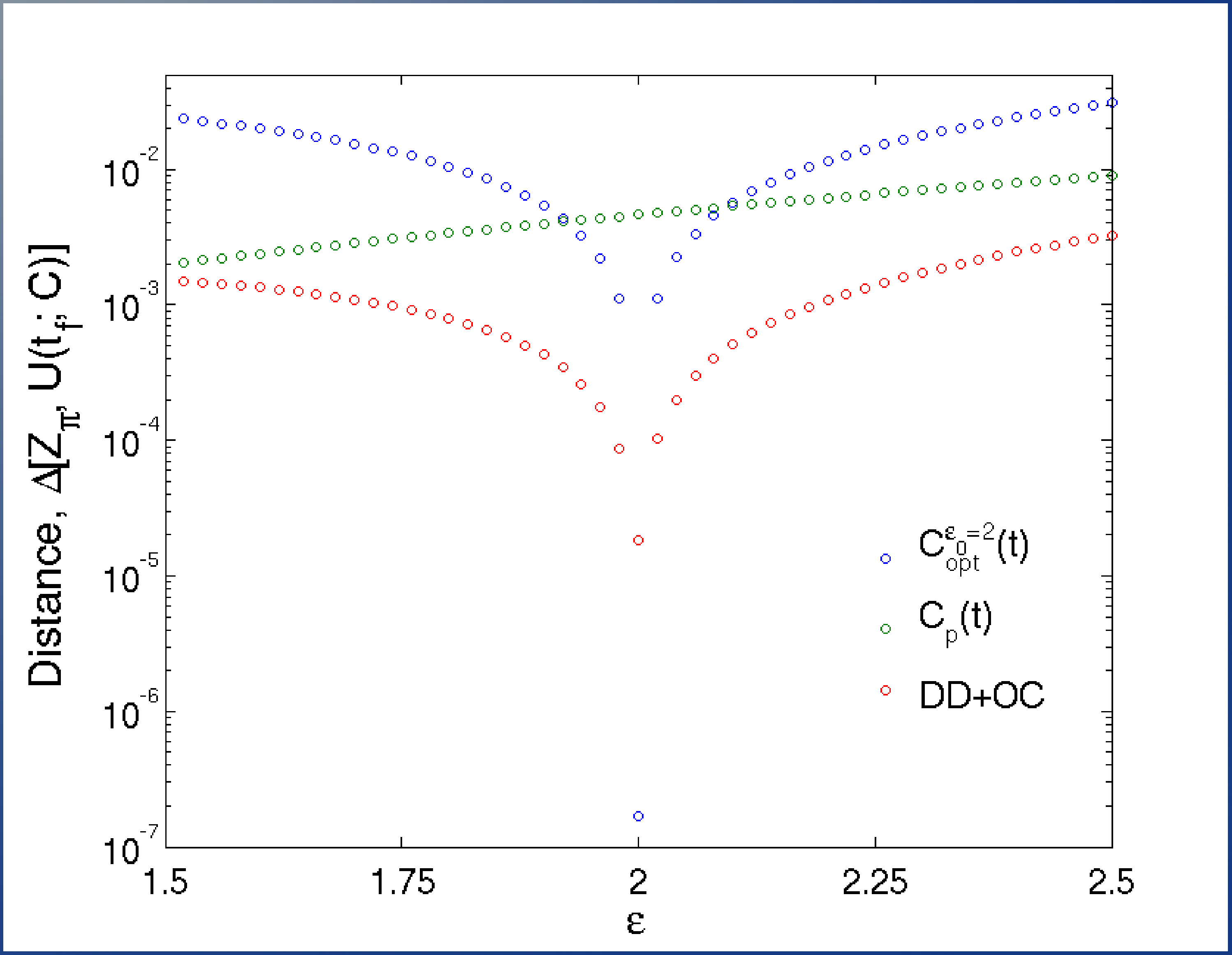
Gate Distances from OC and DD+OC



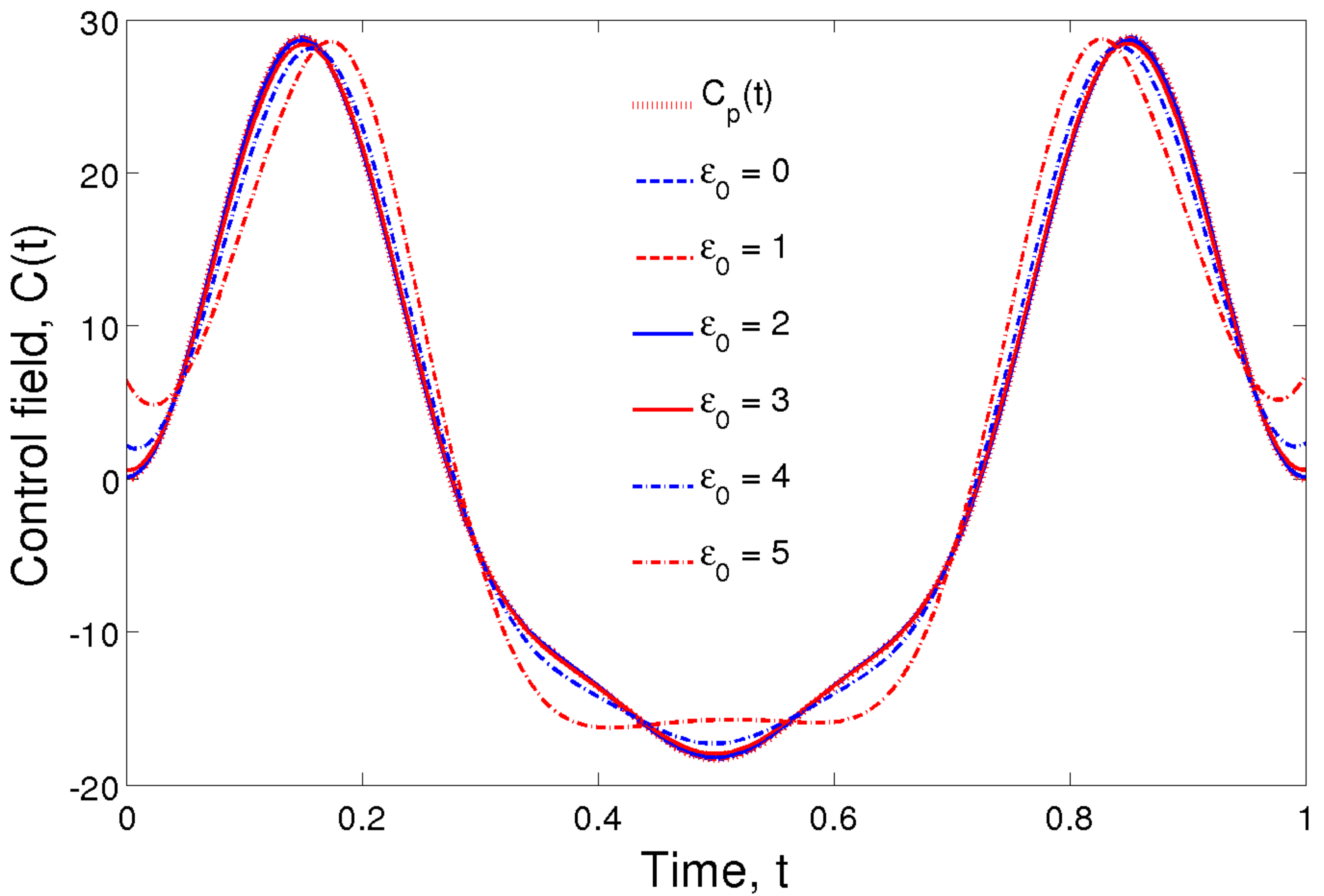
DD+OC Pulses: Improved Robustness 1



DD+OC Pulses: Improved Robustness 2



DD+OC Pulses



$C_p(t)$
$\varepsilon_0 = 0$
$\varepsilon_0 = 1$
$\varepsilon_0 = 2$
$\varepsilon_0 = 3$
$\varepsilon_0 = 4$
$\varepsilon_0 = 5$
Control field, C(t)
Time, t
30
20
10
0
-10
-20
0
0.2
0.4
0.6
0.8
1

Conclusions and Current Work

- Demonstrated dynamical decoupling + optimal control for improved gate fidelity and robustness
- Extend formalism to arbitrary rotation axes and perturbative expansions about arbitrary ϵ.
- Explore robustness to control field variations

Calculating the Distance Measure with the Hilbert-Schmidt Norm

$$\Delta(U,V) = \lambda \min_{\Phi_{\rm e}} \left\{ \| U - (\mathbb{1}_{\rm s} \otimes \Phi_{\rm e}) V \|_{\rm HS} \right\}$$

After some relatively straightforward linear algebra . . .

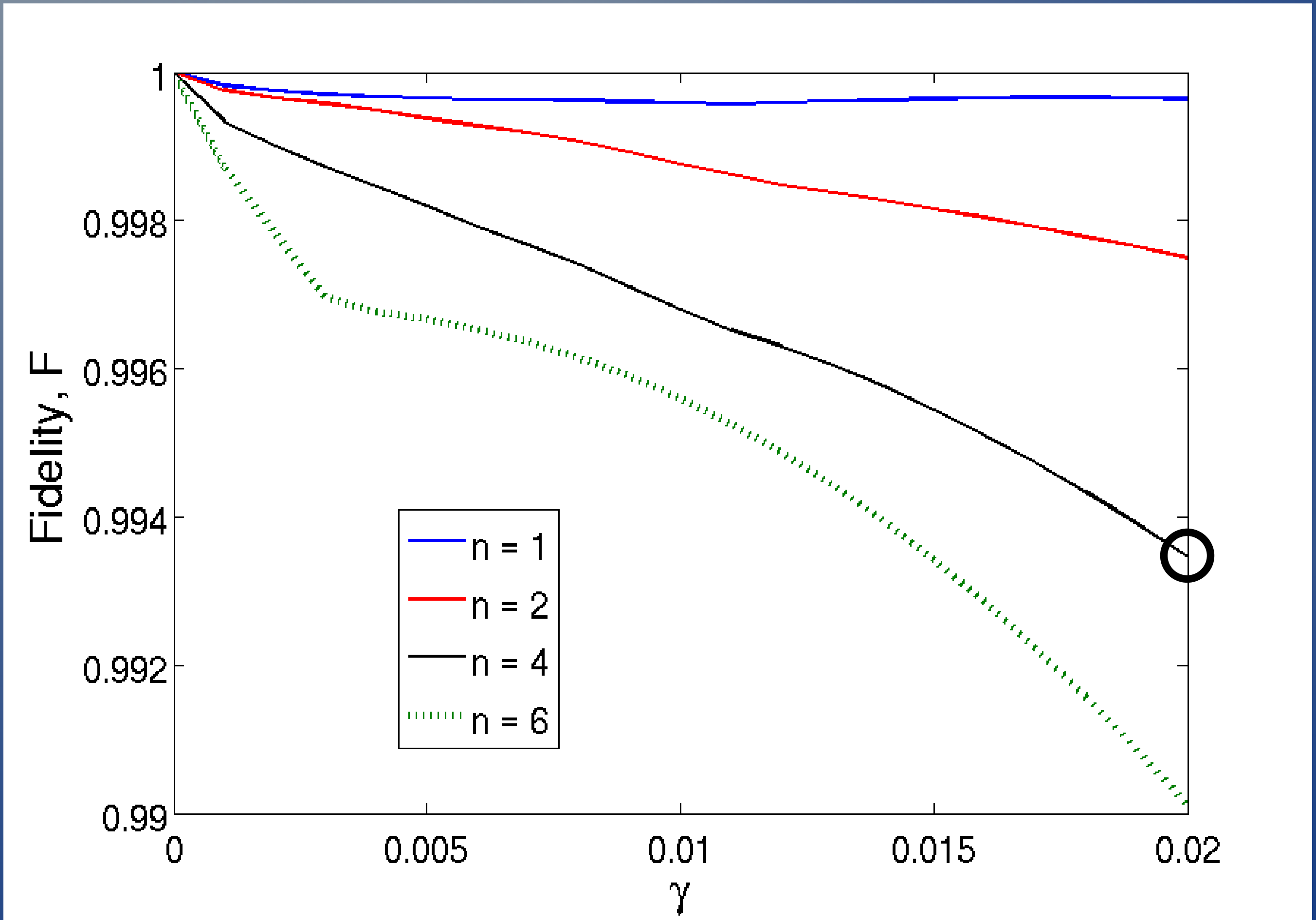
$$= \left[1 - \frac{1}{n} \| \mathrm{Tr}_{\rm s} \left(U V^{\dagger} \right) \|_{\rm Tr} \right]^{1/2}$$

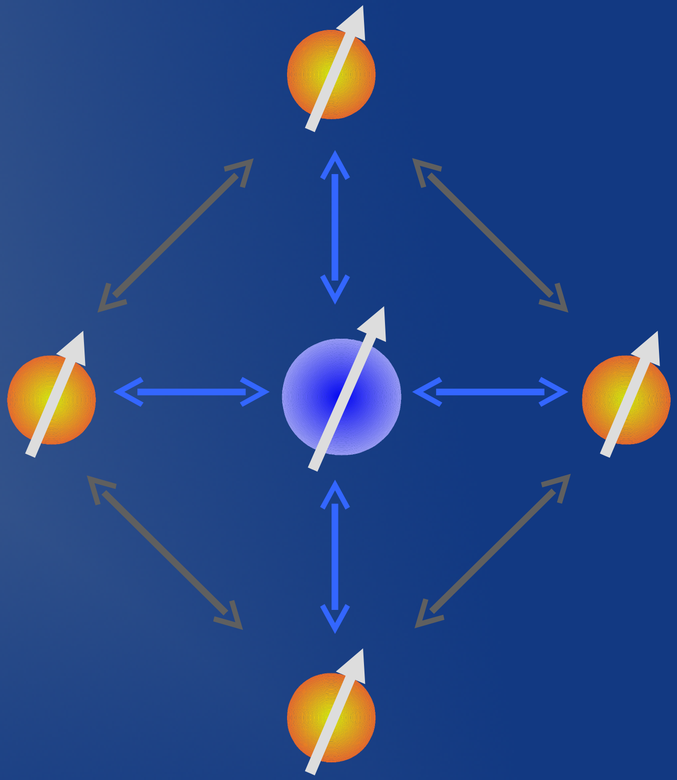
Nice analytical result; solved numerically in practice.

M. Grace, *et al*., New J. Phys., **12** (2010)

Quantum Control Results: One-Qubit Operations

Multiparticle environment: Hadamard gate



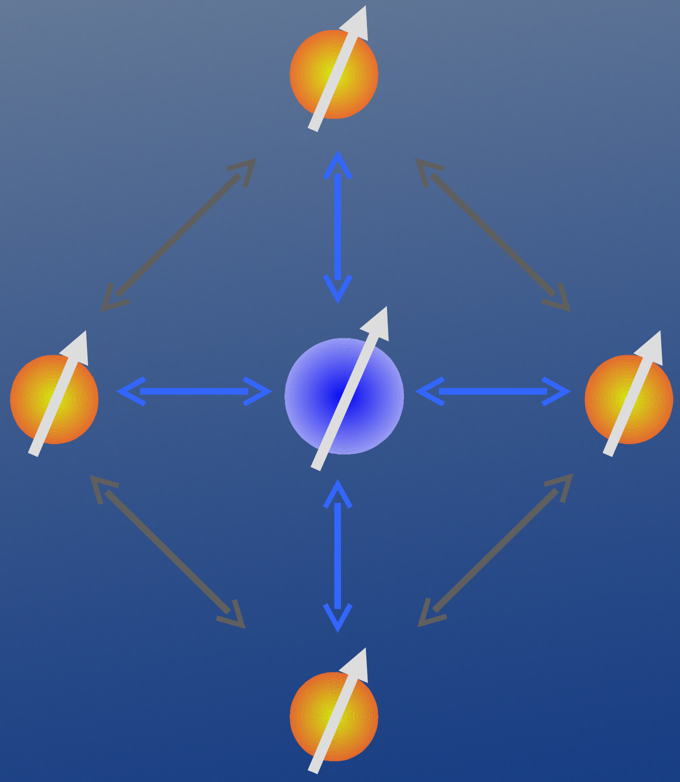


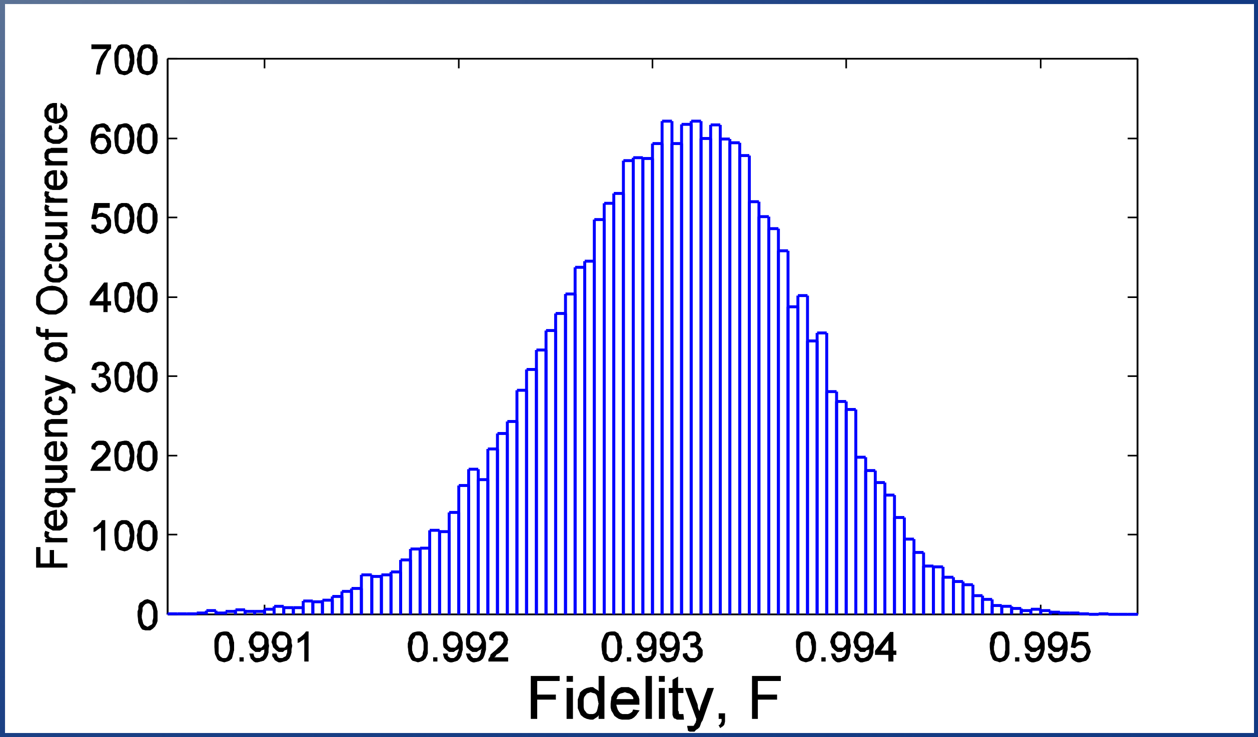
Gate Robustness to System Variations

Optimal Hadamard gate with a four-particle environment:

$\gamma = 0.02$, $\gamma' = 0.0175$, and $F \approx 0.9934$.

This control is applied to an ensemble of systems with random variations in γ and γ' given by $\Delta\gamma/\gamma = 1/8$.





M. Grace, *et al*., J. Mod. Opt., **54** (2007)