Gate Control of Single Electron Spins in Quantum Dots via the Application of the Geometric Phase

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Overview

Background

- ***Introduction and motivation***
- ***Hamiltonian of III-V type Semiconductors***
- ***Effect of bulk inversion symmetry (Dresselhauss Effect)***
- ***Effect of structural inversion symmetry (Rasha Effect)***
- ***Manipulation of electron spin states via geometric phase***

Results:

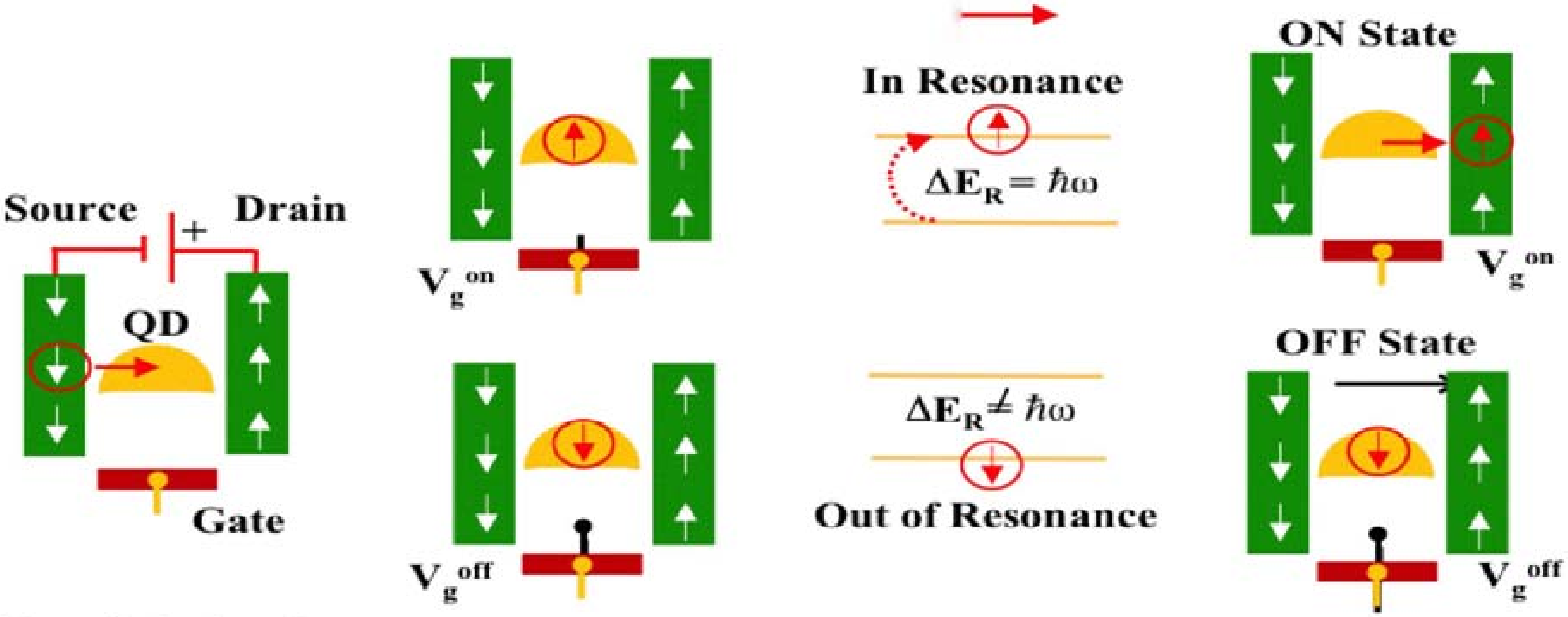
- ***Unitary transformation of electron spin states***
- ***Transition probability of electron spin in quantum dots***
- ***Persistent spin-helix and spin diffusion length***

Summary and Conclusions

Motivation:

•*The importance of gate control in quantum mechanical applications (quantum information processing, low dimensinal nanostructures etc)*

Schematics of spin single electron transistors (SET)



Tunnel injection of one spin-polarized electron per QD

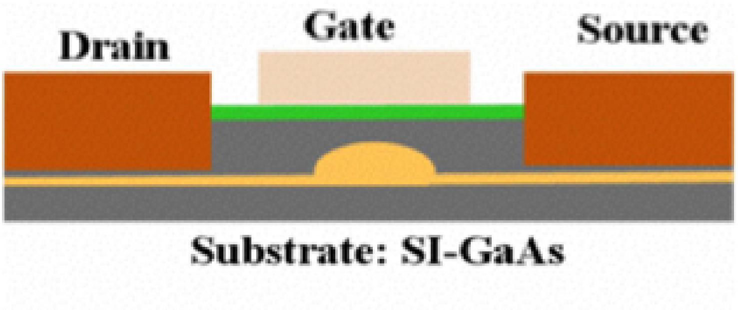
Splitting of QD ground state is controlled by gate voltage V_g

Spins in resonance with RF field will flip (ON State), non-resonant spins will not (OFF State)

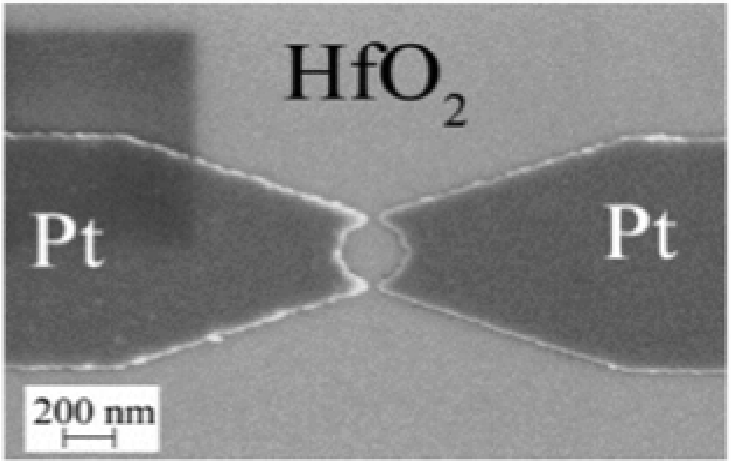
Bandyopadhyay: Phys. Rev. B 61, 13813 (2000)

Possible spin SET prototype

S. Prabhakar and J. Raynolds,
PRB 79, 195307 (2009)



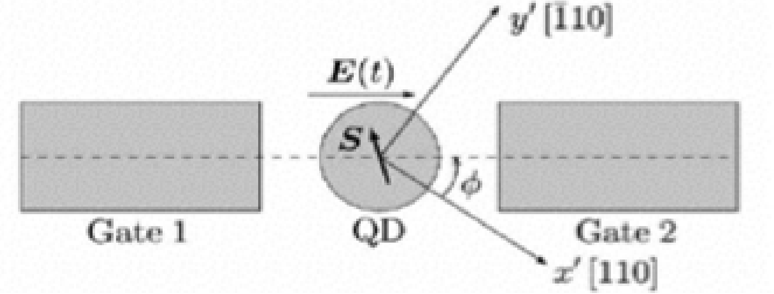
SEM of a 2D-0D heterodimensional prototype



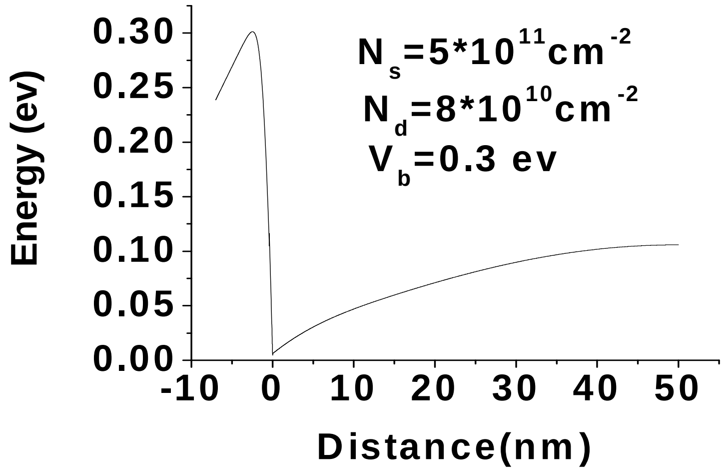
Goal: Development for planar SET prototype using:

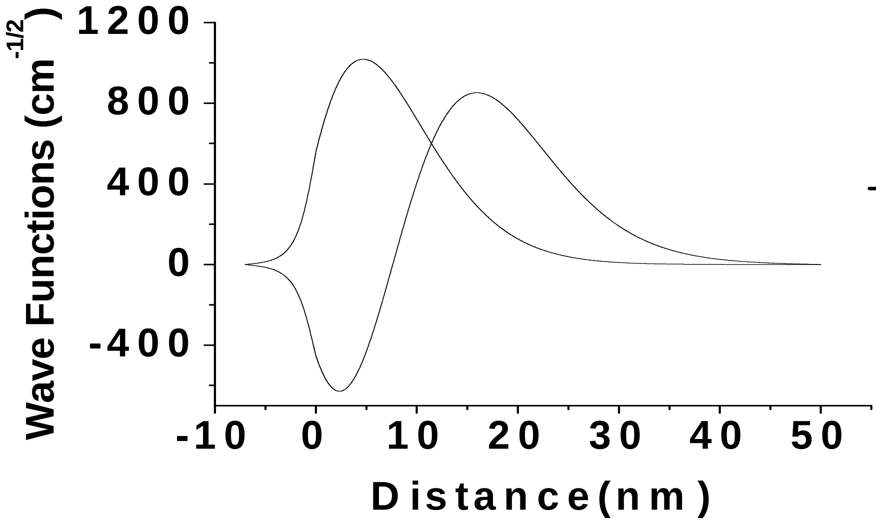
- High-k gate stack on InGaAs/AlGaAs structure
- Hetero-dimensional control of 2D-1D-0D electrons
- Eventually self-assembled InAs QD

Schematic for EDSR spin control
[Loss et al ., PRB 2006]



Two Dimensional Electron Gas (2DEG)





Electron moves in an effective potentials

$$V(z) = -e\,\phi(z) + V_{xc}(z)$$

$$-\frac{\hbar^2}{2}\frac{d}{dz}\frac{1}{m(z)}\frac{d\psi_i(z)}{dz} + V(z)\psi_i(z) = E_i\,\psi_i(z)$$

$$\frac{d}{dz}\varepsilon_0\kappa(z)\frac{d\phi(z)}{dz} = e\sum N_i\,\psi_i^2(z) - \rho_I(z)$$

where

$$\hat{H} = -\frac{\hbar^2}{2}\frac{d}{dz}\frac{1}{m(z)}\frac{d}{dz} + V(z)$$

S. Prabhakar and J. Raynolds, PRB 79, 195307 (2009)
Also See PRB Vol 30,840 (1984)

Hamiltonian of quantum dots in III-V semiconductor

$$H = H_0 + H_z + H_R + H_D$$

$$H_0 = \frac{\vec{P}^2}{2m} + \frac{1}{2} m \omega_0^2 \left(\alpha x^2 + \beta y^2\right) + \tilde{\alpha} x + \tilde{\beta} y + \frac{1}{2} g_0 \mu_B \sigma_z B$$

•The structural inversion asymmetry leads to Rashba spin-orbit coupling

$$H_R = \frac{\alpha_R e E}{\hbar}\left(\sigma_x P_y - \sigma_y P_x\right)$$

•Bulk inversion asymmetry leads to Dresselhaus spin-orbit coupling

$$H_D = \frac{\gamma_D}{\hbar}\left(\frac{2meE}{\hbar^2}\right)^{2/3}\left(-\sigma_x P_x + \sigma_y P_y\right)$$

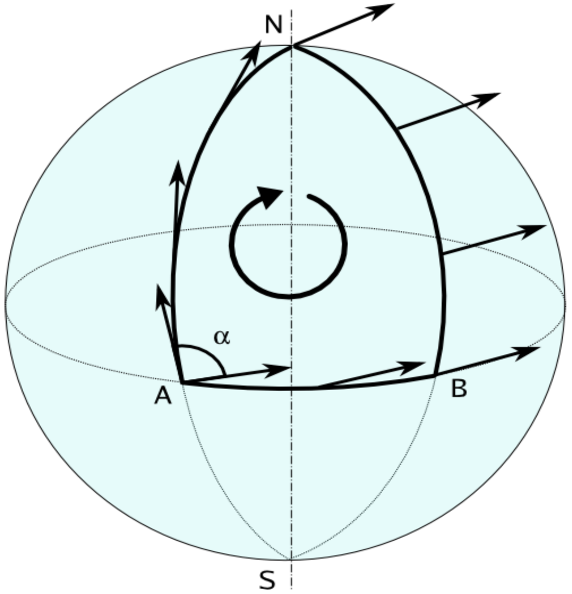
where

$$\omega_0 = \sqrt{\frac{\hbar}{m \ell_0}}, \quad \tilde{\alpha} = eE_x, \quad \tilde{\beta} = eE_y, \quad E$$ are control variables

Geometric phase factors accompanying adiabatic changes (Berry Phase)

According to Schrödinger Equation, the state of the system evolves $\left|\Psi(t)\right\rangle$

V. Vedral, arXiv.org: quant-ph/0212133 (2002).



$$\hat{H}(R(t))\left|\Psi(t)\right\rangle = i\hbar\frac{\partial}{\partial t}\left|\Psi(t)\right\rangle$$

At any instant,

$$\hat{H}(R)\left|n(R)\right\rangle = E_n(R)\left|n(R)\right\rangle$$

$$\left|\Psi(t)\right\rangle = \underbrace{\exp\left\{\frac{-i}{\hbar}\int_0^t dt' E_n(R(t'))\right\}}_{Dynamical\ \ Phase\ \ Factor}\ \underbrace{\exp(i\gamma_n(t))}_{\substack{Geometric\ \ Phase\ \ Factor\\ or\ \ Berry\ \ Phase\ \ Factor}}\left|n(R(t))\right\rangle$$

$$where\quad \gamma_n(T) \neq \gamma_n(0)$$

M. Berry Proc. R. Soc. Lond. A 392,45-57(1984)

Mathematical Expression for the Berry Phase

$$\gamma_n(c) = -\iint_C d\vec{s} \cdot V_n(R)$$

$$V_n(R) = \mathrm{Im} \sum_{l \neq n} \frac{\langle n(R) | \nabla_R \hat{H}(R) | l(R) \rangle \times \langle l(R) | \nabla_R \hat{H}(R) | n(R) \rangle}{\left(E_l(R) - E_n(R)\right)^2}$$

The Hamiltonian of the quantum dot is

$$H = H_0 + H_R + H_D$$

$$H_0 = \frac{\vec{P}^2}{2m} + \frac{1}{2} m \omega_0^2 \left(\alpha x^2 + \beta y^2\right) + \tilde{\alpha} x + \tilde{\beta} y + \frac{1}{2} g_0 \mu_B \sigma_z B$$

where $\tilde{\alpha}$ *and* $\tilde{\beta}$ *are control variables*

Proc. R. Soc. Lond. A 392,45-57(1984); Phys. Rev. B 68, 55330 (2003)

Energy spectrum of a quantum dot

$$H = H_0 + H_R + H_D$$

$$H_R = \frac{\alpha_R e E}{\hbar}\left(\sigma_x P_y - \sigma_y P_x\right) \qquad H_D = \frac{\gamma_D}{\hbar}\left(\frac{2meE}{\hbar^2}\right)^{2/3}\left(-\sigma_x P_x + \sigma_y P_y\right)$$

The energy spectrum of the Zeeman spin splitting part H_0 can be written as

$$\varepsilon_{n_+ n_-} = \left(n_+ + n_- + 1\right)\hbar\omega_+ + \left(n_+ - n_-\right)\hbar\omega_- - G + \frac{1}{2} g_0 \mu_B B \sigma_z$$

where $G = \frac{1}{2}\left(\frac{\tilde{\alpha}^2}{\alpha} + \frac{\tilde{\beta}^2}{\beta}\right)\frac{1}{m\omega_0^2} \qquad \omega_+^- = \frac{1}{2}\left[\omega_c^2 + \omega_0^2\left(\sqrt{\alpha} \pm \sqrt{\beta}\right)^2\right]^{1/2}$

$$s_\pm = \frac{\omega_+}{\omega_c\left[\beta/\alpha\right]^{1/4}}\left[\sqrt{\frac{\beta}{\alpha}} - 1 \pm \sqrt{\frac{\sqrt{\beta}\,\omega_c^2}{\sqrt{\alpha}\,\omega_+^2} + \left(1 - \sqrt{\frac{\beta}{\alpha}}\right)^2}\right]$$

Diagonalization is carried out by applying annihilation and creation operators

$$a_\pm = \frac{1}{\left(s_+ - s_-\right)\left(1 + i\right)}\left[\pm\left(s_\mp + i\right)\frac{\ell}{\hbar}p_x + \left(s_\pm + i\right)\frac{\ell}{\hbar}p_y + \left(1 - s_\mp\right)\frac{1}{\ell}x \pm \left(1 - is_\pm\right)\frac{1}{\ell}y\right]$$

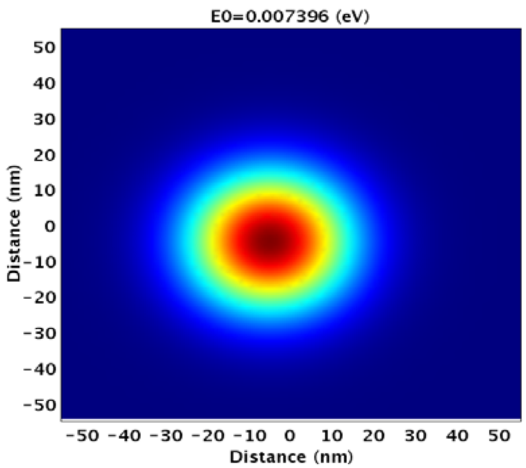
$$a_\pm^\dagger = \frac{1}{\left(s_+ - s_-\right)\left(1 - i\right)}\left[\pm\left(s_\mp - i\right)\frac{\ell}{\hbar}p_x + \left(s_\pm - i\right)\frac{\ell}{\hbar}p_y + \left(1 - s_\mp\right)\frac{1}{\ell}x \pm \left(1 + is_\pm\right)\frac{1}{\ell}y\right]$$

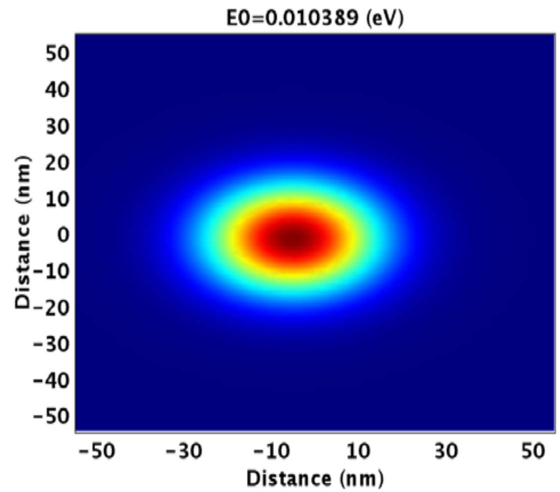
$$\varepsilon_{0,0,+1/2} = \varepsilon_{0,0,+1/2}^{(0)} + \varepsilon_{0,0,+1/2}^{(2)} \qquad \varepsilon_{0,0,-1/2} = \varepsilon_{0,0,-1/2}^{(0)} + \varepsilon_{0,0,-1/2}^{(2)}$$

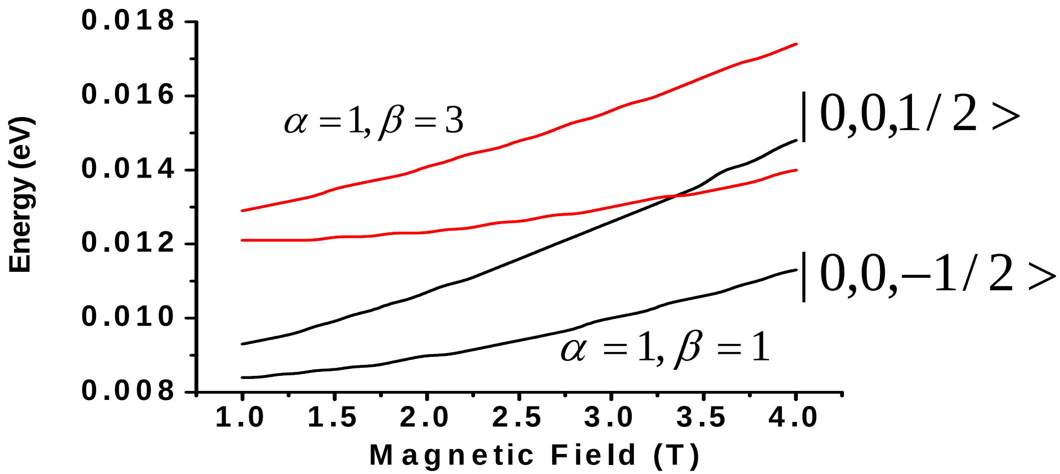
Application:

In-plane wave functions of InAs quantum dot

$\alpha = 1, \beta = 1$ *B=1 T* $\alpha = 1, \beta = 3$







Control parameters:

$$\omega_0 = \sqrt{\frac{\hbar}{m\ell_0}}, \quad B$$

$$\tilde{\alpha} = eE_x, \quad \tilde{\beta} = eE_y, \quad E = E_z$$

In fig., we choose

$$\ell_0 = 20 \ nm$$

$$E = 1.6 \times 10^4 \ V/cm$$

$$E_x = E_y = 10^3 \ V/cm$$

Extension of Berry Phase for degenerate state

For non degenerate state,

$$\left|\Psi_n(T)\right\rangle = \underbrace{\exp\left\{\frac{-\mathrm{i}}{\hbar}\int_0^T E_n(t)\,dt\right\}}_{Dynamical\ Phase\ Factor} \underbrace{\exp\left(\mathrm{i}\gamma_n(C)\right)}_{\substack{Geometric\ Phase\ Factor \\ or\ Berry\ Phase\ Factor}} \left|\Psi_n(0)\right\rangle$$

For degenerate state,

$$\left|\Psi_{n,a}(T)\right\rangle = \exp\left\{\frac{-\mathrm{i}}{\hbar}\int_0^T E(t)\,dt\right\}\hat{U}_{ab}(t)\left|\Psi_{n,b}(0)\right\rangle$$

$\hat{U}_{ad}$ = *Non-Abelean Unitary transformation*

Non-Abelean unitary transformation **can be written as**

$$\hat{U}(t) = T\exp\left(-\frac{i}{\hbar}\int_0^t H_{so}(t')\,dt'\right)$$

$$H_{SO} = 2\alpha\left(P_y S_x - P_x S_y\right) - 2\beta\left(P_x S_x - P_y S_y\right)$$

$$\alpha = \frac{\alpha_R eE}{\hbar}, \quad \beta = \frac{\gamma_c k^2}{\hbar}, \quad k = \left(\frac{2m^* eE}{\hbar}\right)^{1/3}$$

For GaAs, $\alpha_R = 0.044\ nm^2, \quad \gamma_c = 26\times 10^{-3}\,eV\ nm^3$

"S" is the spin operator whose components obey SU(2) Lie algebra

$$[S_+, S_-] = 2S_0, \quad [S_0, S_\pm] = 2S_\pm$$

$$S_\pm = S_x \pm iS_y$$

- ***We are interested to compute the spin propagator of the dot exactly based on the Feynman disentangling technique.***

- ***Another form of spin-orbit Hamiltonian***

$$H_{SO} = 2\alpha\left(P_y S_x - P_x S_y\right) - 2\beta\left(P_x S_x - P_y S_y\right)$$

On the other hand:

$$H_{SO} = H_x(t) S_x + H_y(t) S_y + H_z(t) S_z$$

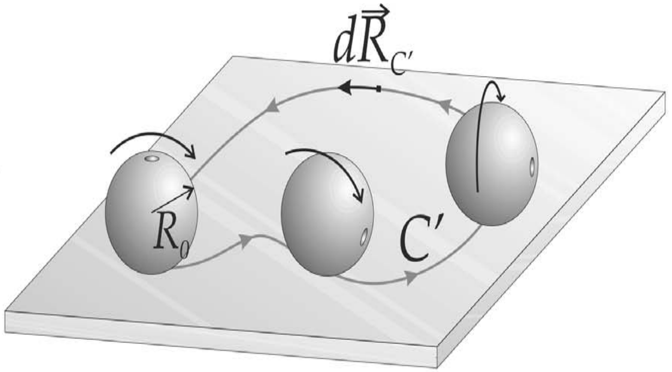
Compare

Therefore

$$H_x(t) = 2\alpha P_y - 2\beta P_x, \qquad H_y(t) = 2\beta P_y - 2\alpha P_x, \qquad H_z(t) = 0$$

$$H_{\pm} = \frac{1}{2}\left(H_x \mp iH_y\right) = \left(\alpha P_y - \beta P_x\right) \mp i\left(\beta P_y - \alpha P_x\right)$$

Consider a quantum dot orbiting in a closed path in the plane of 2DEG



$$P = \frac{d\vec{R}}{dt}, \quad R_x = R_0 \cos \omega t, \quad R_y = R_0 \sin \omega t$$

$$P_x = -R_0 m \omega \sin \omega t \qquad P_y = R_0 m \omega \cos \omega t$$

$$H_{\pm} = (\alpha P_y - \beta P_x) \mp i (\beta P_y - \alpha P_x)$$

$$H_{\pm} = R_0 m \omega (\alpha e^{\mp i \omega t} \mp \beta e^{\pm i \omega t})$$

Pablo San-Jose et al , ***Phys. Rev. B 77, 045305 (2008)***

Non-Abelean unitary transformation **can be written as**

$$\hat{U}_{ad}(t) = T\exp\left(-\frac{i}{\hbar}\int_0^t H_{so}(t^{'})dt^{'}\right) \quad \textit{where} \quad H_{so} = H_+ S_+ + H_- S_-$$

The propagator can be expanded by the method based on Feynman disentangling technique

$$\hat{U}_{ad}(t_0) = \exp(a\hat{S}_+)\exp(b\hat{S}_0)\exp(c\hat{S}_-)$$

$$S_\mu^{'} = \exp\left[\frac{i}{\hbar}\hat{S}_+\int_{t_1}^{t_2} x(t^{'})dt^{'}\right] S_\mu \exp\left[-\frac{i}{\hbar}\hat{S}_+\int_{t_1}^{t_2} x(t^{'})dt^{'}\right]$$

$$a(t^{'}) = -\frac{i}{\hbar}\int_{t_1}^{t_2} x(t^{'})dt^{'}$$

S. Prabhakar, J. E. Raynolds and A. Inomata, and R. Melnik; Phys. Rev. B 82, 195306 (2010); Also see V. S. Popov, Sov. Phys. JETP 35, 687 (1959); R. P. Feynman, Phys. Rev. 84, 108 (1951).

Result:

• ***Based on the Feynman disentangling technique, the evolution operator for spin propagator was solved exactly for three different cases:***

- ***Pure Rashba case***
- ***Pure Dresselhaus Case***
- ***Equal strength of Rashba and Dresselhaus spin-orbit coupling***

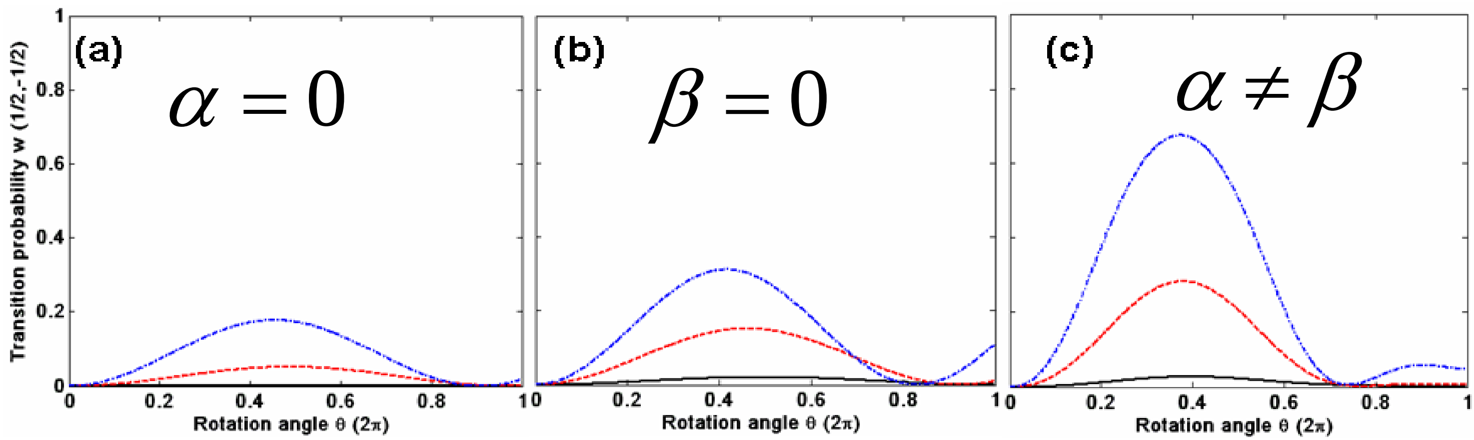
$$U_{ad}(t,0)=\begin{pmatrix} e^{b/2}+ac\,e^{-b/2} & a\,e^{-b/2} \\ c\,e^{-b/2} & e^{-b/2} \end{pmatrix}$$

$$a(t)=\frac{i-1}{\sqrt{2}}\frac{e^{\phi}-1}{e^{\phi}+1} \qquad \phi=\pm 2\sqrt{2}\,iR_0\alpha\left(1+\sin\omega t-\cos\omega t\right)$$

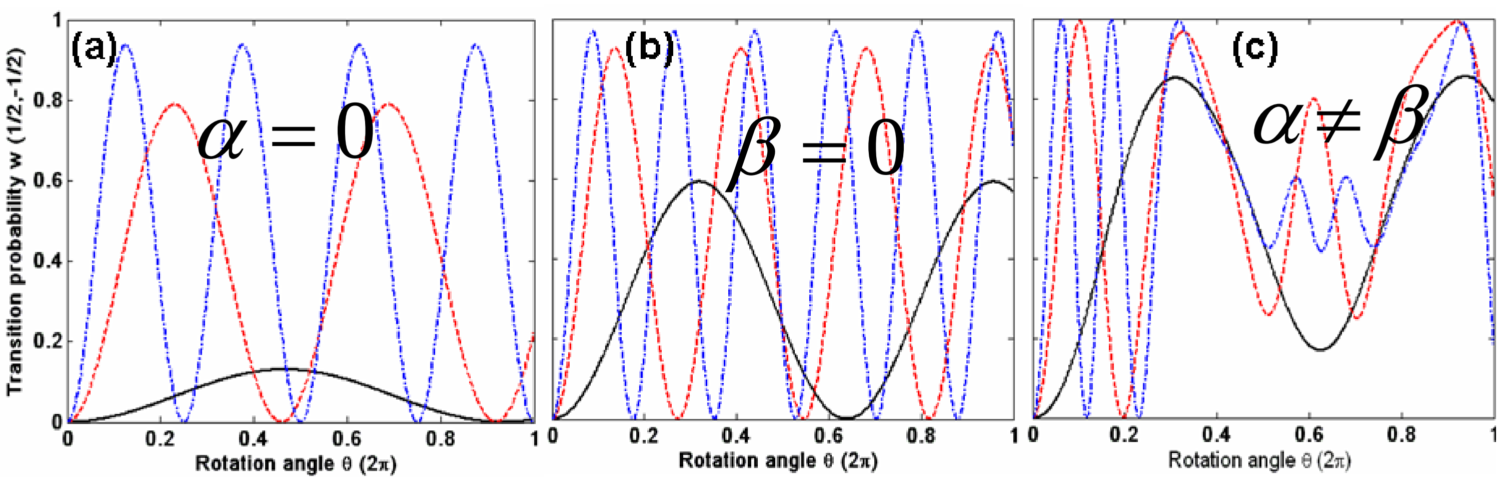
$$e^{b(t)}=\frac{4\,e^{\phi\mp i2\sqrt{2}R_0\alpha}}{\left(1+e^{\phi}\right)^2} \qquad c(t)=\frac{\sqrt{2}\left(1+i\right)}{1+e^{\pm i2\sqrt{2}R_0\alpha}}\frac{1-e^{\phi\mp i2\sqrt{2}R_0\alpha}}{e^{\phi}+1}$$

A single function needed to calculate the transition probability of Electron Spin

$$w_{-1/2,1/2} = |a|^2 \left(1 + |a|^2\right)^{-1} \qquad E = (1,5,10) \times 10^5 \; V/cm$$



$R_0 = 60 \; nm$



$R_0 = 500 \; nm$

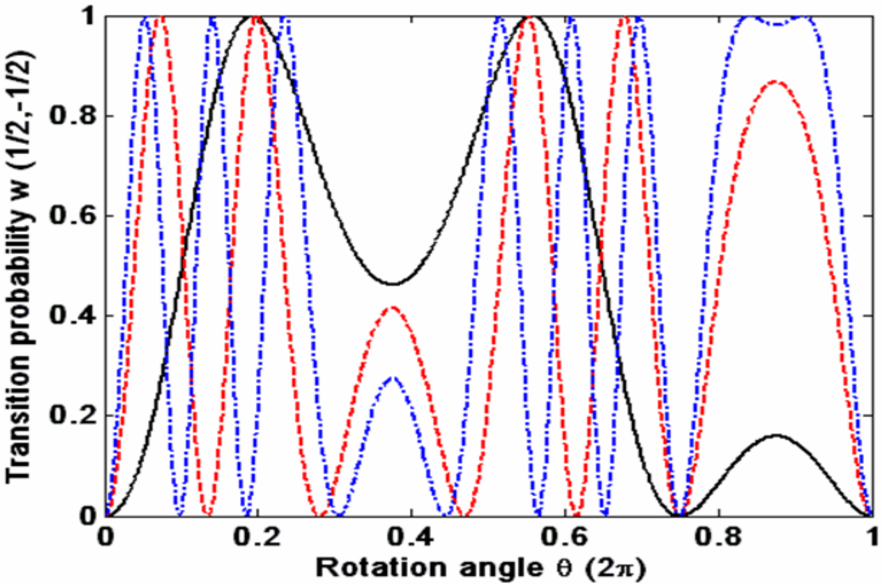
Exact Non-Abelean unitary transformation and Transition Probability of Electron Spin were found for equal strength of Rashba and Dresselhaus spin orbit coupling

- ***Exact Analytical Expression of the coefficient 'a' can be found as***

$$a(\tau) = \frac{e^{-i\theta} - i\,e^{i\theta}}{\sqrt{2}\left[1+\sin\,2\theta\right]^{1/2}}\,\frac{1-e^{i\phi}}{1+e^{i\phi}},\quad \textit{where}\quad \phi = \frac{\sqrt{2}\,m^{*}R_0\,\alpha}{\hbar}\left(\sin\theta-\cos\theta+1\right)$$

- ***Transition probability of electron spin can be found as***

$$w_{-1/2,1/2} = |a|^2\left(1+|a|^2\right)^{-1} = \sin^2\left[\frac{m^{*}R_0\alpha}{\hbar\sqrt{2}}\left(\sin\theta-\cos\theta+1\right)\right]$$



- ***Orbit radius = 60, 175, 250 nm***

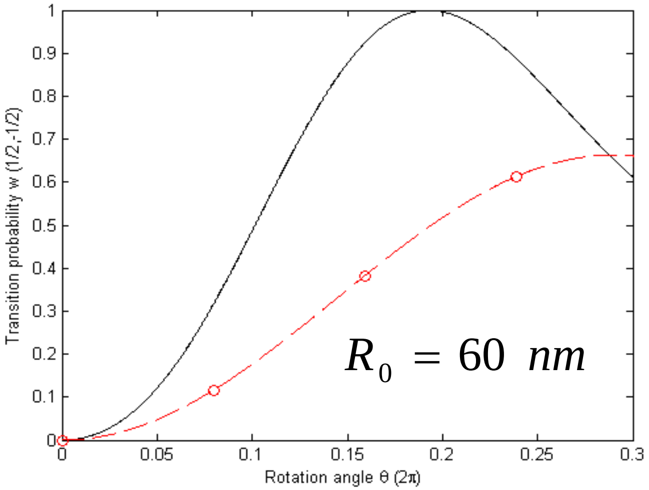
- ***Rashba and Dresselhaus spin-orbit interaction becomes equal at***

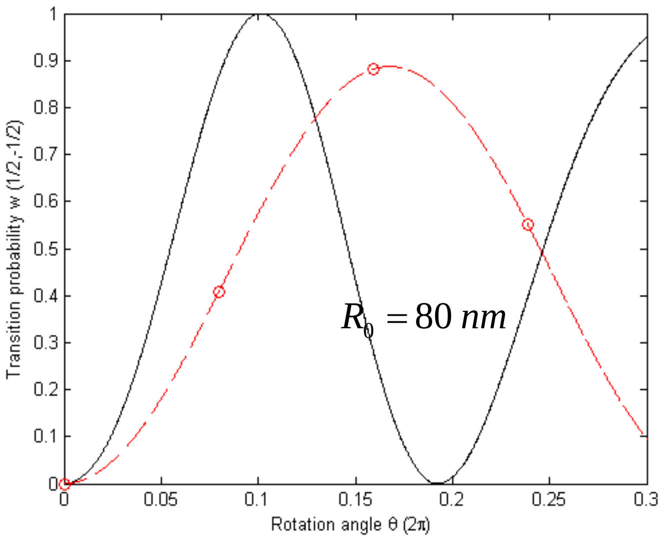
$$E = 3.02\times10^{6}\ V/cm$$

S. Prabhakar, J. E. Raynolds and A. Inomata, and R. Melnik; Phys. Rev. B 82, 195306 (2010)

Diffusion length and persistent spin-helix

•Rashba and Dresselhaus spin orbit interaction becomes equal at $E = 3.02\times 10^{6}\ V/cm$





•Solid Line (Black) is for equal strength of Rashba and Dresselhaus spin-orbit coupling

•Dashed open circles (red) is for either Rashba or Dresselhaus spin-orbit coupling

$$w_{-1/2,1/2} = \sin^2\left[\frac{m^* R_0 \alpha}{\hbar\sqrt{2}}\left(\sin\theta - \cos\theta + 1\right)\right]$$

$$\theta = \varepsilon \approx 0 \ or,\ \theta = \frac{3\pi}{2} - \varepsilon \qquad \sin\theta - \cos\theta + 1 \approx \varepsilon$$

$$w_{-1/2,1/2} = \sin^2\left\{\sqrt{2}\,\alpha R_0 \varepsilon\right\}$$

$$0 \le \sqrt{2}\, R_0 \alpha\varepsilon \le \pi/2$$

Spin diffusion length $$L_s = \frac{R_0\varepsilon}{\pi} = \frac{1}{2\sqrt{2}\,\alpha}$$

$$\alpha = \frac{\alpha_R eE}{\hbar} \text{ for GaAs},\ \alpha_R = 0.044\ nm^2$$

S. Prabhakar, J. E. Raynolds and A. Inomata, and R. Melnik, Phys. Rev. B 82, 195306 (2010); Koralek et al, Nature 458,610 (2009)

Conclusions

- ***Adiabatic control of spin states through geometric phase has been proposed.***
- ***Exact analytical solution of the evolution operator for spin propagator and transition probability has been found.***
- ***It was found that spin-orbit interaction enhances the electron spin transition probability.***
- ***Spin diffusion length for equal strength of Rashba and Dresselhaus spin-orbit coupling has been found through the application of geometric phase.***

Funding Agency:This work is supported by Canada Research Chair Program and NSERC.

•*S. Prabhakar, J Raynolds, A. Inomata, R. Melnik, Phys. Rev. B, 82, 195306 (2010).*
•*S. Prabhakar, R. Melnik, J. Appl. Phys., 108, 064330 (2010).*
•*S. Prabhakar and J. Raynolds, Phys. Rev. B, 79, 195307 (2009).*