# Foundation of Stochastic Analysis September 18-23, 2011

# MEALS

\*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday \*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday

\*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday

Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL).

\*Please remember to scan your meal card at the host/hostess station in the dining room for each meal.

# MEETING ROOMS

All lectures will be held in the new lecture theater in the TransCanada Pipelines Pavilion (TCPL). LCD projector, overhead projectors and blackboards are available for presentations. Note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.

# TENTATIVE SCHEDULE

Sunday	September 18, 2011
16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30 - 19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall
	Beverages and a small assortment of snacks are available on a cash honor system.

Monday	September 19, 2011
7:00 - 8:45	Breakfast, Sally Borden Building
8:45 - 9:00	Introduction and Welcome by BIRS Station Facilitator, Max Bell 159
9:00-9:40	40 minutes Lecture: Yves Lejan
9:50-10:30	40 minutes Lecture: Martin Hairer
10:35-10:55	20 minutes Coffee Break
10:55-11:35	40 minutes Lecture: P.J. Fitzsimmons
11:40 - 13:00	Lunch, Sally Borden Building
13:00 - 14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:05-14:30	25 minutes Lecture: Renming Song
14:35-15:00	25 minutes Lecture: Zoran Vondracek
15:05-15:25	20 minutes Coffee Break
15:25-16:05	40 minutes Lecture: Masayoshi Takeda
16:15-16:55	40 minutes Lecture: Krzysztof Bogdan
17:05-17:30	25 minutes Lecture: Rene Schilling
17:35-18:00	25 minutes Lecture: Mateusz Kwasnicki
18:05 - 19:30	Dinner, Sally Borden Building

Tuesday	September 20, 2011
7:00 - 8:30	Breakfast, Sally Borden Building
8:30-9:10	40 minutes Lecture: Masatoshi Fukushima
9:20-10:00	40 minutes Lecture: Xuemei Li
10:05-10:25	20 minutes Coffee Break
10:25-10:50	25 minutes Lecture: Tadeusz Kulczycki
10:55-11:20	25 minutes Lecture: Panki Kim
11:30-11:55	25 minutes Lecture: Jason Swanson
12:00 - 13:30	Lunch, Sally Borden Building
13:30-14:10	40 minutes Lecture: Steven N. Evans
14:20-15:00	40 minutes Lecture: Jun Kigami
15:05-15:30	25 minutes Coffee Break
15:30-15:55	25 minutes Lecture: David Croydon
16:00-16:25	25 minutes Lecture: Naotaka Kajino
16:30-16:55	25 minutes Lecture: Anita Winter
17:05-17:30	25 minutes Lecture: Soumik Pal
17:35-18:00	25 minutes Lecture: Yanxia Ren
18:05 - 19:30	Dinner, Sally Borden Building

Wednesday	September 21, 2011
7:00 - 8:30	Breakfast, Sally Borden Building
8:30-9:10	40 minutes Lecture: J-D. Deuschel
9:20-10:00	40 minutes Lecture: Nathalie Eisenbaum
10:05-10:30	25 minutes Coffee Break
10:30-10:55	25 minutes Lecture: Jie Xiong
11:00-11:25	25 minutes Lecture: Kainan Xiang
11:30 - 13:00	Lunch, Sally Borden Building
13:00-18:00	Free Afternoon
18:00 - 19:30	<b>Dinner</b> , Sally Borden Building

September 22, 2011
Breakfast, Sally Borden Building
40 minutes Lecture: Richard Bass
40 minutes Lecture: Edwin Perkins
20 minutes Coffee Break
25 minutes Lecture: Elton P. Hsu
25 minutes Lecture: Ben Hambly
25 minutes Lecture: Masha Gordina
Lunch, Sally Borden Building
Group Photo; meet on the front steps of Corbett Hall
40 minutes Lecture: Martin Barlow
25 minutes Lecture: Moritz Kassmann
30 minutes Coffee Break
40 minutes Lecture: Davar Khoshnevisan
25 minutes Lecture: Shui Feng
25 minutes Lecture: Toshihiro Uemura
40 minutes Lecture: Rodrigo Banuelos
Dinner, Sally Borden Building
Reception in honor of Rich Bass, 2nd floor lounge, Corbett Hall

Friday	September 23, 2011
7:00 - 8:40	Breakfast, Sally Borden Building
8:40-9:20	40 minutes Lecture: Karl-Theodor Sturm
9:30-10:10	40 minutes Lecture: Michael Roeckner
10:15-10:45	30 minutes Coffee Break
10:45-11:30	Informal discussion
11:30 - 13:30	Lunch, Sally Borden Building

\*\* Checkout by 12 noon. Participants are welcome to use BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. \*\*

# Foundation of Stochastic Analysis September 18-23, 2011

# ABSTRACTS (in alphabetic order by speaker surname)

## Speaker: Rodrigo Banuelos (Purdue University)

# Title: Martingales and Fourier multipliers, sharp estimates

Abstract: We shall discuss some recent applications of the sharp martingale inequalities of Burkholder to various Fourier multipliers including some that arise from transformations of Lévy processes via the Lévy–Khintchine formula. While these results are of interest on their own right, they are motivated by some well known open problems in analysis and PDE's.

# Speaker: Martin Barlow (University of British Columbia)

## Title: Energy of cutoff functions and applications

Abstract: A well known theorem of Grigoryan and Saloff Coste states that Gaussian heat kernels bounds are equivalent to volume doubling (D) plus Poincare inequalities (PI). In the sub-Gaussian case, which arises on various regular fractals, Rich Bass and I showed that these are equivalent to (D), (PI) and an additional condition, denoted CS(b). The condition CS(b) gives the existence of low energy cutoff functions. In this talk I will give some additional applications of CS(b), which hold without (PI). This is joint work with Sebastian Andres.

# Speaker: Richard F. Bass (University of Connecticut)

## Title: Uniqueness in law for parabolic SPDEs and infinite dimensional SDEs

Abstract: We consider the heat equation on an interval with heat being introduced according to a random mechanism. When the random mechanism is space-time white noise, this equation has been much studied. We look at the case where the white noise is modified by a function A(u)(x) of the current temperatures u and where A is Hölder continuous as a function of u. Unlike other work along these lines, A(u)(x) can depend on the temperatures throughout the interval. Our method involves looking at the Fourier coefficients, which leads to an infinite dimensional system of stochastic differential equations. This is joint work with Ed Perkins.

## Speaker: Krzysztof Bogdan (Wrocław University of Technology, Poland)

#### Title: Schrödinger perturbations

Abstract: I will report joint work in progress with Wolfhard Hansen, Tomasz Jakubowski and Sebastian Sydor on Schrödinger perturbations of integral kernels; ones which produce comparable integral kernels.

- [1] W. Hansen. Global comparison of perturbed Green functions. Math. Ann. 334 (2006), no. 3, 643–678.
- [2] K. Bogdan, W. Hansen, T. Jakubowski. Time-dependent Schrödinger perturbations of transition densities. Studia Math. 189 (2008), no. 3, 235–254.
- [3] T. Jakubowski. On combinatorics of Schrödinger perturbations. Potential Anal. 31 (2009), no. 1, 45–55.
- [4] T. Jakubowski, K. Szczypkowski. Time-dependent gradient perturbations of fractional Laplacian. J. Evol. Equ. 10 (2010), no. 2, 319–339.

## Speaker: David Croydon (University of Warwick, UK)

Title: Mixing time convergence for sequences of random walks on graphs

Abstract: The main conclusion of this work, which is a joint project with Ben Hambly (University of

Oxford) and Takashi Kumagai (Kyoto University), is that the mixing times of the simple random walks on a sequence of graphs converge to the mixing time of a limiting diffusion whenever the corresponding state spaces, invariant measures and heat kernels converge in a suitable Gromov-Hausdorff sense. In addition to presenting this result, I will describe how it can be applied to a number of interesting examples, including some simple lattice models, self-similar fractal graphs with random weights, critical Galton-Watson trees, the Erdos-Renyi random graph at criticality and the range of a random walk in high dimensions.

# Speaker: Jean-Dominique Deusche (Berlin Technical University, Germany)

# Title: Markov chain approximations to non-symmetric diffusions with bounded coefficients

Abstract: We consider a certain class of non-symmetric Markov chains and obtain heat kernel bounds and parabolic Harnack inequalities. Using the heat kernel estimates, we establish a sufficient condition for the family of Markov chains to converge to non-symmetric diffusions. As an application, we approximate non-symmetric divergence forms with bounded coefficients by non-symmetric Markov chains. This extends the results by Stroock-Zheng ([SZ]) to the non-symmetric divergence forms. Joint work with Takashi Kumagai.

[SZ] D.W. Stroock and W. Zheng. Markov chain approximations to symmetric diffusions. Ann. Inst. Henri. Poincaré-Probab. Statist. 33 (1997), 619-649.

# Speaker: Nathalie Eisenbaum (Université Paris VI, France)

Title: Some properties of Gaussian processes

Abstract: We will describe and characterize some properties of Gaussian processes.

# Speaker: Steve Evans (University of California at Berkeley)

## Title: Uplift under Lévystan: Lipschitz minorants of Lévy processes

Abstract: Motivated by problems in statistical density estimation, Groeneboom 1983 established a number of remarkable properties of the greatest convex function dominated by the path of a one-dimensional Brownian motion. Pitman, Bass, Cinlar, Bertoin and others extended these results and showed how they could be derived more easily using techniques such as excursion theory and last exit decompositions. There has been a resurgence of interest in this area with a flurry of recent work on convex minorants of Lévy processes and random walks by various combinations of Abramson, Pitman, Ross and Uribe Bravo. Researchers in non-linear optimization, particularly optimal transport, have developed the notion of cconvexity that greatly generalizes the familiar definition of convex function while keeping many of its important features such as Legendre-Fenchel duality. Within this theory, the set of functions from the reals to the reals with a given upper bound K on their Lipschitz constant share many of the features of the convex functions. I will show that if a Lévy process indexed by the real line has a K-Lipschitz minorant, then the set of times where the process is in contact with its minorant is a stationary regenerative set and hence it is the closed range of a subordinator "made stationary". It is possible to characterize this subordinator in many cases and determine properties such as whether or not its range is countable or has zero Lebesgue measure. If the Lévy process is a Brownian motion with drift, then there are explicit distributions for a number of random variables related to the minorant. This work is joint with Josh Abramson.

## Speaker: Shui Feng (McMaster University)

## Title: An Open Problem Associated with the Two-parameter Dirichlet Process

Abstract: The two-parameter Dirichlet process is a random measure generalizing Ferguson's Dirichlet process. This talk will discuss progresses and difficulties in constructing a reversible diffusion process with the two-parameter Dirichlet process as the reversible measure.

# Speaker: Patrick J. Fitzsimmons (University of California at San Diego)

Title: Zero-Energy Functions of a Symmetric Diffusion

Abstract: Consider a continuous function f on the state space of a symmetric diffusion  $X = (X_t : t \ge 0)$ 

such that  $t \mapsto f(X_t)$  is locally of zero energy. Using time-reversal arguments, I will show that f must be constant along the paths of X. The space-time version of this result will also be discussed. Time permitting, I will examine a representation of general continuous additive functionals of X, extending a result of Tanaka for one-dimensional Brownian motion.

#### Speaker: Masatoshi Fukushima (Osaka University, Japan)

Title: On Brownian motion with darning and Komatu-Leowner equation for multiply connected planar domains

Abstract: Brownian motion with darning (BMD in abbreviation) is applied to the study of conformal mappings of multiply connected planar domains and the associated Komatu-Leowner differential equations. The notion of BMD and its basic properties are presented in Chapter 7 of a forthcoming book [CF] in a more general context. It is a diffusion process obtained from the absorbing Brownian motion on a multiply connected planar domain by rendering each hole of the domain into one point. Especially the zero period property of a general BMD-harmonic function and the conformal invariance of BMD will be used to derive the continuity of some fundamental quantities for the Komatu-Leowner equation. We are motivated by the works [BF1],[BF2],K and [L]. This talk is based on an ongoing joint work with Z.-Q. Chen and S. Rohde.

- [BF1] R.O. Bauer and R.M. Friedrich, On radial stochastic Leowner evolution in multiply connected domains. J. Funct. Anal. 237 (2006), 565-588.
- [BF2] R.O. Bauer and R.M. Friedrich, On chordal and bilateral SLE in multiply connected domains. Math. Z. 258 (2008), 241-265.
- [CF] Z.-Q. Chen and M. Fukushima, Symmetric Markov Processes, Time Change, and Boundary Theory. Princeton University Press, to appear.
- [K] Y. Komatu, On conformal slit mapping of multiply-connected domains. Proc. Japan Acad. 26 (1950), 26-31.
- [L] G. F. Lawler, The Laplacian-b random walk and the Schramm-Leowner evolution. *Illinois J. Math.* **50** (2006), 701-746.

# Speaker: Maria Gordina (University of Connecticut)

# Title: Functional inequalities in infinite dimensions

Abstract: we will talk about recent results on functional inequalities on infinite-dimensional curved spaces and applications of such inequalities. One type of such results concerns integrated Harnack inequalities for the heat kernel measures, and how these inequalities can be used to prove quasi-invariance of heat kernel measures on certain infinite-dimensional groups. These results use geometric data such as lower Ricci bounds. Therefore these techniques is not applicable to sub-elliptic Laplacians. Most of finite-dimensional methods for such heat kernels are dimension-dependent. Nevertheless, recently we were able to prove several types of functional inequalities for a sub-elliptic Laplacian on an infinite-dimensional Heisenberg group. The results of this talk are based on joint work with F. Baudoin, B. Driver, T. Melcher.

#### Speaker: Martin Hairer (University of Warwick, UK)

# Title: Hypoelliptic SDEs driven by rough paths

Abstract: We present a number of recent results extending Hörmander's theorem to a larger class of SDEs driven by Gaussian stochastic processes. In particular, we focus on the case when the driving process is rougher than Brownian motion, using the theory of rough paths to give meaning to the equations. Our main ingredient in the implementation of Malliavin's programme is a deterministic version of Norris's lemma. This relies on a quantitative property of the driving process (a "modulus of Hölder roughness") ensuring that it is sufficiently non-degenerate. One application of the theory is to demonstrate ergodicity of such equations under conditions that are qualitatively similar to those in the classical ergodic theory for SDEs.

Speaker: **Ben Hambly** (University of Oxford) Title: Spectral asymptotics for random fractals Abstract: I will discuss some results concerning the high frequency asymptotics of the eigenvalue counting function for a variety of random fractals. The focus will mainly be on the continuum random tree where we can show the size of the second order term in the asymptotic expansion. We will also discuss other random recursive fractals where a variety of behaviors is possible.

# Speaker: Elton Hsu (Northwestern University)

#### Title: Brownian Motion and Transpotation Inequality on Path Spaces

Abstract: We will show how to use synchronizing Riemannian Brownian motion to prove generalizations of Talagrand's transportation cost inequality to the heat kernel measure on a Riemannian manifold and the Wiener measure on the path space over a Riemannian manifold.

## Speaker: Naotaka Kajino (University of Bielefeld)

Title: Weyl's Laplacian eigenvalue asymptotics for the measurable Riemannian structure on the Sierpinski gasket

Abstract: On the Sierpinski gasket K, Kigami [Math. Ann. 340 (2008), 781–804] has introduced the notion of the measurable Riemannian structure, with which the gradient vector fields of functions, the Riemannian volume measure  $\mu$  and the geodesic metric  $\rho$  are naturally associated. Kigami has also proved in the same paper the two-sided Gaussian bound for the corresponding heat kernel, and I have further shown several detailed heat kernel asymptotics, such as Varadhan's asymptotic relation, in [Potential Anal., in press, doi: 10.1007/s11118-011-9221-5].

In the talk I will talk about the Weyl's Laplacian eigenvalue asymptotics for this case. The correct scaling order for the asymptotics of the eigenvalues is given by the Hausdorff dimension d of the metric space  $(K, \rho)$ , and in the limit of the eigenvalue asymptotics we obtain a constant multiple of the d-dimensional Hausdorff measure  $H^d$ . Moreover, we will also see that this Hausdorff measure  $H^d$  is Ahlfors d-regular with respect to  $\rho$  but that it is singular to the Riemannian volume measure  $\mu$ .

## Speaker: Moritz Kassmann (University of Bielefeld, Germany)

## Title: Harnack's inequality: a new formulation and applications

Abstract: We present a formulation of Harnack's inequality which is applicable to local and nonlocal operators at the same time. We show that this version of Harnack's inequality implies regularity estimates for solutions to several integrodifferential operators. We apply the method to some nonlocal symmetric Dirichlet forms and to generators of jump processes. We discuss how this approach extends known results. The talk is based on three works: [K. 2010], [K.-Mimica 2011] and [Dyda-K. 2011]

## Speaker: Davar Khoshnevisan (University of Utah)

## Title: On the chaotic character of some parabolic SPDEs

Abstract: I will describe how a family of randomly-forced heat equations depends sensitively on the choice of the initial function. Time permitting, we will see a surprising connection, and derivation, of the so called KPZ relation between the spatial and temporal fluctuation exponents of the solution to a family of parabolic Anderson models. This talk is based on joint works with D. Conus, M. Foondun, M. Joseph, and S. Shiu.

# Speaker: Jun Kigami (Kyoto University, Japan)

## Title: Dirichlet forms on a noncompact Cantor set and random walks on its defining tree

Abstract: First we will construct a class of Dirichlet forms on a noncompact Cantor set, which is a generalization of *p*-adic numbers, from prescribed sets of eigenvalues and measures. At the same time, we have concrete expressions of the jump kernel and the transition density. Assuming the volume doubling condition, we construct an intrinsic metric under which estimates of transition density and jump kernel are obtained. Secondly transient random walks on the defining tree of the noncompact Cantor set are shown to induce a subclass of Dirichlet forms discussed in the first part on the noncompact Cantor set as traces.

#### Speaker: Panki Kim (Seoul National University)

Title: Oscillation of unbounded harmonic functions for subordinate Brownian motion and its applications Abstract: In this talk we discuss the oscillation of unbounded harmonic functions for pure-jump subordinate Brownian motion. As an application, we give a probabilistic proof of relative Fatou's theorem for harmonic functions for such subordinate Brownian motion. This is a joint work with Yunju Lee.

Speaker: **Tadeusz Kulczycki** (Polish Academy of Sciences and Wrocław University of Technology, Poland) Title: On jump processes which are traces of reflected Brownian motion

Abstract: Let  $B_t$  be a reflected Brownian motion in a bounded domain  $D \subset \mathbb{R}^d$ . Assume that there exists a part F of a boundary of D which is flat (for example D can be a cylinder and F one of its bases or Dcan be a triangle and F one of its sides). Let us consider the process  $X_t$  which trajectories are traces of the process  $B_t$  on F. More formally  $X_t = B_{\eta_t}$ , where  $\eta_t$  is the inverse of a local time of  $B_t$  on F.

We will show the connection of the process  $X_t$  with the Dirichlet to Neumann map and with the mixed Steklov problem. Such Steklov problem appears very naturally in some problems of hydrodynamics.

It occurs that the jump process  $X_t$  on F shares many properties of a reflected Brownian motion on F. In particular we will show that under some geometric assumptions on D and F the hot spots property holds for the process  $X_t$ . That is, the first non-constant eigenfunction corresponding to the semigroup of the process  $X_t$  attains its maximum and minimum on the boundary of F. We will also study some properties of the process  $X_t$  under continuous deformation of the domain D. Some open questions will be also discussed.

## Speaker: Mateusz Kwasnicki (Wrocław University of Technology, Poland)

#### Title: Boundary Harnack inequality for jump-type processes

Abstract: I will present the results of my recent work with Krzysztof Bogdan and Takashi Kumagai. We consider a Hunt process  $X_t$  in a metric measure space. Under relatively mild assumptions, we prove a boundary Harnack inequality for nonnegative functions harmonic with respect to  $X_t$  in an arbitrary open set. We require:

- (1) existence of the dual process  $X_t^*$ ,
- (2) existence of bump functions in the domains of the Feller generators of  $X_t$  and  $X_t^*$ ,
- (3) boundedness (away from the diagonal) of the jumping kernel and the potential kernel.

Under these assumptions, we prove boundedness near the boundary point  $z \in \partial D$  of the ratio f(x)/g(x) of any two nonnegative harmonic functions f, g in an open set D, given that f and g vanish off D in some neighbourhood of z. More precisely, we prove that in this case, if d(x, z) < r and d(y, z) < r, then

$$\frac{1}{C} \frac{f(x)}{g(x)} \le \frac{f(y)}{g(y)} \le C \frac{f(x)}{g(x)},$$

where C does not depend on D, x, y, f and g, given that f(x) = g(x) = 0 whenever  $x \notin D$  and d(x, z) < 2r.

When  $X_t$  is a symmetric (i.e. self-dual) process which admits local stable-like bounds for the transition density (i.e. heat kernel), then (1) and (3) are satisfied automatically. The only restrictive assumption is in this case (2). In Euclidean setting, (2) holds true if the jumping kernel is sufficiently regular. In general metric measure spaces, (2) can often be proved when there is a sub-Gaussian diffusion process. Under these assumptions, our boundary Harnack inequality is scale-invariant and, in some sense, stable under small perturbations.

The proof involves a new, probabilistic method for proving a priori supremum bounds for nonnegative harmonic functions, which may be of independent interest.

Speaker: **Yves Le Jan** (Université Paris-Sud, France) Title: *The signature of Brownian paths*  Abstract: We show that in dimension larger than two, Brownian paths indexed by [0,T], are determined by their iterated integrals taken up to T.

# Speaker: Xue-Mei Li (University of Warwick, UK)

# Title: The derivative process for SDEs with non-smooth coefficients

Abstract: For an SDE with smooth coefficients, the solution depends continuously on the initial data. The latter is a basic assumption for numerical schemes. In this talk we discuss the construction of a stochastic process, which we call the derivative process, associated to the SDE in the case of the coefficients not differentiable. This is used to give a probabilistic representation for the derivative of the corresponding semi-group. For this construction we also studied the convergence of approximating SDEs and make sense of of an SDE with random coefficients. As a by product we have an existence theorem for the smooth solution flow.

#### Speaker: Soumik Pal (University of Washington)

#### Title: Transportation Cost Inequalities for rank-based models

Abstract: Transportation Cost inequalities are functional inequalities that compare the Wasserstein distance between two probability measures with their relative entropy. Via an argument by Marton these inequalities are powerful methods to show concentration of measure properties. We focus on applications to certain interacting diffusion processes called rank-based models. This is a multidimensional diffusion model where each particle (coordinate index) gets an instantaneous drift and diffusion coefficient depending on its rank among all the particles. It is known that if we exponentiate the coordinates and rescale to have a total sum of one, the ordered values exhibit power law decay in equilibrium. Using Transportation Cost Inequalities we derive uniform Gaussian tail bounds for the empirical least-square estimate of the index of this power law over large intervals of time. Part of this talk is based on joint work with Misha Shkolnikov.

# Speaker: Edwin Perkins (University of British Columbia)

Title: Nonuniqueness for a parabolic SPDE with  $\frac{3}{4} - \epsilon$ -Hölder diffusion coefficients

Abstract: We prove an analogue of the Girsanov examples for SDE's for the parabolic stochastic partial differential equation (SPDE)

$$\frac{\partial u}{\partial t} = \frac{\Delta}{2} u(t,x) + |u(t,x)|^{\gamma} \dot{W}(t,x),$$

with zero initial conditions. Here  $\dot{W}$  is a space-time white noise on  $R_+ \times R$ . More precisely, we show the above stochastic pde has a non-zero solution for  $0 < \gamma < 3/4$ , and hence solutions are not unique in law or pathwise unique. The case  $\gamma = 1/2$  arises as a scaling limit point of a system of branching annihilating random walks. An analogue of Yamada-Watanabe's theorem for SDE's was recently shown by Mytnik and Perkins for SPDE's by establishing pathwise uniqueness of solutions to

$$\frac{\partial u}{\partial t} = \frac{\Delta}{2} u(t, x) + \sigma(u(t, x)) \dot{W}(t, x),$$

where  $\sigma$  is Holder continuous of index  $\gamma > 3/4$ . The situation for the above class of parabolic SPDE's is therefore similar to their finite dimensional counterparts, but with the index 3/4 in place of 1/2. This is joint work with Carl Mueller and Leonid Mytnik.

#### Speaker: Yanxia Ren (Peking University)

#### Title: Small value probabilities for supercritical branching processes with immigration

Abstract: We consider a supercritical Galton-Watson branching process with immigration. It is well known that under suitable conditions on the offspring and immigration distributions, there is a finite, strictly positive and non-degenerate limit for the normalized population size, denoted as W. The main purpose of this paper is to investigate the small value probabilities of W, that is to estimate  $P(W \leq \varepsilon)$ for  $\varepsilon > 0$  small. In comparison with the well-studied results for supercritical Galton-Watson branching process without immigration, precise effects of the balance between offspring and immigration distributions on small value probability of  $\mathcal{W}$ , are obtained. Several illustrative examples are analyzed carefully. They demonstrate the sharpness of our results and the significant effect of the immigration which can cause the near-constancy phenomena even when there is no oscillation in the setting without immigration.

#### Speaker: Michael Röckner (University of Bielefeld, Germany)

Title: Recent extinction results for stochastic porous media equations and applications to self-organized criticality

Abstract: The first part of the talk will recall extinction results for stochastic partial differential equations (with multiplicative noise) of porous media type. These include stochastic fast diffusion equations and more singular cases, where e.g. the nonlinearity is given by a Heaviside or sign function, so is multivalued. The latter describe certain continuum models for the phenomenon of self-organized criticality (SOC). These extinction results have been obtained in the past two years. In the SOC-case, however, extinction was only shown if the underlying spatial domain is one-dimensional and only with positive (though high) probability. The second part of the talk is devoted to very recent results, where for the SOC-case asymptotic extinction was proved for all spatial dimensions and shown to be locally exponentially fast. One main technique in the proofs is to transform the stochastic PDE into a deterministic PDE with a random parameter. The resulting deterministic PDE is of an entirely new type and new methods had to be invented for its analysis.

- J. Ren, M. Röckner, F.-Y. Wang Stochastic generalized porous media and fast diffusion equations, J. Diff. Equations 238 (2007), no. 1, 118–152.
- [2] V. Barbu, G. Da Prato, M. Röckner Finite time extinction for solutions to fast diffusion stochastic porous media equations, C. R. Acad. Sci. Paris – Mathematics 347 (2009), no. 1–2, 81–84.
- [3] V. Barbu, G. Da Prato, M. Röckner Stochastic porous media equation and self-organized criticality, Comm. Math. Phys. 285 (2009), no. 3, 901–923.
- [4] V. Barbu, P. Blanchard, G. Da Prato, M. Röckner Self-organized criticality via stochastic partial differential equations, Theta Series in Advanced Mathematics, "Potential Theory and Stochastic Analysis" in Albac. Aurel Cornea Memorial Volume, 2009, pp. 11–19.
- [5] V. Barbu, M. Röckner On a random scaled porous media equation, BiBoS-Preprint, publication in preparation, 22 pp., 2010.
- [6] V. Barbu, M. Röckner Stochastic porous media equations and self-organized criticality: convergence to the critical state in all dimensions, BiBoS–Preprint, publication in preparation, 21 pp., 2011.

#### Speaker: **Rene Schilling** (Technical University at Dresden, Germany)

#### Title: Sample Path Properties of Feller Processes

Abstract: We present sufficient conditions for the transience and the existence of local times of a Feller process, and the ultracontractivity of the associated Feller semigroup. These conditions are sharp for Lévy processes, and they are based on the local symmetrization technique and a uniform upper bound for the characteristic function of a Feller process. As a byproduct, we obtain for stable-like processes (in the sense of R. Bass) on  $\mathbb{R}^d$  with smooth variable index  $\alpha(x) \in (0, 2)$  a transience criterion in terms of the exponent  $\alpha(x)$ ; if d = 1 and  $\inf_{x \in \mathbb{R}} \alpha(x) \in (1, 2)$ , then the stable-like process admits local times.

This work is joint with Jian Wang (Fujian Normal University and TU Dresden)

## Speaker: Renming Song (University of Illinois)

#### Title: Potential theory of subordinate Brownian motions

Abstract: A subordinate Brownian motion is a Levy process which can be by replacing the time parameter of Brownian motion by an independent subordinator. The subordinator can thought of as "intrinsic" time or "operational" time. Subordinate Brownian motions form a large subclass of Levy processes and they are widely used in applications.

Recently, a lot of progress has been made in the study of the potential theory of subordinate Brownian motions. In this talk, I will give a survey of some recent results on the potential theory of subordinate

Brownian motions without Gaussian components. In particular, I will present a boundary Harnack principle and two-sided estimates on the Green functions of these processes in bounded smooth open sets.

This talk is based on joint works with Panki Kim and Zoran Vondracek.

# Speaker: Karl-Theodor Sturm (University of Bonn, Germany)

#### Title: Optimal Transport from Lebesgue to Poisson

Abstract: We study couplings  $q^{\bullet}$  of the Lebesgue measure and the Poisson point process  $\mu^{\bullet}$ , i.e. measurevalued random variables  $\omega \mapsto q^{\omega}$  s.t. for a.e.  $\omega$  the measure  $q^{\omega}$  on  $\mathbb{R}^d \times \mathbb{R}^d$  is a coupling of  $\mathfrak{L}^d$  and  $\mu^{\omega}$ . For any given  $p \in (0, \infty)$  we ask for a minimizer of the mean  $L^p$ -transportation cost

$$\mathfrak{C}(q^{\bullet}) \;=\; \sup_{B \subset \mathbb{R}^d} \; \frac{1}{\mathfrak{L}^d(B)} \mathbb{E}\left[ \int_{\mathbb{R}^d \times B} |x - y|^p \, dq^{\bullet}(x, y) \right].$$

The minimal mean  $L^p$ -transportation cost turns out to be finite for all p provided  $d \ge 3$ . If  $d \le 2$  then it is finite if and only if p < d/2.

Moreover, in any of these cases we prove that there exist a unique translation invariant coupling which minimizes the mean  $L^p$ -transportation cost. In the case p = 2, this 'optimal coupling' induces a random tiling of  $\mathbb{R}^d$  by convex polytopes of volume 1.

#### Speaker: Jason Swanson (Central Florida University)

#### Title: The calculus of differentials for the weak Stratonovich integral

Abstract: The weak Stratonovich integral is defined as the limit, in law, of Stratonovich-type symmetric Riemann sums. We derive an explicit expression for the weak Stratonovich integral of f(B) with respect to g(B), where B is a fractional Brownian motion with Hurst parameter 1/6, and f and g are smooth functions. We use this expression to derive an Itô-type formula for this integral. As in the case where g is the identity, the Itô-type formula has a correction term which is a classical Itô integral, and which is related to the so-called signed cubic variation of g(B). Finally, we derive a surprising formula for calculating with differentials. We show that if  $\mathbf{d}M = X \, \mathbf{d}N$ , then  $Z \, \mathbf{d}M$  can be written as  $ZX \, \mathbf{d}N$  minus a stochastic correction term which is again related to the signed cubic variation.

# Speaker: Masayoshi Takeda (Tohuku University)

## Title: Feynman-Kac Penalisations of Symmetric Stable Processes

Abstract: B. Roynette, P. Vallois and M. Yor have studied limit theorems for Wiener processes normalized by some weight processes. K. Yano, Y. Yano and M. Yor studied the limit theorems for the one-dimensional symmetric stable process normalized by non-negative functions of the local times or by negative (killing) Feynman-Kac functionals. Our aim is to extend their results on Feynman-Kac penalisations to positive Feynman-Kac functionals of multi-dimensional symmetric  $\alpha$ -stable processes.

## Speaker: Toshihiro Uemura (Kansai University)

#### Title: On multidimentional diffusion processes with jumps

Abstract: Let  $a_{ij}$  and  $b_i$  be coefficients of a second order partial differenctial operator defined on an open set D of  $\mathbb{R}^d$  and k a Levy-type kernel over D. We define a non-symmetric billinear form on L2(D; dx)having the dates above

Under some conditions on the datas, we will show the form becomes a lower bounded semi-Dirichlet form and there exists a diffusion process with jumps associated with the form.

## Speaker: **Zoran Vondracek** (University of Zagreb)

#### Title: Potential theory of subordinate Brownian motions with Gaussian components

Abstract: In this talk I will look at a subordinate Brownian motion with a Gaussian component and a rather general discontinuous part. The assumption on the subordinator is that its Laplace exponent is a complete Bernstein function with a Lévy density satisfying a certain growth condition near zero. The main result that I will present is a boundary Harnack principle with explicit boundary decay rate for

non-negative harmonic functions of the process in  $C^{1,1}$  open sets. I will also discuss an example showing that the boundary Harnack principle fails for processes with finite range jumps. As a consequence of the boundary Harnack principle, one can establish sharp two-sided estimates on the Green function of the subordinate Brownian motion in any bounded  $C^{1,1}$  open set D and identify the Martin boundary of Dwith respect to the subordinate Brownian motion with the Euclidean boundary.

Joint work with Panki Kim and Renming Song

#### Speaker: Anita Winter (Friedrich-Alexander-Universität Erlangen-Nürnberg )

#### Title: Aldous move on cladograms in the diffusion limit

Abstract: A N-cladogram is a semi-labeled, unrooted and binary tree with N leaves labeled  $\{1, 2, ..., N\}$ and with unit edge lengths. Aldous constructed a Markov chain on cladograms and gave bounds on its mixing time.

In this talk we use Dirichlet form methods to construct a continuous tree-valued diffusion and show that Aldous move on cladograms suitably rescaled converges in the weak Gromov topology to this diffusion provided that started in the uniform cladogram.

## Speaker: Kai-Nan Xian (Nankai University, China)

#### Title: An Explicit Schilder-type Theorem for Super-Brownian Motions

Abstract: Like ordinary Brownian motion, super-Brownian motion, a central object in the theory of superprocesses, is a universal object arising in a variety of settings. Schilder-type theorems and Cramér-type theorems are two of the major topics for large-deviation theory. A Schilder-type (which is also a Cramér-type) sample large deviation for super-Brownian motions with a good rate function represented by a variation formula was established in 1993 and 1994 (see [1]-[3]). There have been very valuable contributions for giving an affirmative answer to the question of whether this sample large deviation holds with an explicit good rate function since then. In [4], thanks to previous results on this issue and the Le Gall's Brownian snake, we established such a large deviation for nonzero finite initial measures. Then in [5], we proved the mentioned large deviation holds for infinite initial measures. Those concluded the attacking on the long-standing conjecture proposed in [1].

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- [2] Schied, A. Gro $\beta$ e Abweichungen für die Pfade der Super-Brownschen Bewegung. Bonner Math. Schriften 277, 1995.
- [3] Schied, A. Sample large deviations for super-Brownian motion. Prob. Th. Rel. Fields. 104 (1996), no. 3, 319-347.
- [4] Xiang, K. N. An explicit Schilder-type theorem for super-Brownian motions. Comm. Pure. Appl. Math. 63 (2010), no. 11, 1381-1431.
- [5] Xiang, K. N. An explicit Schilder-type theorem for super-Brownian motions II: infinite initial measures. Preprint, 2011.

#### Speaker: **Jie Xiong** (University of Tennessee)

Title: Uniqueness problems for some measure-valued processes

Abstract: A stochastic partial differential equation (SPDE) is derived for the super Brownian motion regarded as a distribution function valued process. The strong uniqueness for the solution to this SPDE is obtained by a connection between SPDEs and backward doubly stochastic differential equations. Similar results are also proved for the Fleming-Viot process. We also extend the uniqueness result to processes with interaction.