Gradient Random Fields
29th May – 3th June, 2011

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations. Note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.

SCHEDULE

You are welcome to schedule lectures as you see fit, as long as you adhere to the meal times (noted above), coffee break start and end times (noted below) and take into account the ”welcome” on Monday morning, the Banff Centre tour at 1:00 pm, and the group photo at 2:00 pm every Monday afternoon.

Please email your finalized schedule and abstracts to BIRS Station Manager birsmgr@birs.ca by Thursday morning before your arrival (at the latest) in order to allow for printing and posting to the website.

You are also encouraged to e-mail the schedule to your participants. BIRS provides the option of an electronic mail list in order to facilitate communications with your participants. When you login to the Organizer Interface at https://www.birs.ca/orgs, you will be prompted to create an electronic mail list for your workshop. Click ”Yes” to create one and receive instructions, or ”No” to decline. If you would like more information about our electronic mail lists, please e-mail help@birs.ca.

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
	Lecture rooms available after 16:00 (if desired)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall (if desired)
	Beverages and a small assortment of snacks are available on a cash honor system.

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome by BIRS Station Manager, Max Bell 159
9:00–10:00	*Abdelmalek Abdesselam*
10:00–10:20	Coffee Break, 2nd floor lounge, Corbett Hall
10:20–11:20	*Pierluigi Falco*
11:20–12:20	*Margherita Disertori*
12:20–13:20	Lunch
13:20–14:20	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:20	Group Photo; meet on the front steps of Corbett Hall
15:00–15:30	Coffee Break, 2nd floor lounge, Corbett Hall.
15:30–16:30	*Jean-Bernard Bru*
16:30–17:30	*Joseph Conlon*
17:30–19:30	Dinner
19:30	Discussions, open problems, beverages

Tuesday

7:00–8:30	Breakfast
8:30–9:30	*Andrea Braides*
9:30–10:30	*Stefan Luckhaus*
10:30–11:00	Coffee Break, 2nd floor lounge, Corbett Hall
11:00–12:00	*Felix Otto*
12:00–13:30	Lunch
15:00–16:30	Coffee Break, 2nd floor lounge, Corbett Hall.
16:30–17:30	*Takao Nishikawa*
17:30–18:30	*Florian Theil*
18:30–19:30	Dinner
19:30	Discussions, open problems, beverages

Wednesday

7:00–8:30	Breakfast
8:30–9:30	*Tadahisa Funaki*
9:30–10:30	*Christof Kuelske*
10:30–11:00	Coffee Break, 2nd floor lounge, Corbett Hall
11:00–12:00	*Codina Cotar*
12:00–13:30	Lunch
	Free Afternoon
17:30–19:30	Dinner
19:30	Discussions, open problems, beverages

Thursday

7:00–8:30	Breakfast
8:30–9:30	*Ron Peled*
9:30–10:30	*Rick Kenyon*
10:30–11:00	Coffee Break, 2nd floor lounge, Corbett Hall
11:00–12:00	*Oren Louidor*
12:00–13:30	Lunch
15:00–16:30	Coffee Break, 2nd floor lounge, Corbett Hall.
16:30–17:30	*Nicholas Crawford*
17:30–18:30	*Giambattista Giacomin*
18:30–19:30	Dinner
19:30	Discussions, open problems, beverages

Friday

7:00–8:30	Breakfast
8:30–9:30	*Miloš Zahradník*
9:30–10:30	*Noemi Kurt*
10:30–11:00	Coffee Break, 2nd floor lounge, Corbett Hall
11:00–12:00	*Patrick Dondl*
12:00–13:30	Lunch
Checkout by 12 noon.	

** 5-day workshops are welcome to use BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

Gradient Random Fields
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ABSTRACTS
(in alphabetic order by speaker surname)

Abdelmalek Abdesselam (University of Virginia)

The rigorous renormalization group via explicit combinatorial expansions

Abstract: The renormalization group is an important tool for understanding the behavior of systems in statistical mechanics or quantum field theory which involve the interplay of an infinite range of scales. However, mathematical formalisms which rigorously implement renormalization group ideas are rather scarce. The most well known is perhaps the approach of Brydges and coworkers which is based on a polymer representation of integrands for functional integrals, together with a renormalization group action on the space of polymer activities. In this talk I will explain the main ideas of an alternate approach developed in my 1997 thesis and which uses an explicit multiscale cluster expansion in phase space. I will use the Brydges-Mitter-Scoppola phi-four model with modified propagator in three dimensions as a case study.

Andrea Braides (Universita degli studi di Roma Tor Vergata)

Variational problems with percolation

Abstract: the framework of the passage from discrete systems to continuous variational problems offers a perfect environment to model problems with a random dependence (e.g., of the type of interaction between neighboring points on a lattice on a random variable). As a result, we can approximate almost surely the discrete energies by a continuous deterministic energy whose properties may be different above or below a percolation threshold. We apply this scheme to various types of spin systems. The continuous surface tension is described by using homogenization formulas of percolation type, which show a very interesting interaction between variational techniques and percolation arguments.

Jean Bernard Bru (Universidad del Pas Vasco-Ikerbasque Basque Foundation for Sciences)

Diagonalization of Quadratic Operators via Non-Autonomous Evolution Equations

Abstract: I will present a new method to diagonalize, under general conditions, quadratic Hamiltonians acting on a Boson Fock space. Based on the Brocket–Wegner flow, the mathematical novelty of our approach lies in the use of the theory of non-autonomous evolution equations as a key ingredient to diagonalize such operators. In particular, this new technique will be explained through this example.

Joe Conlon (University of Michigan)

Strong Central Limit Theorems in PDE with random coefficients and Euclidean Field Theory.

Abstract: It was shown in 1997 by Naddaf and Spencer that the two point correlation function of a gradient field with uniformly convex action is the expectation of the Green's function for a parabolic PDE with random coefficients. Using this identity and homogenization theorems for PDE with random coefficients, they were able to prove that large scale averages of the correlation function converge to a free Gaussian field correlation function. In this talk we shall discuss recent work of the speaker and Tom Spencer which proves point-wise convergence to free Gaussian field correlation functions. The main tools used in the proof are the Helffer-Sjostrand formula and the continuity of the norms of Calderon-Zygmund operators on L^p and weighted L^p spaces.

Codina Cotar (TU Munich)

Uniqueness of random gradient states

Abstract: We consider two versions of random gradient models. In model A) the interface feels a bulk term of random fields while in model B) the disorder enters though the potential acting on the gradients itself. It is well known that without disorder there are no Gibbs measures in infinite volume in dimension $d = 2$, while there are gradient Gibbs measures describing an infinite-volume distribution for the increments of the field, as was shown by Funaki and Spohn. Van Enter and Kuelske proved that adding a disorder term as in model A) prohibits the existence of such gradient Gibbs measures for general interaction potentials in $d = 2$. Cotar and Kuelske proved the existence of shift-covariant gradient Gibbs measures for model A) when $d \geq 3$ and the expectation with respect to the disorder is zero, and for model B) when $d \geq 2$.

In the current work, we prove uniqueness of shift-covariance gradient Gibbs measures with given tilt under the above assumptions. (this is joint work with Christof Kuelske).

Nicholas Crawford (The Technion)

Localization via Randomness in Classical Statistical Mechanics

Abstract: In this talk we detail recent and ongoing work on randomness induced ordering is statistical mechanics.

The paradigmatic, but by no means only, example is given the $O(2)$ model in d dimensions with an additional local field term in the Hamiltonian given by i.i.d. unit vectors pointing in the North/South direction with equal probability.

The question we are interested in is whether, for any sufficiently small local field strength, there is a temperature low enough so that sufficiently large block averages of spin variables typically have preferred orientations under the corresponding Gibbs measure. If so, what direction(s) are preferred? This question may be viewed as a classical, many body analog of the well known phenomenon of Anderson Localization.

In this talk we will explain why this is a remarkable question even at the classical level and detail partial answers recently obtained.

Margherita Disertori (Université de Rouen)

Anderson localization/delocalization transition for a supersymmetric sigma model

Abstract: We study a lattice sigma model which is expected to reflect the Anderson localization/delocalization transition for real symmetric band matrices in 3d. For this model we prove localization at high temperatures for any dimension $d \geq 1$ and the existence of a diffusive phase in 3 dimensions at low temperature (joint work with T. Spencer and M. Zirnbauer).

Patrick Dondl (University of Heidelberg)

Pinning and depinning in random media

Abstract: We consider a parabolic model for the evolution of an interface in random medium. The local velocity of the interface is governed by line tension and a competition between a constant external driving force $F > 0$ and a heterogeneous random field $f(x, y, \omega)$, which describes the interaction of the interface with its environment. To be precise, let $(\Omega, \mathscr{F}, \mathbf{P})$ be a probability space, $\omega \in \Omega$. We consider the evolution equation

$$\partial_t u(x,t,\omega) = \Delta u(x,t,\omega) - f(x, u(x,t,\omega), \omega) + F$$

with zero initial condition. The random field $f > 0$ has the form of localized smooth obstacles of random strength.

In particular, we are interested in the macroscopic, homogenized behavior of solutions to the evolution equation and their dependence on F. We prove that, under some assumptions on f, we have existence of a non-negative stationary solution for F small enough. This means that all solutions to the evolution equation become stuck if the driving force is not sufficiently large. The proof relies on a percolation argument. Given stronger assumptions on f, but still without a uniform bound on the obstacle strength, we also show that for large enough F the interface will propagate with a finite velocity.

The two results combined show the emergence of a rate-independent hysteresis in systems subject to a viscous microscopic evolution law through the interaction with a random environment.

Joint work with N. Dirr (Bath University) and M. Scheutzow (TU Berlin).

Pierluigi Falco (IAS Princeton)

Interacting Fermions Approach to 2D Critical Models

Abstract: Many two-dimensional models of Statistical Mechanics can be re-casted as a system of interacting fermions: Ising, 6-vertex, 8-vertex, Ashkin-Teller, Interacting Dimers, q-States Potts, Completely Packed Loops, $(1+1D)$ Hubbard, XYZ quantum chain, etc. Their scaling limit is described by the (non-Gaussian) field called Thirring Model. In this presentation I will outline some recent works in which the physicists' ansatz for the exact solution of the Thirring model has been proved; and, from that, the 'critical properties' of a few of the above lattice systems has been studied.

Tadahisa Funaki (University of Tokyo)

Scaling limits for dynamic models of Young diagrams

Abstract: In the first part, the hydrodynamic limits and the fluctuation limits are shown for non-conservative dynamics of $2D$ Young diagrams associated with the uniform or restricted uniform grandcanonical ensembles. These dynamic results have close connections to static ones such as Vershik curves (LLN) and CLT due to Vershik, Yakubovich and others. Our dynamics define evolutions of decreasing interfaces of SOS type and their gradients are described by 1D particle systems on a positive integer lattice with stochastic reservoirs at 0. In the second part, I will state a conjecture on the hydrodynamic limit for a particle system related to a conservative dynamic associated with the canonical ensemble (surface diffusion model). As a step to prove such conjecture, the equivalence of ensembles (or local equilibriums) under a spatially inhomogeneous conditioning is shown. The relation of our result to the Vershik curve will be discussed. If time permits, I will discuss the $3D$ case and a dynamic of honeycomb dimers. Some parts of the talk are joint works with M. Sasada, M. Sauer and B. Xie.

Giambattista Giacomin (Université Paris Diderot)

Transitions in active rotator systems

Abstract: The large scale behavior of a system of many finite dimensional stochastic dynamical systems may be very difficult to predict by looking at the dynamics of the isolated systems. We attack this issue by considering active rotators with mean field gradient interactions: by 'active rotators' we mean that the dynamics of the isolated rotator is non-trivial. It is well known that the large scale interacting dynamics is accurately described by a Fokker-Planck equation and, in the limit of 'inactive dynamics', the model reduces to a particular case of the Kuramoto synchronization PDE, for which we have identified a stable normally hyperbolic manifold of stationary solutions (in the so-called synchronized regime). We then use the robustness of hyperbolic structures to deal with the case of active rotators. This approach yields a complete description of the phase diagram of the active rotators model, at least for moderate activity, thus identifying for which values of the parameters (notably, noise intensity and/or coupling strength) the system is in an excited or in an inhibited state. Thanks also to the explicit character of some of our expressions, the approach brings a new insight into the combined effect of active dynamics, noise and interaction.

Rick Kenyon (Brown University)

Conformal invariance of double-dimer loops

Abstract: A double-dimer configuration on a graph G is a union of two dimer covers of G. We introduce quaternion weights in the dimer model and show how they can be used to study the homotopy classes (relative to a fixed set of faces) of loops in the double dimer model on a planar graph. As an application we prove that, in the scaling limit of the "uniform" double-dimer model on Z^2 the loops are conformally invariant.

Christof Kuelske (University Bochum)

Discrete approximations to massless models

Abstract: Consider two different ways of discretizing classical continuous spin models: 1) First look at the continuous spin Gibbs measure, and then apply a local discretization map to the measure. 2) First apply the local discretization map to the spins, and then compute the Boltzmann weights with the given Hamiltonian.

We will present our results on the existence and non-existence of effective interactions for the resulting discrete spin-measures in 1) and compare to existing results for 2). We also discuss continuous symmetry breaking and Kosterlitz-Thouless behavior in the discretized rotor-model in this light.

Noemi Kurt (TU-Berlin)

Pinning of a Laplacian interface model

Abstract: In this talk, we consider the centered Gaussian field on the d-dimensional integer lattice whose covariance matrix is given by the inverse of the discrete bi-Laplacian (as opposed to the discrete Laplacian in the case of the classical gradient interface model). We give a reward to the interface if it touches the 0-hyperplane, and ask how this reward changes the behaviour of the interface. As opposed to the gradient model, we don't have harmonic analysis and corresponding random walk representations at our disposal, even though we can relate some of the quantities to intersections of random walk. In this talk, we derive representations for the covariances of the field under the pinning constraint, and show that as an easy consequence the free energy in the supercritical dimensions is positive. Using our representations, we can make some steps towards proving an exponential decay of correlations of the pinned field in the supercritical dimensions.

Oren Louidor (UCLA)

Fixation for distributed clustering processes

Abstract: We study a discrete-time resource flow in Z^d, where wealthier vertices attract the resources of their less rich neighbors. For any translation-invariant probability distribution of initial resource quantities, we prove that the flow at each vertex terminates after finitely many steps. This answers (a generalized version of) a question posed by Van den Berg and Meester in 1991. The proof uses the mass-transport principle and extends to other graphs. With Marcelo R. Hilario, Charles M. Newman, Leonardo T. Rolla, Scott Sheffield and Vladas Sidoravicius.

Stefan Luckhaus (University of Leipzig)

Interpolation and large deviations for elasticity hamiltonians

Abstract: The talk is about deriving nonlinear elastic free energies from lattice based Hamiltonians. It is concentrating on the technical aspects of interpolation (coupling).The estimates at this point are strong enough to deal with a moderately higher growth from above than below for the Hamiltonian. (with R.Kotecký)

Takao Nishikawa (Nihon University)

Hydrodynamic limit for the Ginzburg-Landau $\nabla\phi$ interface model with a conservation law and the Dirichlet boundary condition

Abstract: We discuss the hydrodynamic scaling limit for the interface model with a conservation law, that is, the stochastic dynamics of the interface preserving its sum. For the dynamics on the torus, that is, the dynamics under the periodic boundary condition, the nonlinear fourth-order partial differential equation is derived as the macroscopic equation in [N. 2002]. The aim of this talk is to discuss the behavior of the interface motion with the conservation law and the Dirichlet boundary condition, and to derive the macroscopic equation.

Felix Otto (Max Planck Institute for Mathematics in the Sciences, Leipzig)

Optimal error estimates in stochastic homogenization

Abstract: In applications like water flow through a porous medium, one has to solve an elliptic equation $-\nabla \cdot (a(x)\nabla u) = f$ with heterogeneous coefficients $a(x)$ that vary on a characteristic length scale ℓ much smaller than the domain size. The coefficients $a(x)$ are typically characterized in statistical terms: Their statistics are assumed to be stationary (i. e. translation invariant) and to decorrelate over distances large compared to ℓ. In this situation, it is known from the theory of stochastic homogenization that the solution operator, i. e. the inverse of the elliptic

operator $-\nabla \cdot a(x)\nabla$, behaves—on scales large compared to ℓ—like the inverse of $-\nabla \cdot a_{\text{hom}}\nabla$ with homogeneous, deterministic coefficients a_{hom}. This is a major reduction in complexity. Hence the relation between the statistics of the heterogeneous coefficients $a(x)$ and the value of the homogenized coefficient a_{hom} is of practical importance. Stochastic homogenization also provides a formula for a_{hom} in terms of the ensemble of $a(x)$. It involves the solution of the "corrector problem" $-\nabla \cdot (a(x)(\nabla\phi\xi + \xi)) = 0$ in the whole space R^d for a given direction ξ. The formula then is given by $a_{\text{hom}}\xi = \langle a(\nabla\phi\xi + \xi)\rangle$, where $\langle\cdot\rangle$ denotes the ensemble average. Despite its simplicity, this formula has to be approximated in practice:

1) The corrector problem can only be solved for a small number of realizations of the coefficients $a(x)$. Thus, appealing to ergodicity, the ensemble average has to be replaced by a spatial average over a region of large diameter L.

2) The corrector problem can only be solved in a finite domain of large diameter L, thereby introducing some artificial boundary conditions.

In this talk, we present optimal estimates on both errors for the simplest possible model problem: We consider a discrete elliptic equation on the d-dimensional lattice Z^d with random coefficients a, which are identically distributed and independent from edge to edge. This makes a connection with the area of "random walks in random environments". We establish the optimal scaling of both errors in the ratio $L/\ell \gg 1$ (where the correlation length ℓ is unity in our model problem). It turns out that the scaling is the same as in the case of coefficients $a(x)$ that are very close to the identity (where the corrector problem can be linearized). Hence with respect to the error scaling, the highly nonlinear relation between $a(x)$ and a_{hom} behaves like the linearized one.

Our methods involve spectral gap estimates and estimates on the Green's function which only depend on the ellipticity ratio λ of a.

This is joint work with Antoine Gloria, INRIA Lille.

Ron Peled (Tel Aviv University)

High-dimensional Lipschitz functions are typically flat

Abstract: We consider integer-valued functions on the discrete torus Z_n^d constrained to change by at most 1 between adjacent vertices. This model can be viewed as a discrete model for Lipschitz functions. We will explain why in high dimensions a typical function from this class, suitably normalized, exhibits long range order and is nearly constant on either the even or odd sublattice. Thus the function is very flat, having bounded variance at each point and taking few values overall. Additionally, exploiting a relation with proper colorings, we deduce that a uniformly sampled proper 3-coloring in high dimensions will take predominantly one color on either the even or odd sublattice. Consequently, we establish for the first time the existence of a phase transition in high dimensions for the anti-ferromagnetic 3-state Potts model. Our results have consequences also in 2 dimensions, where they give one side of a conjectured roughening transition. The talk will be illustrated by many pictures and we will mention many related conjectures and open questions concerning proper colorings and random functions. Part of this work is joint with Ohad N. Feldheim.

Florian Theil (University of Warwick)

Crystallization in three dimensions: The FCC case

Abstract: We study the asymptotic behaviour of minimizers of atomistic pair interaction systems of Lennard-Jones type in the limit wher the number of particles tends to infinity. For a large class of pair interaction potential it can be shown rigorously that the minimizers converge to a rigid fcc lattice. Novel applications of discrete geometry and rigidity estimates are key ingredients of the proof.

Miloš Zahradník (Charles University)

Cluster expansion around massless gaussian, for a special class of perturbing potential wells

Abstract: Cluster expansion of a (continuous spin) partition function around a positive definite gaussian field is a standard tool of mathematical physics. We consider the case of semidefinite, "massless" gaussian corresponding to

the standard Dirichlet form on three dimensional lattice. The perturbing potential U will be given in a special way, as a Laplace transform of gaussians

$$e^{-U(y)} = 1 - \varepsilon \int_0^1 d\nu(\lambda)\, e^{-\lambda y^2}$$

where ν is a distribution of the type $d\nu(\lambda) = \phi(\lambda)\, d^2/d\{\lambda\}^2$ and ϕ is a function on interval $[0,1]$ such that $\phi(\lambda \approx \lambda$ for small λ and then ϕ decays to zero at $\lambda = 1$. Using Wick formula we represent the partition function of such a system by a polymer partition function, where polymers are "connected conglomerates" of "dotted", closed walks ("circles"). The dots are suitable "point decorations" of the walks, naturally grouped (in the corresponding sites of the underlying lattice) into "clips" glueing the circles of the system together into one conglomerate. The weights of the clips are given by coefficients from the expansion of function $e^{-U(x_i)}$. The special choice of the potential U allows a reasonable renormalization of cluster expansion of such a model, by erasing circles having enough simple structure (namely those with "only one dot") thus creating enough mass for the remainder, so that it becomes absolutely summable. This is a report on a work which is still in progress.