KAM theory and Geometric Integration
June 5 to June 10, 2011

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations. Note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.

SCHEDULE

Sunday	
16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
	Lecture rooms available after 16:00 (if desired)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall (if desired)
	Beverages and a small assortment of snacks are available on a cash honor system.

Monday	
7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome by BIRS Station Manager, Max Bell 159
9:00–10:00	**Dario Bambusi** (Univ. Milano)
	Asymptotic stability of solitary waves in dispersive equations
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:30	**Ernst Hairer** (Univ. Geneva)
	Modulated Fourier expansions
11:30–13:00	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00	Group Photo; meet on the front steps of Corbett Hall
16:00–17:00	**Mohammed Lemou** (CNRS & Univ. Rennes)
	Orbital Stability of Spherical Galactic Models
17:00–18:00	**Carles Simó** (Universitat de Barcelona)
	Jet transport and applications
18:00–19:30	Dinner

Tuesday	
7:00–8:30	Breakfast
8:30–9:30	**Chus Sanz-Serna** (Univ. Valladolid)
	Numerical mathematics and the method of averaging
09:30–10:30	**Laurent Thomann** (Univ. Nantes)
	Resonant dynamics for the quintic non linear Schrödinger equation
10:30–11:00	Coffee Break
11:00–12:00	**Florian Méhats** (Univ. Rennes)
	Stroboscopic averaging for highly oscillating nonlinear Schrodinger equations
12:00–13:30	Lunch
15:00	Coffee
17:00–18:00	**Thomas Kappeler** (Univ. Zürich)
	NLS & KAM
18:00–19:30	Dinner

Wednesday	
7:00–8:30	Breakfast
8:30–9:30	**Weizhu Bao** (Univ. Singapore)
	Modeling, analysis and simulation for degenerate dipolar quantum gas
9:30–10:30	**Thierry Paul** (CNRS & Ecole Polytechnique)
	Convergence of a quantum normal form and an exact quantization formula
10:30–11:00	Coffee Break
11:00–12:00	**Stephen Gustafson** (Univ. Vancouver)
	Global symmetric Schrödinger maps
12:00–13:30	Lunch
	Free Afternoon
17:30–19:30	Dinner

Thursday	
7:00–8:30	Breakfast
8:30–9:30	**Melvin Leok** (Univ. California at San Diego)
	General Techniques for Constructing Variational Integrators
09:30–10:30	**Philippe Guyenne** (Univ. Delaware)
	A Hamiltonian higher-order NLS equation for surface gravity waves
10:30–11:00	Coffee Break
11:00–12:00	**Zaijiu Shang** (Acad. Sci. Beijing)
	Numerical Stability of Hamiltonian Systems by Symplectic Integration
12:00–13:30	Lunch
15:00	Coffee
16:00–17:00	**Fleur McDonald** (Massey Univ.)
	Travelling Wave Solutions for Multisymplectic Discretisations of Wave Equations
17:00–18:00	**Yannick Sire** (Univ. Marseille)
	KAM theory for whiskered tori on lattices
18:00–19:30	Dinner

Friday	
7:00–8:30	Breakfast
8:30–9:30	**Alexander Ostermann** (Univ. Innsbruck)
	Meshfree integration of evolution equations
9:30–10:30	**Renato Calleja** (Univ. McGill)
	KAM theory for dissipative systems: from rigorous results to numerics
10:30–11:00	Coffee break
11:00–12:00	**Rafael de la Llave** (Univ. Texas)
	An a-posteriori KAM theorem for whiskered tori for some ill-posed Hamiltonian PDE
12:00–13:30	Lunch
Checkout by 12 noon.	

** 5-day workshops are welcome to use BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

Name of 2011 5-day Workshop
Date of 2011 5-day Workshop

ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Weizhu Bao** (National University of Singapore)
Title: *Modeling, analysis and simulation for degenerate dipolar quantum gas*
Abstract: In this talk, I will present our recent work on mathematical models, asymptotic analysis and numerical simulation for degenerate dipolar quantum gas. As preparatory steps, I begin with the three-dimensional Gross-Pitaevskii equation with a long-range dipolar interaction potential which is used to model the degenerate dipolar quantum gas and reformulate it as a Gross-Pitaevskii-Poisson type system by decoupling the two-body dipolar interaction potential which is highly singular into short-range (or local) and long-range interactions (or repulsive and attractive interactions). Based on this new mathematical formulation, we prove rigorously existence and uniqueness as well as nonexistence of the ground states, and discuss the existence of global weak solution and finite time blowup of the dynamics in different parameter regimes of dipolar quantum gas. In addition, a backward Euler sine pseudospectral method is presented for computing the ground states and a time-splitting sine pseudospectral method is proposed for computing the dynamics of dipolar BECs. Due to the adaption of new mathematical formulation, our new numerical methods avoid evaluating integrals with high singularity and thus they are more efficient and accurate than those numerical methods currently used in the literatures for solving the problem. In addition, new mathematical formulations in two-dimensions and one dimension for dipolar quantum gas are obtained when the external trapping potential is highly confined in one or two directions. Numerical results are presented to confirm our analytical results and demonstrate the efficiency and accuracy of our numerical methods. Some interesting physical phenomena are discussed too.

References

[1] W. Bao, Y. Cai and H. Wang, Efficient numerical methods for computing ground states and dynamics of dipolar Bose-Einstein condensates, J. Comput. Phys., 229 (2010), pp. 7874-7892.

[2] W. Bao, N. Ben Abdallah and Y. Cai, Gross-Pitaevskii-Poisson equations for dipolar Bose-Einstein condensate with anisotropic confinement, preprint.

[3] Y. Cai, M. Rosenkranz, Z. Lei and W. Bao, Mean-field regime of trapped dipolar Bose-Einstein condensates in one and two dimensions, Phys. Rev. A, 82 (2010), article 043623.

[4] A. Griesmaier, J. Werner, S. Hensler, J. Stuhler and T. Pfau, Bose-Einstein condensation of Chromium, Phys. Rev. Lett., 94 (2005), article 160401.

[5] B. Xiong, J. Gong, H. Pu, W. Bao and B. Li, Symmetry breaking and self-trapping of a dipolar Bose-Einstein condensate in a double-well potential, Phys. Rev. A, 79 (2009), article 013626.

Speaker: **Dario Bambusi** (University of Milano)
Title: *Asymptotic stability of solitary waves in dispersive equations.*
Abstract: We consider the subcritical Hamiltonian NLS in $\mathbb{R}^3$; it is well known that under suitable assumptions on the nonlinearity it admits a family of travelling solitary waves which are orbitally stable. We prove that generically they are asymptotically stable.

The result was known when the Floquet spectrum of the soliton has no non trivial eigenvalues. It is here extended to the general case.

The proof (which is developed in an abstract framework) is based on the combination of Hamiltonian and dispersive techniques. The main technical difficulties one has to face are related to the fact that the generators of the symmetry are unbounded operators. This obliges to develop Marsden-Weinstein reduction theory when the group action is only continuous and normal form theory when the generating vector field is not smooth. This also causes some difficulties for dispersive estimates. Such difficulties are solved using recent results by Parelman and Beceanu on Strichartz estimates for time dependent potentials.

Speaker: **Renato Calleja** (University of Delaware)
Title: *A numerically accessible criterion for the breakdown of quasi-periodic solutions*
Abstract: We formulate and justify rigorously a numerically efficient criterion for the computation of the analyticity breakdown of quasi-periodic solutions in Symplectic maps and 1-D Statistical Mechanics models. Depending on the physical interpretation of the model, the analyticity breakdown may correspond to the onset of mobility of dislocations, or of spin waves (in the 1-D models) and to the onset of global transport in symplectic twist maps. The criterion we propose here is based on the blow-up of Sobolev norms of the hull functions. The justification of the criterion suggests fast numerical algorithms that we have implemented using Fourier methods in several examples.

Speaker: **Stephen Gustafson** (University of British Columbia)
Title: *Global symmetric Schrödinger maps*
Abstract: I will describe some results on singularity (non-)formation and stability, in the energy-critical 2D setting, for a nonlinear Schroedinger equation of geometric and physical (ferromagnetism) origin – the Schroedinger map. In particular, radial solutions are global. This is joint work with Eva Koo.

Speaker: **Philippe Guyenne** (University of Delaware)
Title: *A Hamiltonian higher-order NLS equation for surface gravity waves*
Abstract: We present a systematic and consistent Hamiltonian approach to nonlinear modulation of surface water waves on arbitrary depth, both in two and three dimensions. It is based on the reduction of the problem to a lower-dimensional system involving surface variables alone. This is accomplished by introducing the Dirichlet–Neumann operator which gives the normal fluid velocity at the free surface, and expressing it as a Taylor series in terms of the surface elevation. In this framework, we derive new Hamiltonian envelope models describing the weakly nonlinear modulation of quasi-monochromatic surface gravity waves both on finite and infinite depth. In particular, we derive Hamiltonian versions of Dysthe's equation which is valid at one order higher than the cubic NLS equation. In the deep-water case, we analyze the stability properties of our Hamiltonian Dysthe equation regarding the Benjamin–Feir instability of a Stokes wave, and compare them with existing non-Hamiltonian results. We also perform numerical simulations using a symplectic time integrator, to test these stability results as well as to check the conservation of the Hamiltonian.

This is joint work with W. Craig (McMaster University) and C. Sulem (University of Toronto).

Speaker: **Ernst Hairer** (University of Geneva)
Title: *Modulated Fourier expansions*
Abstract: The theory of modulated Fourier expansions has its origin in the study of the long-time behaviour of numerical integrators when standard backward error analysis cannot directly be applied due to the presence of high oscillations. The main idea is to separate fast oscillatory motion from slow dynamics in the solution. It is successfully applied to yield information over long times for the analytic solution of the differential equation as well as for the numerical solution obtained by suitable discretizations.

New insight is gained for the numerical energy conservation in Hamiltonian systems that are perturbations of highly oscillatory harmonic oscillators. In the case of several high frequencies, resonance plays an important role. Closely connected is the near conservation of oscillatory energies.

Modulated Fourier expansions also allow to explain long-time regularity of solutions for non-linearly perturbed wave equations. The techniques carry over to numerical discretizations, which results in an understanding of the long-time near-conservation of energy, momentum, and harmonic actions. The ideas can also be applied to get insight into the distribution of mode energies over long times, when the initial data are small and concentrated in one Fourier mode.

Yet another application of the technique of modulated Fourier expansions is for the Fermi-Pasta-Ulam (FPU) problem. Insight into the long-time dynamics is obtained for small initial data, where only a few low frequency modes are excited. Suitable numerical discretizations retain the correct qualitative behaviour.

This is a joint-work with Christian Lubich. Parts of it are in collaboration with David Cohen, Ludwig Gauckler, and Daniel Weiss.

Speaker: **Thomas Kappeler** (University of Zürich)
Title: *NLS & KAM*
Abstract: In this talk I will survey recent results on the normal form of the defocusing and focusing NLS and its applications.

Speaker: **Mohammed Lemou** (CNRS & University of Rennes 1)
Title: *Orbital Stability of Spherical Galactic Models.*
Abstract: We consider the three dimensional gravitational Vlasov Poisson system which is a canonical model in astrophysics to describe the dynamics of galactic clusters. A well known conjecture is the stability of spherical models which are nonincreasing radially symmetric steady states solutions. This conjecture was proved at the linear level by several authors in the continuation of the breakthrough work by Antonov in 1961. In a previous work, we derived the stability of anisotropic models under spherically symmetric perturbations using fundamental monotonicity properties of the Hamiltonian under suitable generalized symmetric rearrangements first observed in the physics literature. In this work, we show how this approach combined with a new generalized Antonov type coercivity property implies the orbital stability of spherical models under general perturbations.

Speaker: **Melvin Leok** (University of California - San Diego)
Title: *General Techniques for Constructing Variational Integrators*
Abstract: The numerical analysis of variational integrators relies on variational error analysis, which relates the order of accuracy of a variational integrator with the order of approximation of the exact discrete Lagrangian by a computable discrete Lagrangian. The exact discrete Lagrangian can either be characterized variationally, or in terms of Jacobi's solution of the Hamilton–Jacobi equation. These two characterizations lead to the Galerkin and shooting-based constructions for discrete Lagrangians, which depend on a choice of a numerical quadrature formula, together with either a finite-dimensional function space or a one-step method. We prove that the properties of the quadrature formula, finite-dimensional function space, and underlying one-step method determine the order of accuracy and momentum-conservation properties of the associated variational integrators. We also illustrate these systematic methods for constructing variational integrators with numerical examples.

Speaker: **Rafael de la Llave** (University of Texas)
Title: *An a-posteriori KAM theorem for whiskered tori for some ill-posed Hamiltonian PDE.*
Abstract: We develop a framework to study whiskered tori in some Hamiltonian PDE. We formulate an equation that is satisfied by the parameterization of the solution and its whiskers and show that if there is aan approximate solution, that satisfy some non-degeneracy conditions, then there is a true solution close by. The abstract theory applies to several ill-posed equations that were proposed as models for water waves by Boussinesq. This is joint work in progress with Yannick Sire.

Speaker: **Fleur McDonald** (Massey University)
Title: *Travelling Wave Solutions for Multisymplectic Discretisations of Wave Equations.*
Abstract: Symplectic integrators for Hamiltonian ODEs have been well studied over the years and a lot is known about these integrators. They preserve the symplecticity of the system which automatically preserves other geometric properties of the system, such as a nearby Hamiltonian and periodic and

quasiperiodic orbits. It is then natural to ask how this generalises to Hamiltonian PDEs, which leads us to the concept of multisymplectic integration. We ask how well do multisymplectic integrators capture the long time dynamics of multi-Hamiltonian PDEs? As multi-Hamiltonian PDEs possess travelling wave solutions, we wish to see how well multisymplectic integrators preserve these types of solutions. This will give us an idea of how well the multisymplectic integrator is replicating the dynamics of the PDE.

Speaker: **Florian Méhats** (University of Rennes 1)
Title: *Stroboscopic averaging for highly oscillating nonlinear Schrodinger equations*
Abstract: We present a numerical method that enables to integrate highly oscillating nonlinear Schrodinger equations without resolving the fast oscillations in time. This method is based on the so-called stroboscopic averaging, which constructs an averaged dynamics that possesses the following properties : this differential system is still Hamiltonian and its solution coincides with the solution of the initial problem at the stroboscopic points. The stroboscopic averaging method (SAM) integrates numerically the averaged system without using its analytical expression. This is a joint work with F. Castella, P. Chartier and A. Murua

Speaker: **Alexander Ostermann** (University of Innsbruck)
Title: *Meshfree integration of evolution equations*
Abstract: For the numerical solution of time-dependent partial differential equations, a class of meshfree exponential integrators is proposed. These methods are of particular interest in situations where the solution of the differential equation concentrates on a small part of the computational domain which may vary in time. For the space discretization, radial basis functions with compact support are suggested. The reason for this choice are stability and robustness of the resulting interpolation procedure. The time integration is performed with an exponential Rosenbrock method or an exponential splitting method. The required matrix functions are computed by Newton interpolation based on Leja points. The proposed integrators are fully adaptive in space and time. Numerical examples that illustrate the robustness and the good stability properties of the method are given. This is joint work with Marco Caliari, Verona and Stefan Rainer, Innsbruck.

Speaker: **Thierry Paul** (CNRS & Ecole Polytechnique)
Title: *Convergence of a quantum normal form and an exact quantization formula*
Abstract: We condider the quantization of the linear flow of diophantine frequencies ω over the torus $\mathbb{T}^l$, $l > 1$, namely the Schrödinger operator $-i\hbar\omega\cdot\nabla$ on $L^2(\mathbb{T}^l)$, perturbed by the quantization of a function $\mathcal{V}_\omega : \mathbb{R}^l \times \mathbb{T}^l \to \mathbb{R}$ of the form $\mathcal{V}(z \circ \mathcal{L}_\omega(\xi), x)$, $\quad \mathcal{L}_\omega(\xi) := \omega_1\xi_1 + \ldots + \omega_l\xi_l$, real-holomorphic. We prove that the corresponding quantum normal form converges uniformly with respect to $\hbar \in [0, 1]$. Since the quantum normal form reduces to the classical one for $\hbar = 0$, this result simultaneously yields an exact quantization formula for the quantum spectrum, as well as a convergence criterion for the Birkhoff normal form, valid for a class of perturbations holomorphic away from the origin.

Speaker: **Chus Sanz-Serna** (University of Valladolid)
Title: *Numerical mathematics and the method of averaging*
Abstract: We shall explain ho to perform averaging analytically throught the combinatorial techniques now used to study the properties of numerical integrators. The novel approach systmatizes the derivation of high-order averaged systems. This is a joint work with Ph Chartier and A Murua.

Speaker: **Zaijiu Shang** (Chinese Academy of Sciences, Beijing)
Title: *Numerical Stability of Hamiltonian Systems by Symplectic Integration*
Abstract: Symplectic numerical integration theory for Hamiltonian systems has been developed rapidly in recent twenty five years. The recent monographs [1] and [2] summarize the main developments and important results of this theory. Qualitative behavior of symplectic integrators applied to Hamiltonian systems has been investigated by many authors. Some stability results either in the spirits of the KAM

theory or based on the backward analysis have been well established. The typical stable dynamics of Hamiltonian systems, e.g., quasi-periodic motions and their limit sets — minimal invariant tori, can be topologically preserved and quantitatively approximated by symplectic integrators. In this talk I give a brief review about old results and some new studies. The main emphasis is on the understanding of stability of Hamiltonian systems by symplectic numerical integration in the framework of KAM theory and backward analysis theory.

References

[1] E. Hairer, C. Lubich, and G. Wanner, Geometric Numerical Integration—Structure-Preserving Algorithms for Ordinary Differential Equations, Spinger-verlag Berlin, 2002

[2] K. Feng and M. Qin, Symplectic Geometric Algorithms for Hamiltonian Systems, Zhejiang Science & Technology Press Hangzhou 2003 (Chinese version), Zhejiang Science & Technology Publishing House, Hangzhou and Springer-Verlag Berlin 2010 (English revised version).

Speaker: **Carles Simó** (University of Barcelona)
Title: *Jet transport and applications*

Abstract: Many problems in dynamics require the knowledge of the local dependence of some orbits on the changes of initial conditions and/or parameters. As an example we can mention bifurcations, integrability conditions, checking KAM conditions, etc. This can require some symbolic manipulation to obtain, e.g., a suitable normal form and it is relatively easy when dealing with the behavior around a given point.

However, when this is desired around some orbit which is known only numerically, like a periodic orbit not known analytically, one has to obtain information from variational equations and, possibly, higher order variational equations. This can be systematically carried out by transporting a jet at the desired order along the orbit. The method can be based on any numerical integrator, but it is specially convenient to use high order Taylor methods.

Some applications will be done to checking integrability conditions, to the applicability of KAM theorem and to the next passage of asteroid Apophis. Part of this work has been done with R. Martínez [3,4] and T. Kapela [2] and other coworkers [1]. The computations can be converted into rigorous proof by using standard CAP (Computer Aided Proofs) techniques.

References

[1] E.M. Alessi, A. Farrés, À. Jorba, C. Simó, A. Vieiro, Efficient Usage of Self Validated Integrators for Space Applications, European Space Agency, the Advanced Concepts Team, *Ariadna Final Report 07-5202 2008*.

[2] T. Kapela, C. Simó, Rigorous KAM results around arbitrary periodic orbits for Hamiltonian Systems, in preparation.

[3] R. Martínez, C. Simó, Non-integrability of Hamiltonian systems through high order variational equations: Summary of results and examples, *Regular and Chaotic Dynamics* **14**, 323–348, 2009.

[4] R. Martínez, C. Simó, Non-integrability of the degenerate cases of the Swinging Atwood's Machine using higher order variational equations, *Discrete and Continuous Dynamical Systems A* **29**, 1–24 (2011).

Speaker: **Yannick Sire** (University of Marseille)
Title: *KAM theory for whiskered tori on lattices*

Abstract: I will report on some joint work with E. Fontich and R. de la Llave about the construction of quasi-periodic and almost-periodic solutions on lattices. I will develop an a posteriori KAM theory, which does not require the reduction to action-angle variables and does not need transformation theory.

Speaker: **Laurent Thomann** (University of Nantes)

Title: *Resonant dynamics for the quintic non linear Schrödinger equation*

Abstract: We consider the quintic nonlinear Schrödinger equation on the circle. We prove that the solution corresponding to an initial datum built on four Fourier modes which form a resonant set have a non trivial dynamic that involves periodic energy exchanges between the modes initially excited. It is notable that this nonlinear phenomena does not depend on the choice of the resonant set. The dynamical result is obtained by calculating a resonant normal form up to order 10 of the Hamiltonian of the quintic NLS and then by isolating an effective term of order 6. Notice that this phenomena can not occur in the cubic NLS case for which the amplitudes of the Fourier modes are almost actions, i.e. they are almost constant.