

Only homology cycles in an index  $n_1 n_2$  subgroup  $\mathcal{H}$  of  $H_1(\mathbb{T}^2)$  can pass through the region  $S$ .

Parallel transport along  $\Gamma$  induces an automorphism of  $\mathcal{H}$  and there is a basis of  $\mathcal{H}$  in which the automorphism is represented by the matrix  $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ .

In a basis of  $H_1(\mathbb{T}^2)$  the last matrix takes the form  $\begin{pmatrix} 1 & 0 \\ \frac{1}{n_1 n_2} & 1 \end{pmatrix}$ .